The Zitter Electron Model and the Anomalous Magnetic Moment

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Abstract: We introduce a classical geometric model of the electron in which a primary circular Zitterbewegung motion with radius equal to the Compton radius is combined with a secondary helical component to generate a toroidal–solenoidal structure for the internal charge and current distributions. Within this geometry, we construct electromagnetic fields consistent with the local helical kinematics and evaluate the associated energy density, momentum flow, and current density over the toroidal volume. These quantities uniquely determine the minor radius $r = R\sqrt{2\alpha/\pi}$. The resulting magnetic moment reproduces μ_B and yields a correction factor $g = \sqrt{1+\alpha/\pi}$. The analysis suggests that several intrinsic properties of the electron may admit a coherent interpretation in terms of an underlying toroidal electromagnetic geometry.

1 Introduction

Quantum mechanics (QM) is considered the most accurate physical theory available today. Since its conception, however, QM has generated controversy. This controversy lies not in the theory's results but in its physical interpretation.

One of the most controversial interpretations of QM was postulated by Bohr and Heisenberg. The "Copenhagen Interpretation" described QM as a system of probabilities that became definite upon the act of measurement. This interpretation was heavily criticized by many of the physicists who had participated in the development of QM, most notably Albert Einstein. Because of its probability features, Einstein believed that QM was only valid for analyzing the behavior of groups of particles and that the behavior of individual particles must be deterministic. In a famous quote from a 1926 letter to Max Born, Einstein stated, "He (God) does not play dice with the universe".

When QM is applied to individual particles, it leads to logical contradictions and paradoxical situations (e.g., the paradox of Schrödinger's Cat). Einstein believed that QM was incomplete and that there must be a deeper theory based on hidden variables that would explain how subatomic particles behave individually. Einstein and his followers were unable to find a hidden-variable theory compatible with QM, so the Copenhagen Interpretation eventually became the standard interpretation. If we assume that Einstein was correct and that QM applies only to groups of particles, then it would be necessary to develop a new deterministic theory to explain the behavior of individual particles.

2 Ring Electron Model

2.1 de Broglie Frequency

In 1924, de Broglie presented his famous doctoral thesis "Research on the Theory of the Quanta" [1], in which he proposed that the electron possesses wave-like properties. His work begins with a simple idea: de Broglie equates Einstein's two well-known expressions for energy, obtaining an extremely high intrinsic frequency for the electron (1.23×10^{20} Hz).

$$E = mc^2 = hf \tag{1}$$

$$f_B = \frac{mc^2}{h} \tag{2}$$

The presence of this internal frequency implies that the electron has an intrinsic oscillatory mechanism and an associated proper time (8×10^{-21} s).

$$T = \frac{1}{f} = \frac{h}{mc^2} \tag{3}$$

2.2 Parson's Magneton

A few years earlier, in 1915, Parson [2] had proposed a novel model of the electron as a ring-shaped structure, in which a unitary charge circulates around the ring, generating a magnetic field. In this model, the electron acts not only as the unit of electric charge but also as the unit of magnetic moment, or magneton. Several notable physicists—including Webster [3] [4], Lewis [5], Grondahl [6] and Compton [7]—conducted studies in support of Parson's Ring Electron Model. All of these works were compiled by Allen in 1918 in "The Case for a Ring Electron" [8] and discussed at a meeting of the Physical Society of London:

> *"There are many reasons why it is preferable to assume that the electron is in the form of a current circuit which can produce magnetic effects. Then the electron, in addition to exerting electrostatic forces, behaves like a small magnet. In its simplest form the magnetic electron may be looked upon as a circular anchor ring of negative electricity which rotates about its axis with a*

velocity which is certainly large, and is perhaps comparable with that of light."

The most significant study of the Ring Electron was conducted by Compton, who published a series of papers [9] [10] [11] demonstrating that the Compton effect was better explained using Parson's Ring Electron Model than the classical spherical model:

"The phenomena of scattering were found to be quantitatively accounted for, within the probable errors of observation, if the electron was considered to be a flexible ring of electricity with a radius of 2 pm."

2.3 Uniform Circular Motion

We model the electron as a ring in which an electric charge moves at the speed of light in uniform circular motion (UCM). The equations describing this motion are as follows,

Position:

$$\begin{cases} x(t) = R\cos wt \\ y(t) = R\sin wt \end{cases} \quad (4)$$

$$\phi = wt \quad (5)$$

Velocity:

$$\begin{cases} x(t) = -Rw\sin wt \\ y(t) = Rw\cos wt \end{cases} \quad (6)$$

$$v_r = Rw = c \quad (7)$$

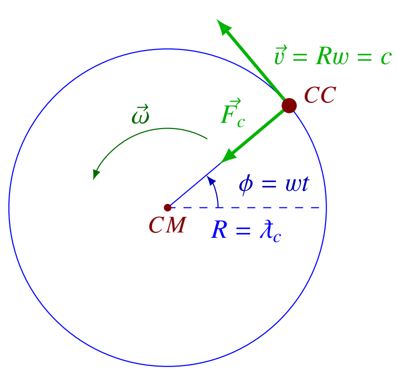

Figure 1: Uniform Circular Motion

Assuming de Broglie's frequency as the electron's rotational frequency, the radius of the ring matches the reduced Compton wavelength (0.386 pm), and the circumference matches the Compton wavelength (2.42 pm).

