ER $\neq$ EPR : A Machine Readable ZKP

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Abstract

We rigorously follow the foundational principles of Einstein, Podolsky, Rosen, and Sommerfeld to their natural conclusions and present an operational critique of the ER = EPR conjecture. Employing the bedrock axioms of symmetry and repeatability, which demand that identical experiments yield consistent, directly measurable results; we evaluate the observable claims of both theories without introducing new formalisms. Our analysis reveals that the ER = EPR equivalence is internally inconsistent: the Einstein-Rosen bridge, as a consequence of general relativity's Lorentzian foundation, implies the physical reality of transformed coordinates, which we demonstrate violates repeatability. In contrast, the EPR criterion of reality and Sommerfeld's 1909 spherical model align strictly with direct observability. Via the CST appendix we demonstrate that Sommerfeld's geometric framework naturally satisfies the EPR criterion, reformulating entanglement as deterministic antipodal symmetry on a sphere and offering a collapse-free model of quantum measurement. The results expose a fundamental paradox within Einstein's own legacy, showing that his 1935 papers on ER and EPR advance irreconcilable interpretations of physical reality under the constraints of symmetry and repeatability. This work does not propose new ideas but reveals that the natural conclusions of these classic works are logically incompatible, challenging the coherence of the ER=EPR conjecture.

Part I
Proof of ER $\neq$ EPR

1 Introduction

From Galileo's ship to Feynman's lectures, the great teachers of physics have always emphasized one simple truth: nature cannot contradict herself. Two identical experiments, performed under identical conditions, must yield identical results. This principle of symmetry and repeatability is older than relativity and deeper than quantum theory. It is the silent backbone of every physical law and forms a universal skeleton key for testing theory.

In this work we investigate the widely discussed conjecture $ER = EPR$. Our approach is intentionally minimal. We do not rely on the technical machinery of relativity or quantum field theory. Instead, we evaluate the conjecture using only the claims that competing theories make about observables, and we test these claims against the supremacy of symmetry and repeatability. In this sense our reasoning is akin to a zero-knowledge proof: we demonstrate incompatibility without requiring detailed knowledge of the underlying constructions.

1.1 Brief Theoretical Background

The ER and EPR ideas began with Einstein's 1935 papers: the Einstein-Rosen (ER) bridge introduced non-traversable wormholes in general relativity, while the Einstein-Podolsky-Rosen (EPR) paradox challenged the completeness of quantum mechanics via entanglement. Though developed independently, the ER=EPR conjecture proposed in 2013 links these by suggesting entangled particles are connected through microscopic wormholes, providing a geometric interpretation of quantum correlations. This conjecture is grounded in advances in holographic duality and quantum information theory but challenges operational principles like symmetry and repeatability. The present work explores this tension, testing ER=EPR against direct measurability axioms from foundational physics.

2 Axioms

We adopt two axioms that guide all subsequent reasoning:

A1: Symmetry. Identical experiments performed under identical privileges must yield identical results. Nature cannot selectively implement different observables for the same setup.

A2: Repeatability. The output of any experiment must be a directly measurable quantity that can be repeated in the same frame with the same apparatus.

These axioms are methodological filters applied with a no compromise policy.

3 Repeatability Test: Establishing the Method

To clarify the repeatability test, consider Einstein's 1905 formulation of special relativity. Einstein writes [1] [2]:

> *"Thus for a coordinate system moving with the earth the mirror system of Michelson and Morley is not shortened, but it is shortened for a coordinate system which is at rest relatively to the sun."*

Here, Einstein asserts that Lorentz-transformed coordinate (x') corresponds to physical reality in the sun frame (rest frame). Yet within the rest frame, repeated experiments will only yield the same originally measured (x). The claim that (x') is physically real therefore conflicts with repeatability.

By contrast, Sommerfeld (1909) proposed a spherical model of velocity composition [3]:

- Transformations are dispensable.
- Predictions are compatible with directly observed (x). Refer proof **A.3** of CST appendix.
- The so-called "spherical defect" is not an anomaly but a geometric invariant. Refer proof **A.4** of CST appendix.

This model passes the repeatability test: identical experiments always yield (x).

4 Sommerfeld's Compliance with Relativistic Postulates

A critical test of any relativistic model is its adherence to SR's two foundational postulates. We now demonstrate that Sommerfeld's spherical model strictly upholds both, while remaining compatible with the axiom of repeatability.

Postulate 1 (Principle of Relativity): The laws of physics are identical in all inertial frames.

