Calculation of the longitudinal mass from Bertozzi's experiment

Jorma Jormakka
jorma.o.jormakka@gmail.com

Abstract: The article shows that the energy-momentum relation is incorrect. Kinetic energy does not grow with velocity. Bertozzi's experiment only show that there is an energy component that does grow with velocity, it may be thermal energy. The article looks at Bertozzi's measurements. It is seen that Bertozzi's experiment refutes relativistic kinetic energy. Bertozzi's experiment gives the longitudinal mass of an electron in kinetic energyas roughly $m = \gamma^0.5m_0$ for the Van der Graaf accelerator supporting the new concept of weakening of force.

1. Introduction

The term relativistic mass is not any longer used as it has been realized that it is not mass, mass does not increase with velocity. Despite of that, terms like relativistic momentum, relativistic kinetic energy and the energy-momentum relation are considered correct and verified by many experiments.

This article proves in Section 2 that the energy-momentum relation and all its variants in the form of relativistic kinetic energy, relativistic momentum, relativistic mass and so on are wrong. Kinetic energy and momentum do not grow with velocity. However, there is an energy component that does grow with velocity, it may be thermal energy. Section 3 looks at Bertozzi's measurements that are supposed to have verified relativistic kinetic energy. It is seen that this is not the case, Bertozzi's experiment actually refuted relativistic kinetic energy.

The terms longitudinal mass and transverse mass for apparent mass in the situations when the force is parallel or perpendicular to the velocity of the mass are old terms from the time of Lorentz and Kaufmann and they are valid unlike the later relativistic mass concepts. The article shows in Section 4 that Bertozzi's measurements support the formula $m = \gamma^2 m_0$ for the longitudinal mass in a linear accelerator.

2. Theoretical refutation of the energy-momentum relation

The energy-momentum relation in the Special Relativity Theory is

$$E = \sqrt{(pc)^2 + (m_0c^2)^2} = mc^2 \quad \text{where} \quad m = \gamma m_0 \quad \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}. \tag{1}$$

If (1) is true, then kinetic energy is

$$E_k(v) = mc^2 - m_0c^2 = (\gamma - 1)m_0c^2. \tag{2}$$

A simple theorem proves that (2) is impossible.

Theorem 1. *If the trajectory of a mass m_0 is determined by minimizing energy, the following two claims hold. If the kinetic energy depends only on velocity, it must have the form $E_k(v) = \frac{1}{2}m_0 v^2$. The mass m_0 is constant.*

Proof: Let $x = 0$ and $v = dx/dx = 0$ when $t = 0$. Let $E_p(x)$ be potential energy that only depends on x. Let the mass m_0 be on the potential $E_p(h)$ at time $t = 0$. Let the mass fall in the potential field to the potential $E_p(s)$ where $s = h - x$. Then $E_p(x) = E_p(h) - E_p(s)$. Conservation of energy states that $E = E_p(x) + E_k(v)$ is constant, thus $E = E_p(h)$. Therefore for all $t \geq 0$ holds

$$E_k(v) = E_p(h) - E_p(s) = E_p(x). \tag{3}$$

The theorem assumes that the trajectory of the mass m_0 in the potential field of E_p is calculated by minimizing energy, i.e., minimizing the action integral

$$S = \int E(x, dx/dt)dt = \int E_p(x) + E_k(dx/dt)dt. \tag{4}$$

This is done in calculus of variations by solving the Euler-Lagrange equation

$$\frac{\partial}{\partial x}E_p(x) - \frac{d}{dt}\frac{\partial}{\partial(dx/dt)}E_k(dx/dt) = 0. \tag{5}$$

A short calculation proves the claim of the Theorem:

$$\frac{\partial}{\partial x}E_p(x) = \frac{d}{dt}\frac{\partial}{\partial v}E_k(v) \tag{6}$$

$$\frac{d}{dx}E_p(x) = \frac{d}{dt}\frac{d}{dv}E_k(v) \tag{7}$$

$$\frac{d}{dx}E_k(v(x)) = \frac{dv}{dt}\frac{d}{dv}\frac{d}{dv}E_k(v) \tag{8}$$

$$\frac{dt}{dx}\frac{dv}{dt}\frac{d}{dv}E_k(v) = \frac{dv}{dt}\frac{d}{dv}\frac{d}{dv}E_k(v). \tag{9}$$