$$w = 2\pi f = \frac{mc^2}{\hbar} \quad (8)$$

$$R = \frac{c}{w} = \frac{\hbar}{mc} = \lambdabar_c \quad (9)$$

$$l = 2\pi R = \frac{h}{mc} = \lambda_c \quad (10)$$

This wavelength appears experimentally in the well-known Compton effect.

$$\triangle\,\lambda = \lambda_c\,(1 - \cos\phi) \quad (11)$$

If we multiply de Broglie's frequency by the Compton wavelength, we obtain the speed of light. This indicates that these three parameters of the electron are strongly interrelated.

$$f_B\,\lambda_c = c \quad (12)$$

2.4 Center of Mass and Center of Charge

The geometry of the Ring Electron Model reconciles experiments showing a size below 10^{-18} meters with others, like the Compton effect, that suggest an effective scale around 10^{-12} meters.

It also shows several other notable features:

- The electron has a ring-shaped geometry roughly 500 times larger than a proton.
- The mass of the electron originates from its internal energy and is distributed within the surrounding electromagnetic field.
- The Center of Mass (CM) is defined as a single point located at the geometric center of the ring.
- The Center of Charge (CC) is defined as the point at which the entire electric charge of the electron can be considered to be concentrated, analogous to the center of mass in a mass distribution
- The CC undergoes uniform circular motion at the speed of light along a well-defined ring centered on the CM.
- The CC moves continuously around the CM without energy loss. The electron behaves like a superconducting ring with a persistent current.
- The electron is considered to be at rest when the CM is at rest; in that case, the charge undergoes only rotational motion with no translational motion. The emission or absorption of energy involves acceleration of the CM.
- The electron's ring functions as a circular antenna. In such antennas, the resonance frequency matches the circumference of the ring. In the electron's case, the resonance frequency coincides with the Compton frequency.
- The CC has no mass, allowing it to have extremely small size without collapsing into a black hole, and to move at the speed of light without violating relativity. In fact, for the electron's properties to be relativistically invariant, the speed of the CC must be exactly equal to the speed of light.

2.5 Electron Magnetic Moment

Every electron has an associated magnetic moment of $-9.28\times 10^{-24} J/T$, a value known as one Bohr Magneton (μ_B). This property of the electron has been experimentally confirmed in numerous experiments, such as the Stern-Gerlach experiment, the Zeeman effect or the electron paramagnetic resonance (EPR) spectroscopy.

According to Maxwell's equations, magnetic moments do not exist alone. Any magnetic moment arises from an electric current, that is, from the movement of an electric charge. The presence of the electron's magnetic moment implies the existence of an internal electric current within the electron.

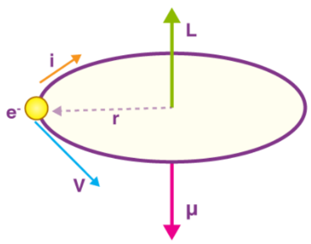

Figure 2: Magnetic Moment

In the Ring Electron Model, the motion of the charge generates an electric current of 19.83 A.

$$I = ef = \frac{emc^2}{h} \tag{13}$$

The circular motion of this current generates a magnetic moment with a magnitude of one Bohr Magneton (μ_B).

$$\mu_m = I\,S = ef\,\pi R^2 = \frac{e\hbar}{2m} = \mu_B \tag{14}$$

A magnetic moment of one Bohr magneton corresponds to circular motion carrying an angular momentum of $\hbar$. Uniform circular motion at the speed of light, with a radius equal to the reduced Compton wavelength, produces exactly this angular momentum.

$$L = mcR = \hbar \tag{15}$$

The ratio between magnetic moment and angular momentum, known as the gyromagnetic ratio, has the value $e/2m$.

$$\mu_m = \frac{e}{2m} L \tag{16}$$

This value is consistent with the magnetic moment produced by an electric current circulating around a closed loop.

2.6 Electron self-rotation

In 1926, Uhlenbeck and Goudsmit [12] proposed that the electron possesses an intrinsic self-rotation, now known as spin.

This hypothesis had previously been proposed by Kronig, but Pauli rejected it. His argument was straightforward: if the electron has a size equal to or smaller than the classical electron radius, it would have to rotate at over 100 times the speed of light to account for its observed angular momentum.

$$L = mr_e v_r = \hbar \quad \Rightarrow \quad v_r > 100c \tag{17}$$

Pauli's argument is valid only for a spherical model of the electron, but in the case of a ring-shaped electron, the calculated rotational speed is precisely the speed of light. He was so convinced that the fourth quantum number could not be described as a geometric property of the electron that he referred to it as the "classically non-describable two-valuedness". Unfortunately, Pauli never considered the possibility of a ring-shaped electron.

Despite his initial objections, in 1927 Pauli eventually accepted the hypothesis of spin. He proposed that the angular momentum and magnetic moment are intrinsic properties of the electron, unrelated to any actual spinning motion. Eventually, Pauli's view became the currently accepted interpretation of spin.