- Sommerfeld's velocity composition law, derived from spherical trigonometry, is a geometric law. Its form is independent of the chosen inertial frame; the geometry of the sphere and the relationships between points A, B', and C are absolute. The prediction of the null result is therefore invariant.

Postulate 2 (Invariance of c): The speed of light in vacuum is constant and independent of the motion of the source.

- The model is constructed upon the emission of a signal (e.g., light) from a point A. The fundamental identity $\sin(AB')/\sin(B'C) = 1$ (Appendix A) and the constant path length $AB' + B'C = \pi$ for the symmetric case are geometric expressions of this postulate. The geometry ensures that the effective two-way speed, which is the only empirically measurable quantity, is constant for any orientation, directly explaining the null result of Michelson-Morley experiments without recourse to contracting coordinates.

Thus, Sommerfeld's framework successfully derives the empirical predictions of SR (e.g. Michelson-Morley null result) from first principles. However, it achieves this while making a fundamentally different ontological claim: it attributes physical reality only to the directly measurable quantity (x) within a frame, rejecting the physical reality of the Lorentz-transformed coordinate (x'). It predicts the MM null result *without* the contraction that conflicts with repeatability.

5 Mapping the Theories

We now construct the following logical map:

5.1 Repeatability and Its Consequences

- SR asserts reality of (x') in the rest frame.
- Sommerfeld asserts (x) is physically real.

Thus:

$$SR \neq \text{Sommerfeld} \tag{1}$$

5.2 SR and Its Consequences

- SR asserts reality of (x') in the rest frame.
- General relativity (GR) is locally built upon SR (tangent spaces assume Lorentz invariance) [4].
- ER bridges are solutions of GR [5].

Thus:

$$ER = GR, \quad GR \supset SR. \tag{2}$$

5.3 Sommerfeld and Its Consequences

- Sommerfeld asserts (x) is real.
- EPR (1935) likewise insists that only directly predictable, directly measurable outcomes count as elements of reality [6].
- CST proof **A.3** ensures the total path length is constant; π radians (i.e., antipodal points). This geometrically corresponds to maximal separation and symmetry; naturally modeling entanglement as correlated antipodal states [7].

Thus:

$$EPR = \text{Sommerfeld}. \tag{3}$$

A detailed proof is presented in part II.

6 Deriving the Contradiction

From the above:

$$SR \neq \text{Sommerfeld}, \quad GR \neq \text{Sommerfeld}, \; ER \neq \text{Sommerfeld}, \quad EPR = \text{Sommerfeld}. \tag{4}$$

Therefore:

$$ER \neq EPR. \tag{5}$$

But the Maldacena-Susskind conjecture asserts [8]:

$$ER = EPR. \tag{6}$$

This yields a paradox; two truths cannot contradict [9]. By Galileo's principle, either the one or both axioms must be abandoned, or the conjecture $ER = EPR$ must be rejected.

7 Conclusion

We have shown, using only symmetry and repeatability, that the conjecture $ER = EPR$ is incompatible with the observable claims of SR and Sommerfeld. Einstein's own words commit SR to the physicality of (x'), violating repeatability, while Sommerfeld and EPR restrict themselves to (x).

This result does not disprove the mathematical connections explored by Maldacena and Susskind. Instead, it demonstrates that their conjecture cannot be reconciled with a strict operational interpretation of physics. The fault line exposed is not in the mathematics but in the interpretation of what constitutes physical reality. Either the bridge is real and repeatability is violated, or the elements of reality are direct observables and the ER=EPR equivalence fails.

This paradox is not external to Einstein but internal to his own work. His two 1935 papers: the ER bridge, inseparably tied to general relativity and its Lorentzian inheritance; and the EPR criterion of reality, which aligns with Sommerfeld in demanding that only directly measurable outcomes count as real. Under the supremacy of symmetry these two legacies are irreconcilable: ER requires privileging transformed coordinates, while EPR forbids it. Thus our result reveals a deep paradox at the heart of Einstein's thought. One cannot coherently hold both ER and EPR to be true if symmetry and repeatability are to remain supreme.

Part II
Sommerfeld = EPR (Detailed Proof)

8 Introduction

Einstein, Podolsky, and Rosen (EPR, 1935) introduced the "criterion of reality" to argue that quantum mechanics is incomplete. Einstein emphatically supported determinism stating, "God does not play dice". Arnold Sommerfeld (1909), in his treatment of velocity composition via spherical trigonometry, provided a geometric model that we reinterpret in the CST framework. The claim under examination is:

Sommerfeld's spherical model ≡ EPR criterion of reality.