Writing $y(v) = \frac{d}{dv}E_k(v)$ and $1/v = dt/dx$, and multiplying both sides with dt/dv

$$\frac{1}{v}y = \frac{d}{dv}y \tag{10}$$

$$\frac{dv}{v} = \frac{dy}{y}. \tag{11}$$

Thus, $y = Cv$ where C is constant and $E_k(v) = (1/2)Cv^2$. Taking the limit $v \to 0$ we can identify $C = m_0$. EOP

There is no weak point in Theorem 1: we can select $E_p(x)$ for any $E_k(dx/dt)$, conservation of energy must hold, and (5) is the correct mathematical equation for finding the minimum of S. The only argument that might be used is that for elementary particles it is not always clear if they can be treated as masses that move along a minimum energy trajectory, but for electrons in a vacuum tube or a particle accelerator this seems to be the case.

Let us apply the theorem to (2). The kinetic energy (2) is not of the only possible form, so it is wrong. We also get self-contradictions: calculating momentum and force from kinetic energy as they come from the second term of the Euler-Lagrange equation for the kinetic energy in (2) we get

$$p = \frac{\partial}{\partial(dx/dt)} E_k(v) = \frac{\partial}{\partial v}(\gamma - 1)m_0 c^2 = \gamma^3 m_0 v \tag{12}$$

$$F = \frac{d}{dt} p = \frac{d}{dt}\gamma^3 m_0 v = \gamma^5 \left(1 + 2\frac{v^2}{c^2}\right) m_0 \frac{dv}{dt}. \tag{13}$$

Expressions (12) and (13) are self-contradictions in Special Relativity Theory. In (1) the relativistic momentum is given as $p = \gamma m v$ and in Einstein's "proof" of $E = mc^2$ he starts with $F = \frac{d}{dt}(mv)$ and gets to $E = mc^2$ which gives (2), but(2) refutes $F = \frac{d}{dt}(mv)$ because it gives F as in (13).

The reason for selecting $U_p(x)$ in Theorem 1 is that the force from this potential $F = \frac{d}{ds} E_p(x)$ is completely similar to the force from work W, $F = \frac{d}{ds} W$. We can formulate Theorem 1 as:

Theorem 2. *If any component $f(v)$ of energy does not have the form $f(v) = Cv^2$ for a constant C, then the work that this energy can do does not equal the energy that is has, i.e., it violates conservation of energy.*

Notice that Theorem 2 states that the relativistic kinetic energy $T_k(v) = (\gamma - 1)m_0 c^2$ violates conservation of energy and is therefore impossible. Let us make the calculation in Theorem 1 explicitely for this case. Let W be work that T_k can do. Then W should equal $T_k(v)$, but calculating

$$F = \frac{d}{ds} W = \frac{d}{dt}\frac{d}{dv} T_k \tag{14}$$

$$\frac{d}{ds} T_k = \gamma^5 \left(1 + 2\frac{v^2}{c^2}\right) m_e \frac{dv}{dt} \tag{15}$$

$$\frac{dt}{ds}\frac{dv}{dt}\frac{d}{dv}\gamma c^2 = \gamma^5 \left(1 + 2\frac{v^2}{c^2}\right) \frac{dv}{dt} \tag{16}$$

$$\frac{1}{v}\gamma^3 \frac{v}{c^2} c^2 = \gamma^5 \left(1 + 2\frac{v^2}{c^2}\right) \tag{17}$$

$$\gamma^{-2} = 1 + 2\frac{v^2}{c^2}. \tag{18}$$

Therefore relativistic kinetic energy is not possible. The equations (2) and (1) are incorrect. Indeed, if (2) were correct, we could build a perpetual motion machine because if (2) is correct, work that kinetic energy can do is larger than kinetic energy. We can lift a stone up, let it fall on the ground and collect energy, there will be more energy than required for lifting the stone up.