2.7 Quantum Hall Resistance

The energy of the electron is very low, but the frequency of oscillation is extremely large, which results in a significant power of about 10 megawatts:

$$P = \frac{mc^2}{T} = hf^2 \tag{18}$$

The electric potential can be calculated as the electron energy per unit of electric charge, resulting in a value of approximately half a million volts:

$$V = \frac{E}{e} = \frac{hf}{e} \tag{19}$$

Multiplying the voltage of about 500.000 V by the current of about 20 A, we obtain a power, again, of about 10 megawatts.

$$P = V\,I = hf^2 \tag{20}$$

Applying Ohm's law, we obtain a value for the impedance of the electron equal to the value of the Quantum Hall Resistance.

$$R = \frac{V}{I} = \frac{\phi_m}{\phi_e} = \frac{h}{e^2} \tag{21}$$

This value is quite surprising, since it is observable at the macroscopic level and was not discovered experimentally until 1980.

2.8 Quantum Magnetic Flux

According to Faraday's Law, voltage is the variation of the magnetic flux per unit of time. So, in a period of rotation, we obtain a magnetic flux value which coincides with the quantum of magnetic flux, another macroscopically observable value.

$$V = \frac{\phi_m}{T} \tag{22}$$

$$\phi_m = V\,T = \frac{hf}{e}\frac{1}{f} = \frac{h}{e} \tag{23}$$

The electron behaves as a superconducting ring, and it is experimentally known that the magnetic flux in a superconducting ring is quantized.

From this result, we infer that the electron is formed by two indivisible elements: a quantum of electric charge and a quantum of magnetic flux, the product of which is equal to Planck's constant.

$$\phi_e\,\phi_m = h \tag{24}$$

2.9 Quantum LC Circuit

Both the electrical current and the voltage of the electron are frequency dependent. This means that the electron behaves as a Quantum LC circuit, with a Capacitance (C) and a Self Inductance (L). We can calculate these coefficients for an electron at rest, obtaining values $L = 2.08 \times 10^{-16} H$ and $C = 3.13 \times 10^{-25} F$:

$$L = \frac{\phi_m}{I} = \frac{h}{e^2 f} \tag{25}$$

$$C = \frac{\phi_e}{V} = \frac{e^2}{hf} \tag{26}$$

Applying the formulas of the LC circuit, we can obtain the values of impedance and resonance frequency, which coincide with the previously calculated values of impedance and natural frequency of the electron:

$$Z = \sqrt{\frac{L}{C}} = \sqrt{\frac{h/e^2 f}{e^2/hf}} = \frac{h}{e^2} \tag{27}$$

$$f = \frac{1}{\sqrt{LC}} = \sqrt{\frac{1}{(h/e^2 f)(e^2/hf)}} = f \tag{28}$$

To calculate the electric energy, we can multiply the electric flux by the voltage, or equivalently, the capacitance by the square of the voltage. Similarly, to calculate the magnetic energy, we can multiply the magnetic flux by the current, or the inductance by the square of the current. All four calculations yield the same value, $E = hf$, which corresponds to the electron's energy.

$$E_e = \phi_e V = CV^2 = hf \tag{29}$$

$$E_m = \phi_m I = LI^2 = hf \tag{30}$$

We assume that the electron's total energy is symmetrically divided between its electric and magnetic energy.

$$\frac{E_e}{2} + \frac{E_m}{2} = hf \tag{31}$$

2.10 Vector Potential

The magnetic flux can also be calculated as the integral of the vector potential over a closed curve. In this way, we obtain the value of the vector potential.

$$\phi_m = \oint |\vec{A}|\,dl = 2\pi R\,|\vec{A}| = \frac{h}{e} \tag{32}$$

$$|\vec{A}| = \frac{mc}{e} \tag{33}$$

Both the magnetic flux and the vector potential calculated for an electron are consistent with the Aharonov–Bohm effect.

$$\Delta\varphi = \frac{e\phi_m}{\hbar} = 2\pi \tag{34}$$

2.11 Linear Momentum

The Ring Electron Model defines two distinct points (CM and CC). Consequently, it also defines two types of velocity and linear momentum: a translational linear momentum associated with the motion of the CM, and a rotational linear momentum arising from the circular motion of the CC around the CM. We define the total linear momentum as the sum of both linear momenta.

$$\vec{P_T} = \vec{p_t} + \vec{p_r} \tag{35}$$

We propose interpreting this total momentum as the canonical linear momentum, typically defined as:

$$\vec{P_T} = \vec{P_c} = m\vec{v} + e\vec{A} \tag{36}$$

The translational component corresponds to the conventional linear momentum, while the rotational component is linked to the vector potential term 'eA' and the motion of the Center of Charge (CC) at the speed of light.

$$\vec{p_t} = m\vec{v} \tag{37}$$

$$\vec{p_r} = m\vec{v_r} = e\vec{A} \tag{38}$$

The electron is considered to be at rest when the center of mass (CM) is at rest. In this case of an electron at rest ($\vec{p_t} = m\vec{v} = 0$), the equality becomes:

$$\vec{P_c} = \cancel{m\vec{v}} + e\vec{A} = mc \tag{39}$$

The linear momentum generates an angular momentum with a value exactly equal to the reduced Planck constant, as we have calculated previously.

$$\vec{L} = \vec{r} \times \vec{P_c} = mcR = \hbar \tag{40}$$

2.12 Uncertainty Principle

Since the charge moves at the speed of light along a circular trajectory, its instantaneous position oscillates with an extremely high angular frequency. Without precise knowledge of the phase of this internal motion, the charge's position at any given moment becomes inherently uncertain and can only be described in probabilistic terms.