Our goal is to reconstruct this claim formally, beginning with EPR, then CST, and finally combining them. We also discuss how CST accounts for entanglement and eliminates the wavefunction collapse.

9 The EPR Reality Criterion

The EPR paper states:

> If, without in any way disturbing a system, we can predict with certainty (probability = 1) the value of a physical quantity, then there exists an element of physical reality corresponding to that quantity.

Formally:

$$P(x) : \text{Quantity } x \text{ can be predicted with certainty without disturbance.''} \tag{7}$$

$$R(x) : \text{Quantity } x \text{ corresponds to an element of physical reality.''} \tag{8}$$

The EPR criterion can thus be expressed:

$$\forall x; [P(x) \to R(x)]. \tag{9}$$

10 Sommerfeld's Spherical Model and CST

Sommerfeld replaced Einstein's Lorentzian addition law with a geometric law based on spherical trigonometry. In the CST formulation (see Appendix), the key features are:

- Emission at point A always yields a deterministic null result at point C.
- The mapping $A \mapsto C$ is a fixed π-rotation about a reference point Q.
- No measurement is needed to establish this mapping; outcomes are guaranteed by geometry.
- Between A and C lies an infinite locus of possible reflection points (superposition), but these do not affect the certainty of the $A \mapsto C$ prediction.

Let us formalize:

$$S(x) : \text{“Quantity } x \text{ is fixed by CST geometry.”} \tag{10}$$

From CST:

$$\forall x; [S(x) \to P(x)]. \tag{11}$$

11 Composition: CST Implies EPR

We now combine:

$$S(x) \to P(x), P(x) \to R(x) \tag{12}$$

Therefore:

$$\forall x; [S(x) \to R(x)]. \tag{13}$$

Thus, within this logical framework, CST directly satisfies the EPR criterion: any state determined by Sommerfeld's geometry corresponds to an element of reality.

12 Entanglement via Antipodal Points

The CST geometry also provides a natural representation of entanglement:

- On a sphere, every point has an antipodal partner. If B is a reflection point, then B' is its antipode.
- States at antipodal points are deterministically correlated: the outcome at one fixes the outcome at the other.
- This mirrors the EPR scenario, where entangled particles yield perfectly correlated results despite spatial separation.

Thus entanglement appears as a manifestation of antipodal symmetry on the CST sphere.

13 The Infinite-Arm Interferometer (IAI)

In the IAI construct, the Michelson–Morley interferometer is generalized to infinite arms distributed over all angles $0 \leq \theta \leq 2\pi$. The circle of reflection points around origin Q encodes all possible paths simultaneously:

- The locus of reflections forms a continuous circle—a stationary geometric superposition.
- Every emission at A deterministically yields a null result at C, independent of orientation.
- Since the end state is always predictable via π-rotation, no wavefunction collapse is required.

Thus the IAI realizes a collapse-free quantum information system in purely geometric terms.

14 Qubit Realization via the IAI

The IAI construct provides more than a mere analogy; it yields a deterministic geometric realization of a quantum bit (qubit). On the Bloch sphere i.e. the fundamental representation of a qubit's state space, the IAI's geometry manifests directly:

- The emission point A represents the qubit's prepared state vector $|\psi\rangle$.

- The deterministic π-rotation $A \mapsto C$ represents a unitary operation (e.g., a bit-flip from $|0\rangle$ to $|1\rangle$).
- The infinite circle of possible reflection points B' represents the continuous superposition of states inherent in the qubit's definition, $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$.
- The guaranteed null result at C signifies a deterministic measurement outcome based solely on initial conditions.

This framework satisfies the EPR criterion for the qubit's value: the outcome at C is predictable with certainty ($P(x)$) from the geometry of A, without any disturbance or collapse. Therefore, by the EPR definition, the measurement result is an element of physical reality ($R(x)$) from the moment of preparation. The IAI thus models a qubit not as a probabilistic object awaiting collapse, but as a geometric entity with a predetermined, measurable outcome encoded antipodally.

15 Interpretation

Internal Validity

Within CST framework :

- A (emission) and C (result) are deterministically connected.
- No disturbance, no measurement, no collapse are needed.
- Entanglement is represented as antipodal symmetry, not probabilistic correlation.
- The IAI encodes superposition geometrically, yet always yields a certain outcome.

Hence: Sommerfeld's CST = EPR criterion plus compatibility with both of SR's postulates.