The next section looks at the experiment by William Bertozzi that finally cemented the wrong view that (1) is correct and empirically verified. Bertozzi's article [2] does not state that (1) is correct. It merely makes the correct observations that an electron cannot be speeded up to a higher speed than c and that (2) approximately matches with the measurements. We will see in the next section that (2) does not match the measurements sufficiently well.

3. What did Bertozzi find?

Bertozzi's measurement gave the following values (the value for $EK/m_0c^2 = 15$ is not of interest here)

Table 1. If (2) holds, $E_k/m_ec^2 = \gamma - 1$. In the table $\gamma - 1$ is calculated from the measured v^2/v^2.

E_k/m_ec^2	measured v^2/c^2	calculated $\gamma - 1$
1	0.752	1.008
2	0.828	1.411
3	0.922	2.581
9	0.974	5.202

We notice from Table 1 that the calculated $\gamma - 1$ is not a very good approximation to the measured E_k/m_ec^2. This is not caused by measurement errors. Bertozzi describes how the measured values were obtained. The value for v^2/c^2 was calculated from the time electrons took to move from anode to the aluminium plate. This time was measured by two pulses shown on an oscilloscope and the flight time is accurate at least to the second significant digit. As Bertozzi gives three digits for $v^2 - c^2$, he considered all three digits valid. The kinetic energy E_k was measured from temperature rise in the aluminimum plate and Bertozzi's paper says that the first digit is correct. How then $\gamma - 1$ calculated from v^2/c^2 can differ in the first significant digit from E_k/m_ec^2 for $E_k/m_ec^2 = 9$? It could not if (2) is the accurate formula. Especially, it would not be possible that measured E_k is clearly higher than the theoretical kinetic energy $(\gamma - 1)m_ec^2$.

We can see the same issue when comparing the measured v^2/c^2 with one calculated form E_k/m_ec^2 assuming that $\gamma - 1 = E_k/m_ec^2$ in Table 2.

Table 2. Calculated v^2/c^2 is calculated by assuming that $E_k/m_ec^2 = \gamma - 1$ holds.

E_k/m_ec^2	measured v^2/c^2	calculated v^2/c^2
1	0.752	0.750
2	0.828	0.889
3	0.922	0.938
9	0.974	0.990

The value of measured v^2/c^2 is correct to at least second digit and the value for the measured E_k/m_ec^2 is correct to the first digit. The differences with calculated v^2/c^2 cannot be explained by measurement errors. The formula (2) does not hold.

Let us move to what actually must have happened in Bertozzi's measurement. The kinetic energy of an electron is $E_k = (1/2)m_e v^2$ and v does not exceed c. Bertozzi measured that the electrons have more energy. We can go around Theorem 1 by proposing that the total energy of an electron has a third part that does not produce a force, or produces a so small force that it can be ignored:

$$E(t, x, v, T) = E_p(t, x, v) + E_k(v) + E_T(t, T). \tag{19}$$

In (14) the additional energy component is given as $E_T(t, T)$ suggesting that it is temperature: the electrons get heated because force from the particle accelerator cannot increase their speed to c. Energy from the accelerator was used and it must have gone somewhere. Bertozzi measured heating energy that electrons gave to the aluminum plate. We can conclude that electrons do get the energy that the accelerator tries to give them, but this energy will not change the momentum of the electron, it will not appear as kinetic energy of the electron. It will appear as energy that excites the electron, or heats it, or changes the wavefunction of the electron. We can describe this energy as some kind of thermal energy in the electron, as it is measured by Bertozzi as heating the plate. This energy does not produce a force. Heat energy does not produce a force as heat causes movement of electrons to all directions and there is no force vector. This is why the energy component $E_T(t, T)$ disappears from the Euler-Lagrange equations in partial derivations. Lorentz noticed something of this type in his measurements: the electron changes its shape in high velocities by flattening in the direction of the velocity. Or maybe it expands in the direction perpendicular to the velocity and stores thermal energy to its wavefunction.

4. Evaluating the longitudinal mass in the kinetic energy formula

Lorentz [6] proposed that a fast moving electron has two different apparent masses: a longitudinal mass in the direction of the velocity and a transverse mass in the direction perpendicular to the velocity. Direct measurements of transverse mass were made by Kaufmann [5] and others in early 20th century. These measurements confirm that the classical formula is not correct and show that the transverse mass, mass seen by a force that is perpendicular to the velocity of the mass, is $m = \gamma m_0$.