From this perspective, the Heisenberg Uncertainty Principle can be interpreted as a direct consequence of the electron's internal structure: knowing its linear momentum precisely ($p = mc$) implies complete uncertainty about the location of the center of charge along its orbit, and vice versa.

The resulting uncertainties satisfy the following relations, correctly capturing the order of magnitude.

$$\Delta x \sim \frac{\hbar}{mc}, \quad \Delta p \sim mc \quad \Rightarrow \quad \Delta x \cdot \Delta p \sim \hbar, \tag{41}$$

2.13 Schrödinger equation

The circular motion can be conveniently represented in the complex plane using Euler's identity:

$$\psi(t) = x(t) - i\, y(t) \tag{42}$$

$$\psi(t) = R \cos wt - i\, R \sin wt \tag{43}$$

By convention, we will use a negative sign to denote a clockwise rotation of the motion, which ensures that we obtain positive energies later on.

$$\psi(t) = Re^{-iwt} \tag{44}$$

When we take the time derivative of the exponential function, we obtain the same function multiplied by a constant. In this sense, the exponential function is an eigenfunction of the derivative operator, and the constant ($-iw$) is its corresponding eigenvalue.

$$\frac{\partial \psi}{\partial t} = -iw\, Re^{-iwt} = -iw\psi \tag{45}$$

If we multiply both sides of the equation by the factor $i\hbar$ and substitute the energy as $E = \hbar w$, we obtain an expression similar to the Schrödinger equation.

$$i\hbar \frac{\partial \psi}{\partial t} = \hbar w \psi, \qquad i\hbar \frac{\partial \psi}{\partial t} = E\psi \tag{46}$$

3 Helical Electron Model

3.1 Zitterbewegung

The Dirac equation [13] superseded the Schrödinger equation by introducing a mathematically consistent framework that incorporated both the electron's spin and the principles of special relativity.

$$i\hbar \frac{\partial \psi}{\partial t} = \left(-i\hbar c\, \vec{\alpha} \cdot \nabla + \beta mc^2\right) \psi \tag{47}$$

Schrödinger [14] analyzed the Dirac equation and discovered a surprising result: the equation could be decomposed into a classical motion combined with a rapid oscillatory motion, which he called Zitterbewegung.

$$\hat{x}(t) = \underbrace{\hat{x}(0) + c^2 \hat{p} H^{-1} t}_{\text{Classic}} + \underbrace{\frac{i\hbar c}{2} H^{-1} \left(\vec{\alpha}(0) - c\hat{p}H^{-1}\right) e^{-2iHt/\hbar}}_{\text{Zitter}} \tag{48}$$

Assuming $H = mc^2$ and $p = mv$, it is straightforward to see that this oscillatory motion occurs at the speed of light, with a radius on the order of the electron's Compton wavelength.

$$\hat{x}(t) = \underbrace{x_0 + vt}_{\text{Classic}} + \underbrace{\frac{i\lambda\!\!\!^{-}_c}{2} (\vec{\alpha}_0 - \beta)\, e^{-2iwt}}_{\text{Zitter}} \tag{49}$$

Although Schrödinger initially believed that Zitterbewegung was merely a mathematical artifact, Dirac considered it to be a real and unavoidable physical effect, as he explained in his 1933 Nobel Prize [15] acceptance speech:

> *"It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment."*

The oscillatory motion known as Zitterbewegung is commonly interpreted as resulting from interference between positive and negative energy states. Although this view remains dominant, a classical interpretation has continued to attract some advocates. In 1952, Kerson Huang [16] proposed an interpretation identical to that previously suggested by the Ring Electron Model:

> *"It is shown that the well-known Zitterbewegung may be looked upon as a circular motion about the direction of the electron spin, with a radius equal to the Compton wavelength (divided by 2π)*

of the electron. It is further shown that the intrinsic spin of the electron may be looked upon as the "orbital angular momentum" of this motion. The current produced by the Zitterbewegung is seen to give rise to the intrinsic magnetic moment of the electron."

In the 1980s, several prestigious researchers, such as Barut [17] [18] [19] [20] and Hestenes [21], began to take Zitterbewegung seriously as a mechanism for interpreting the Dirac equation classically.

Hestenes coined the term Zitter to refer to this interpretation of Zitterbewegung as a real physical phenomenon, and proposed a "Zitterbewegung Interpretation of Quantum Mechanics" [22]. He also named "Zitter Electron Model" to any electron model that interprets the Zitter as an actual internal motion at the speed of light.