A The Conjoined Spherical Triangle Framework

A.1 Definition

Let the points A, Q, and C lie on the surface of a unit sphere:

- A: fixed start point.
- C: fixed target point.
- Q: movable evolution point along the arc AC.

Let B' be defined such that $\triangle AB'Q$ and $\triangle CB'Q$ share the side $QB' = h$ (the *hinge*). Angles:

$$\angle B'AQ = A, \quad \angle AQB' = \theta, \quad \angle AB'Q = i,$$

$$\angle B'CQ = C, \quad \angle CQB' = \pi - \theta, \quad \angle CB'Q = r.$$

A.2 Equation 1: Fundamental Identity

Applying the spherical law of sines to both triangles:

$$\frac{\sin(AB')}{\sin\theta} = \frac{\sin h}{\sin A} = \frac{\sin(AQ)}{\sin i},$$

$$\frac{\sin(B'C)}{\sin(\pi-\theta)} = \frac{\sin h}{\sin C} = \frac{\sin(QC)}{\sin r}.$$

With $AQ = QC$ under symmetry, we obtain:

$$\frac{\sin(AB')}{\sin(B'C)} = \frac{\sin i}{\sin r} = \frac{\sin A}{\sin C} = 1.$$

A.3 Path Law

From symmetry:

$$AB' + B'C = \pi.$$

A.4 Area Law

For any spherical triangle with internal angles (A, B, C) the area is given by

$$\text{Area}(\triangle) = A + B + C - \pi.$$

Applying this to the two CST triangles, we obtain

$$\text{Area}(\triangle AB'Q) = A + i + \theta - \pi,$$

$$\text{Area}(\triangle CB'Q) = C + r + (\pi - \theta) - \pi = C + r - \theta.$$

Summing both contributions yields

$$\text{Area}(\triangle AB'Q) + \text{Area}(\triangle CB'Q) = (A + i + \theta - \pi) + (C + r - \theta).$$

The θ terms cancel, leaving

$$A + C + i + r - \pi.$$

Using the CST relation $A + C + i + r = 2\pi$, the total area becomes

$$\text{Area}(\triangle AB'Q) + \text{Area}(\triangle CB'Q) = \pi.$$

Thus the combined area of the two triangles in the symmetric CST is always equal to π steradians, establishing the Area Law.

A.5 Duality Law

By polar triangle conjugation:

$$\text{Polar}(\triangle AB'Q) = \triangle CB'Q,$$

preserving area and curvature.

A.6 Informational Tension Law

Defining informational tension across the hinge:

$$T_{\text{CST}} = \frac{h}{\lambda}.$$

B Generalization via the Asymmetry Parameter α

B.1 Definition

Let $AQ = L_1$, $QC = L_2$, and define:

$$\alpha = \frac{QC}{AQ}.$$

$\alpha = 1$ gives symmetry; $\alpha \neq 1$ encodes curvature imbalance.

B.2 Generalized Laws

- Path:
$$AB' + B'C = \Phi(\alpha) = \pi \cdot \frac{2\min(\alpha, 1)}{1+\alpha}.$$
- Area:
$$A_{\text{total}} = \pi \cdot \Psi(\alpha), \quad \Psi(\alpha) = \frac{4\alpha}{(1+\alpha)^2}.$$
- Duality:
$$\Omega(\alpha) = \frac{2\sqrt{\alpha}}{1+\alpha}, \quad \text{Polar}_\alpha(\triangle AB'Q) = \triangle^* CB'Q(\alpha).$$
- Informational tension:
$$T_{\text{CST}}(\alpha) = \frac{h(\alpha)}{\lambda}, \quad h(\alpha) = h_0 \frac{1+\alpha^2}{2\alpha}.$$

C CST in Physical Contexts

C.1 Stationary Parallax

A and C = observer positions, B' = star. Parallax encoded by $\angle AQB' - \angle CQB'$.

C.2 Michelson–Morley

A = emission, C = recombination, B' = mirror, Q = apparatus evolution. Null result explained by spherical invariance.

D Rotational Symmetry and Superposition

- Rotational invariance under $SO(3)$.
- Infinite superposition principle: many CST cycles coexist independently.

E Projection Effects: Escherian Distortions

When projected into 3D:

- Distortions grow with baseline-to-distance ratio b/d.
- Distortions increase with relative velocity v.
- Only symmetric case $\theta = \pi/2$ projects without distortion.

F Compatibility with EPR

- Determinism: outcomes fully fixed by geometry.
- No collapse: measurement is a readout, not a selection.
- Completeness: every element of reality is geometrically encoded.

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