Einstein in his book [3], based on his lectures in Princeton, attributes the formula $m = \gamma m_0$ to Lorentz, without mentioning that for Lorentz this was the formula for the transverse mass while he had proposed a different formula for the longitudinal mass and that he considered both of these masses as apparent masses.

Bertozzi's measurements [2] confirm that electrons cannot be speeded to higher velocity than c with an electric field and that apparent mass, longitudinal mass in his experiment, corresponds to real energy that can be measured with a calorimeter.

After Einstein's Special Relativity Theory (SRT) [4] was accepted, there was no interest in measuring the formula for the longitudinal mass: according to SRT there was only one relativistic mass $m = \gamma m_0$, but Section 2 proves that the relativistic mass concept it invalid and the longitudinal mass is an interesting concept to measure. Lorentz made a guess that the longitudial mass in the force formula would be $m = \gamma^3 m_0$, while [1] proposes by a theoretical calculation that is should be $m = \gamma^2 m_0$ in the force formula. The presented article evaluates the formula for the longitudinal mass in the kinetic energy formula from Bertozzi's experiment [2]. How the longitudinal mass in the force formula and the kinetic energy formula are related is not inversigated in this article.

The theoretical mechanism that gives the formula $m = \gamma^2 m_0$ for the logitudinal mass in the force formula in [1] is a simple model for an interaction between a field and a mass. In this model the apparent mass arises from force weakening. The mechanism of force weakening is similar to what happens when pushstarting a car: first the car accelerates nicely, but when the speed of the car equals the running speed of the pushing person, he cannot any more accelerate the car. The reason is not that the mass of the car has grown to infinity. It is that the car cannot feel the force from the pushing person's hands when it moves with the same speed as the person. That is, if the field creating the force to accelerate a mass has a propagation speed c and it cannot accelerate the mass above this speed. In high speed the mass does not feel the force from the field. The field makes work, but this work does not speed up the mass. Where does this work go? From Bertozzi's measurements we can conclude that an accelerated electron does get this work and stores it somewhere as energy. This energy is not kinetic energy and will not change the solution to the Euler-Lagrange equation. It will not increase the mass of the electron, but the energy shows in the calorimeter.

Weakening of force can be described as

$$\gamma^{-\alpha} F = F' = m_0 a \tag{20}$$

$$\gamma^{-\alpha_1} p = p' = m_0 v \tag{21}$$

$$\gamma^{-\alpha_2} E_k = E'_k = \frac{1}{2} m_0 v^2. \tag{22}$$

Notice that F', p' and E'_k in (20), (21), (22) satisfy the requirements

$$F' = \frac{d}{dt} p' \qquad p' = \frac{\partial}{\partial v} E'_k \tag{23}$$

and the function γ in (20), (21) and (22) does not appear in the Euler-Lagrange equation. This is correct: the equation of motion that gives the trajectory of the mass m_0 is determined by the forces that the mass feels.

We can interpret weakening of force as apparent mass

$$F = ma = \gamma^{\alpha} a \tag{24}$$

$$p = mv = \gamma^{\alpha_1} v \tag{25}$$

$$E_k = \frac{1}{2}\gamma^{\alpha_2} v^2 \tag{26}$$

and this apparent E_k is then the "kinetic energy" that Bertozzi measured. It is a sum of the two last terms in (19), $E_k = E_k(v) + E_T(t,T)$. With this apparent kinetic energy, we can describe the whole energy as

$$E = m_e c^2 + \frac{1}{2}\gamma^{\alpha_2} m_e v^2 \tag{27}$$

and find α_2 by equating it with (1) for a test what α_2 would be if (1) is true

$$E_k = (\gamma - 1) m_e c^2 = \frac{1}{2}\gamma^{\alpha_2} m_e v^2. \tag{28}$$

$$\alpha_{2,SRT} = \frac{\ln 2 + \ln(\gamma - 1) - \ln(v^2/c^2)}{\ln \gamma} \tag{29}$$

We can also calculate what α is from the measurements that Bertozzi made.

$$\alpha_{2,measured} = \frac{\ln 2 + \ln(E_k/(m_e c^2)) - \ln(v^2/c^2)}{\ln \gamma} \tag{30}$$

The following table gives the results.