3.2 Helical Motion

The electron is considered to be at rest if the CM is at rest, since in that case the electric charge has only rotational movement without any translational movement. In contrast, if the CM moves with a constant velocity (v), then the CC moves in a helical motion around the CM.

The electron's helical motion is analogous to the observed motion of an electron in a homogeneous external magnetic field.

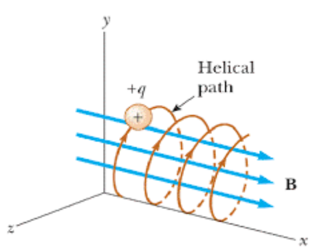

Figure 3: Helical Electron Model.

The direction of the center of mass (CM) displacement is independent of the direction of the angular momentum. However, in this analysis we will restrict ourselves to the case in which the motion is collinear with the angular momentum, since the resulting trajectory is a pure helix and the calculations are greatly simplified.

This motion can be parameterized as:

$$\begin{cases} x(t) = R\cos(wt), \\ y(t) = R\sin(wt), \\ z(t) = vt. \end{cases} \tag{50}$$

The electron's helical motion can be deconstructed into two orthogonal components: a rotational motion and a translational motion. The velocities of rotation and translation are not independent; they are constrained by the electron's tangential velocity that is constant and equal to the speed of light.

Using the Pythagorean Theorem, the relationship between these three velocities is:

$$c^2 = v_r^2 + v^2. \tag{51}$$

Then the rotational velocity of the center of charge is:

$$v_r = c\sqrt{1-(v/c)^2} = c/\gamma. \tag{52}$$

Where gamma is the coefficient of the Lorentz transformation, the base of the Special Relativity Theory:

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}}. \tag{53}$$

When the electron is at rest, its rotational velocity is equal to the speed of light. As the translational velocity increases, the rotational velocity must decrease. At no time can the translational velocity exceed the speed of light.

3.3 Linear Momentum

Three velocities also imply three linear momenta. To calculate the linear momenta, we multiply the three velocity components by the factor $(\gamma m)^2$

$$(\gamma m)^2 c^2 = (\gamma m)^2 v_r^2 + (\gamma m)^2 v^2 \tag{54}$$

Substituting the value of the rotational velocity ($v_r = c/\gamma$)

$$(\gamma mc)^2 = (mc)^2 + (\gamma mv)^2 \tag{55}$$

We interpret the resulting expression as the canonical linear momentum, consisting of a translational component and a rotational component. The translational part corresponds to the usual linear momentum ($p = \gamma mv$), whereas the rotational part is associated with the vector potential 'eA'.

$$\vec{P_c} = \vec{p_r} + \vec{p_t} = e\vec{A} + \gamma m\vec{v} \tag{56}$$

Where the total canonical momentum and the associated total wave number ($p = \hbar k$) are:

$$|\vec{P_c}| = \frac{E}{c} = \gamma mc, \quad k_T = \frac{P_c}{\hbar} = \frac{\gamma mc}{\hbar} \tag{57}$$

3.4 Relativistic Energy

Three velocities also imply three energies. Multiplying equation (55), by the factor c^2 and substituting the value of the linear momentum ($p = \gamma mv$)

$$(\gamma mc^2)^2 = (mc^2)^2 + (pc)^2. \tag{58}$$

Results in the relativistic energy equation:

$$E = \gamma mc^2 = \sqrt{(mc^2)^2 + (pc)^2} \tag{59}$$

In classical physics, the relativistic energy is interpreted as the sum of the rest energy (E_0) and the kinetic energy (E_k).

$$E = E_0 + E_k \tag{60}$$

$$E_0 = mc^2, \quad E_k = (\gamma - 1)\, mc^2 \approx \frac{1}{2} mv^2 \tag{61}$$

But we can offer an alternative interpretation. This time we multiply the three velocity components by the factor (γm)

$$(\gamma m)c^2 = (\gamma m)v_r^2 + (\gamma m)v^2 \tag{62}$$

By rearranging the factors, we once again obtain the same value for the total relativistic energy ($E_T = \gamma mc^2$) as the sum of two energies, but different from the previous case. In this case, we can interpret these two energies as the Rotational Energy and the Translational Energy.

$$\gamma mc^2 = \frac{mc^2}{\gamma} + \gamma mv^2 = E_r + E_t \tag{63}$$

Assuming that each energy corresponds to an associated speed and an associated frequency according to the formula $E = hf$, we can analyze the three energies.

$$E_T = hf_r + hf_t = hf_T = \gamma mc^2 \tag{64}$$

The frequency associated to the total energy is,

$$f_T = \frac{\gamma mc^2}{h} \tag{65}$$

The wavelength associated to the total energy is the Compton wavelength for a relativistic mass.

$$f_T \times \lambda_T = c, \qquad \lambda_T = \frac{h}{\gamma mc} \tag{66}$$

If the electron is at rest ($v = 0$, $\gamma = 1$), the energy, frequency and wavelength revert to those initially calculated for the Ring Electron Model.

	v	E	f	λ	p	k
Rot.	$v_r = c/\gamma$	$\frac{mc^2}{\gamma}$	$\frac{mc^2}{\gamma h}$	$\frac{h}{mc}$	eA	$\frac{1}{\lambda_c}$
Trans.	v	γmv^2	$\frac{\gamma mv^2}{h}$	$\frac{h}{\gamma mv}$	γmv	$\frac{1}{\lambda_B}$
Total	c	γmc^2	$\frac{\gamma mc^2}{h}$	$\frac{h}{\gamma mc}$	γmc	$\frac{\gamma mc}{\hbar}$

Table 1: Rotational and Translational components

3.5 Rotational Component

The rotational frequency is,

$$f_r = \frac{E_r}{h} = \frac{mc^2}{\gamma h} \tag{67}$$

If we look for the rotational wavelength corresponding to the rotational frequency and rotational velocity, we obtain the Compton wavelength, which corresponds to the circumference of the rotational motion.

$$f_r \times \lambda_r = v_r = \frac{c}{\gamma}, \qquad \lambda_r = \frac{h}{mc} = \lambda_c \tag{68}$$

The rotational momentum component and the associated rotational wave number are:

$$|\vec{p_r}| = eA = mc, \quad k_r = \frac{p_r}{\hbar} = \frac{1}{\lambda_c} \tag{69}$$

By solving for the vector potential, we once again obtain the same value calculated from the vector potential (33):

$$|\vec{A}| = \frac{mc}{e} \tag{70}$$

3.6 Translational Component

Finally, the translational frequency is:

$$f_t = \frac{E_t}{h} = \frac{\gamma mv^2}{h} \tag{71}$$

If we calculate the translational wavelength corresponding to the translational frequency and velocity, we obtain the de Broglie wavelength

$$f_t \times \lambda_t = v, \qquad \lambda_t = \frac{v}{f_t} = \frac{h}{\gamma mv} = \lambda_B \tag{72}$$

When an electron collides with a surface, there is not only an exchange of kinetic energy but also the translational energy of the electron comes into play, which manifests as a specific frequency and wavelength, producing the effect observed in the Davisson-Germer experiment.

The translational momentum component and his associated translational wave number are:

$$|\vec{p_t}| = \gamma mv, \qquad k_t = \frac{p_t}{\hbar} = \frac{1}{\lambda_B} \tag{73}$$

3.7 Cylindrical Helix

Although all these frequencies and wavelengths may manifest physically under certain conditions, they do not necessarily correspond to actual geometric frequencies or lengths of the electron.

If the electron moves at a constant velocity along a direction collinear with its angular momentum, its trajectory takes the form of a cylindrical helix. The geometry of the helix is defined by two constant parameters: the radius of the helix (R) and the helical pitch (H).

The frequency of this helix matches the rotational frequency calculated previously ($f_r = mc^2/\gamma h$). The radius of the helix remains constant at ($R = \hbar/mc$), while the helical pitch corresponds to the distance between two turns of the helix.

$$H = \frac{v}{f_e} = v\,\frac{\gamma h}{mc^2} = \gamma\beta\lambda_c. \tag{74}$$

The electron's helical motion can be interpreted as a wave motion with a wavelength equal to the helical pitch and a frequency equal to the electron's natural frequency. Multiplying the two factors results in the electron's translational velocity:

$$H \times f_e = v, \tag{75}$$

We can also calculate the curvature (κ) and the torsion (τ) of the cylindrical helix, where $h = H/2\pi = \gamma\beta\bar{\lambda}_c$:

$$\begin{cases} \kappa = \dfrac{R}{R^2 + h^2} = \dfrac{1}{\gamma^2 R}, \\[2ex] \tau = \dfrac{h}{R^2 + h^2} = \dfrac{\beta}{\gamma R}. \end{cases} \tag{76}$$

According to Lancret's Theorem, the necessary and sufficient condition for a curve to be a helix is that the ratio of curvature to torsion must be constant. This ratio is equal to the tangent of the angle between the osculating plane with the axis of the helix:

$$\tan\alpha = \frac{\kappa}{\tau} = \frac{1}{\gamma\beta}. \tag{77}$$

4 Electromagnetic Fields

4.1 Fields at the Center of Mass

The Ring Electron Model, the Biot-Savart Law can be applied to calculate the magnetic field at the center of the ring, resulting in a magnetic field of 30 million Tesla, equivalent to the magnetic field of a neutron star. For comparison, the magnetic field of the Earth is 0.000005 T, and the largest artificial magnetic field created by man is only 90 T.

$$B_0 = \frac{\mu_0 I}{2R} = \alpha\frac{m^2c^2}{e\hbar} = 3.23 \times 10^7\ T \tag{78}$$

The electric field in the center of the ring matches the value of the magnetic field multiplied by the speed of light ($E = cB$).

$$E_0 = \frac{1}{4\pi\epsilon_0}\frac{e}{R^2} = \alpha\frac{m^2c^3}{e\hbar} = 9.61 \times 10^{15}\ V/m \tag{79}$$

4.2 Zitter Force

The Ring Electron Model implies the existence of a centripetal acceleration and a centripetal force of 0.212 N. This force, which we refer to as the Zitter Force, counteracts the centrifugal force of the charge orbiting around its center of mass.