Table 3.

v^2/c^2	γ	$\alpha_{2,SRT}$	$\alpha_{2,measured}$
0.752	2.008	0.491	0.537
0.828	2.411	0.509	0.653
0.922	3.587	0.481	0.523
0.974	6.202	0.382	0.470

Table 3 shows that there is a difference between the values of α_2 from (1) and from Bertozzi's measurement, strengthening the conclusion that Bertozzi's experiment did not verify (1), it refuted (1). Again we notice that more energy was measured than the relativistic kinetic energy formula predicts.

The second column in Table 3 is not interesting since Theorem 2 in Section 1 proves that (1) is not correct. The third column is interesting as it reflects measurements.

If apparent kinetic energy is caused by weakening of force, then the way the force weakens depends on the situation: there is no single correct value for α. Lorentz noticed it as he defined longitudinal and transverse masses and consided both as apparent masses. Therefore the longitudinal mass in Bertozzi's experiment can be as Table 3 suggests that in the Van der Graaf linear accelerator the longitudial mass for kinetic energy is about $m = \gamma^{\frac{1}{2}} m_e$.

It is natural to require that α and α_2 in (20) and (22) are related by the formulas

$$F = \frac{d}{dt}p \qquad p = \frac{\partial}{\partial v}E_k. \tag{31}$$

Then

$$E_k = \gamma^{\alpha_2} \frac{1}{2} m_0 v^2. \tag{32}$$

$$p = \frac{t}{dv} \gamma^{\alpha_2} \frac{1}{2} m_e v^2 = m_0 v \gamma^{\alpha_2} \left(1 + \frac{\alpha_2}{2} \frac{v^2}{c^2}\right) \tag{33}$$

$$p = m_0 v \gamma^{\alpha_2 + 2} \left(1 - \left(1 - \frac{\alpha_2}{2} \frac{v^2}{c^2}\right)\right) \tag{34}$$

$$F = \frac{d}{dt} p = m_0 \frac{dv}{dt} \gamma^{\alpha_2} \left(1 + \alpha_2 \frac{v^2}{c^2} \gamma^2 + \frac{3}{2} \alpha_2 (\alpha_2 + 2) \frac{v^4}{c^4} \gamma^4\right). \tag{35}$$

The first order approximation of (35) is

$$F = m_0 a \gamma^{3\alpha_2} + O\left(\frac{v^2}{c^4}\right). \tag{36}$$

The first order approximation of (34) is

$$p = m_0 v \gamma^{2\alpha_2} + O\left(\frac{v^2}{c^4}\right). \tag{37}$$

Table 3 gives α_2 over 0.5, below 0.6. This value should give the longitudinal mass for a force as something like $\alpha = 1.6$. It is not exactly $\alpha = 2$ what [1] proposes, but the Van der Graaf accelerator does not give a constant force that is parallel to the velocity of an electron. As the relativistic kinetic mass is impossible because of Theorem 1 and as Bertozzi's measurements are not in conflict with weakening of force, one can say that Bertozzi's measurements do support weakening of force and the logitudinal mass for a force in the Van der Graaf accelerator is about $m = \gamma^{1.6} m_0$.

7. References

1. Jormakka, J. "Refutation that experiments verify the relativity theory and a more reasonable proof of $E = mc^2$." 2025. Available at the ResearchGate.

2. Bertozzi, W. "Speed and Kinetic Energy of Relativistic Electrons". 1964, *American Journal of Physics*, 32(7):551-555.

3. Einstein, A. *The meaning of relativity.* original 1922, last edition 1955. Princeton University Press.

4. Einstein, A. "Zur Elektrodynamik bewegter Körper". 1905, *Annalen der Physik*, 322 (10):891-921.

5. Kaufmann, W. "Die elektromagnetische Masse des Elektrons". 1902, *Göttinger Nachrichten* (5):291-296.

6. Lorentz, H.A. "Über die scheinbare Masse der Ionen", 1900, *Physikalische Zeitschrift*, 2(5):78-80.