$$a_c = \frac{v_r^2}{R} = \frac{c^2}{R} = \frac{mc^3}{\hbar} \tag{80}$$

$$F_c = ma = \frac{m^2c^3}{\hbar} \tag{81}$$

Since we have consistently assumed that the electron is governed solely by electromagnetic fields, the Zitter force must necessarily be a Lorentz force.

$$\vec{F}_c = e\vec{E} + e\,\vec{v}\times\vec{B} \tag{82}$$

4.3 Schwinger Limits

We can calculate the electric and magnetic fields that produce this Zitter force and we obtain the following values.

$$E_s = \frac{|\vec{F}_c|}{e} = \frac{m^2c^3}{e\hbar} = 1.32 \times 10^{18}\ V/m \tag{83}$$

$$B_s = \frac{|\vec{F}_c|}{ec} = \frac{m^2c^2}{e\hbar} = 4.41 \times 10^9\ T \tag{84}$$

These values are identical to the well-known values referred to as the 'Schwinger limits' in Quantum Electrodynamics [23]. These values also match the values of E and B at the center of the ring, scaled by the fine-structure constant.

$$E_0 = \alpha E_s, \qquad B_0 = \alpha B_s \tag{85}$$

4.4 Fields at the Center of Charge

To generate the Zitter Force required by the model, we assume that at the location of the Center of Charge there exists a magnetic field $\vec{B}$ perpendicular to the plane of rotation, capable of inducing the necessary rotational acceleration. This magnetic field produces a cyclotron-type acceleration of the charge.

A perpendicular electric field must also be associated with this magnetic field. The electric field must have only a radial component, aligned with the centripetal force. The two fields must also satisfy the condition $\vec{E} = c\vec{B}$. Using the Larmor radius formula for this field, taking the reduced Compton wavelength as the orbital radius and the speed of light as the rotational velocity, we obtain

$$B_s = \frac{mc}{eR} = \frac{m^2c^2}{e\hbar}, \qquad E_s = cB_s = \frac{mc^2}{eR} = \frac{m^2c^3}{e\hbar} \tag{86}$$

These values correspond to the previously calculated Schwinger limits.

4.5 Cylindrical Coordinates

To describe the electromagnetic fields, we will use cylindrical coordinates, with radial (ρ), tangential (ϕ), and vertical (z) components.

$$\vec{G} = G_\rho \hat{\rho} + G_\phi \hat{\phi} + G_z \hat{z} \tag{87}$$

Where the expressions for the divergence and curl of a generic vector G are given by:

$$\nabla \cdot \vec{G} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho G_\rho\right) + \frac{1}{\rho}\frac{\partial G_\phi}{\partial \phi} + \frac{\partial G_z}{\partial z} \tag{88}$$

$$\begin{aligned}\nabla \times \vec{G} = &\left(\frac{1}{\rho}\frac{\partial G_z}{\partial \phi} - \frac{\partial G_\phi}{\partial z}\right)\hat{\rho} + \left(\frac{\partial G_\rho}{\partial z} - \frac{\partial G_z}{\partial \rho}\right)\hat{\phi} \\ &+ \frac{1}{\rho}\left(\frac{\partial}{\partial \rho}(\rho G_\phi) - \frac{\partial G_\rho}{\partial \phi}\right)\hat{z}\end{aligned} \tag{89}$$

Since the center of charge moves on a circular orbit:

$$\rho(t) = R, \qquad \phi(t) = wt \qquad z(t) = 0 \tag{90}$$

The cylindrical unit vectors satisfy:

$$\frac{\partial \hat{\rho}}{\partial \phi} = \hat{\phi}, \quad \frac{\partial \hat{\phi}}{\partial \phi} = -\hat{\rho} \tag{91}$$

so, with $\phi = wt$:

$$\frac{\partial \hat{\rho}}{\partial t} = w\hat{\phi}, \quad \frac{\partial \hat{\phi}}{\partial t} = -w\hat{\rho} \tag{92}$$

We evaluate the E and B fields only at the position of the center of charge, which means evaluating the fields along its circular trajectory. The magnitudes of both fields do not oscillate; the only oscillation present is the spatial oscillation of the center of charge during its circular motion. We define these electromagnetic fields restricted to the circular path by introducing the Euler factor $e^{-\phi t}$.

$$\vec{E} = E_s\, e^{-i\phi}\hat{\rho}, \qquad \vec{B} = B_s\, e^{-i\phi}\hat{z} \tag{93}$$

4.6 Vector Potential

We recover the value of the vector potential that we have calculated both from the magnetic flux and from the linear momentum (33). Knowing that the direction of the vector potential is collinear with the rotation velocity, we obtain:

$$\vec{A} = \frac{mc}{e}e^{-i\phi}\hat{\phi} \tag{94}$$

We calculate the curl of the vector potential and obtain the magnetic field vector, just as expected.

$$\nabla \times \vec{A} = \frac{1}{R}\left(\frac{\partial}{\partial \rho}(\rho A_\phi)\right)\hat{z} = \frac{A_\phi}{R}\frac{\not\partial \not\rho}{\not\partial \rho}\hat{z} = \frac{mc}{eR}e^{-iwt}\,\hat{z} \tag{95}$$

$$\nabla \times \vec{A} = \vec{B} \tag{96}$$

Both the electric and magnetic fields, as well as the vector potential, satisfy the Schrödinger equation as valid wavefunction solutions.

$$i\hbar\frac{\partial \vec{E}}{\partial t} = \hbar w\vec{E}, \qquad i\hbar\frac{\partial \vec{B}}{\partial t} = \hbar w\vec{B}, \qquad i\hbar\frac{\partial \vec{A}}{\partial t} = \hbar w\vec{A} \tag{97}$$

4.7 Scalar Potential

We identify the scalar potential as the electron's voltage.

$$V = -\frac{hf}{e}e^{-i\phi} = -\frac{mc^2}{e}e^{-i\phi} \tag{98}$$

Computing the gradient of this voltage, we obtain:

$$\nabla V = \frac{\not\partial V}{\not\partial \rho}\hat{\rho} + \frac{1}{R}\frac{\partial V}{\partial \phi}\hat{\phi} + \frac{\not\partial V}{\not\partial z}\hat{z} = i\frac{mc^2}{eR}e^{-iwt}\hat{\phi} \tag{99}$$

We calculate the time derivative of the vector potential using the product rule.

$$\frac{\partial \vec{A}}{\partial t} = \frac{\partial (A_\phi \hat{\phi})}{\partial t} = \frac{\partial A_\phi}{\partial t}\hat{\phi} + A_\phi\frac{\partial \hat{\phi}}{\partial t} \tag{100}$$

$$\frac{d\vec{A}}{dt} = -iw\frac{mc}{e}e^{-iwt}\hat{\phi} - \frac{mc}{e}e^{-iwt}w\,\hat{\rho} \tag{101}$$

$$\frac{\partial \vec{A}}{\partial t} = -i\frac{mc^2}{eR}e^{-iwt}\hat{\phi} - \frac{mc^2}{eR}e^{-iwt}\hat{\rho} \tag{102}$$

We obtain the electric field as a function of the vector and scalar potentials, as expected.

$$\vec{E} = -\frac{\partial \vec{A}}{dt} - \nabla V \tag{103}$$

4.8 Lorenz gauge condition

We calculate the divergence of the vector potential and the temporal derivative of the scalar potential (voltage).

$$\nabla \cdot \vec{A} = \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} = -i\frac{mc}{eR}e^{-iwt} \tag{104}$$

$$\frac{\partial V}{\partial t} = iw\frac{mc^2}{e}e^{-iwt} = i\frac{mc^3}{eR}e^{-iwt} \tag{105}$$

As we can see, the Lorenz gauge condition is satisfied.

$$\nabla \cdot \vec{A} + \frac{1}{c^2}\frac{\partial V}{\partial t} = 0 \tag{106}$$

4.9 Poynting Vector

The cross product of the electromagnetic fields yields the Poynting vector, whose direction is parallel to the rotational velocity.

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} \tag{107}$$

$$\vec{S} = \frac{mc^3}{4\pi\alpha R^3}\, e^{-i2wt}\hat{\phi} \tag{108}$$

4.10 Magnetic Gauss's Law

We now proceed to verify whether the given expressions for the electric field E and the magnetic field B are consistent with Maxwell's equations. This idea was originally proposed by C.A.M. dos Santos in [24].

The simplest of Maxwell's equations to verify is Gauss's law for magnetism.

$$\nabla \cdot \vec{B} = 0 \tag{109}$$

We verify that the law is satisfied since the magnetic field has only a z-component, and its derivative with respect to z is zero.

$$\nabla \cdot \vec{B} = \frac{\partial B_z}{\partial z} = 0 \tag{110}$$

4.11 Faraday's Law

Faraday's law relates the curl of the electric field to the time derivative of the magnetic field.

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{111}$$

The calculation of the magnetic field's time derivative is straightforward.

$$\frac{\partial \vec{B}}{\partial t} = -iwB_s\, e^{-iwt}\hat{z} \tag{112}$$

Since the electric field has only a radial component, its curl has only two components. As in the case of Gauss's law for magnetism, the electric field does not depend on the z-coordinate, so that term vanishes, leaving:

$$\nabla \times \vec{E} = \cancel{\frac{\partial E_\rho}{\partial z}}\hat{\phi} - \frac{1}{R}\frac{\partial E_\rho}{\partial \phi}\hat{z} = \frac{iE_s}{R}e^{-iwt}\hat{z} \tag{113}$$

By equating both terms, we verify that Faraday's law is also satisfied.

$$\frac{iE_s}{R}e^{-iwt}\hat{z} = iwB_s e^{-iwt}\hat{z} \tag{114}$$

$$\frac{E_s}{B_s} = Rw = c \tag{115}$$

4.12 Electric Gauss's Law

Gauss's law for electricity states that the divergence of the electric field corresponds to the electric charge density (ρ_e).

$$\nabla \cdot \vec{E} = \frac{\rho_e}{\varepsilon_0} \tag{116}$$

The divergence of the electric field can be calculated using equation (88).

$$\nabla \cdot \vec{E} = \frac{1}{R}\frac{\partial}{\partial \rho}(\rho E_\rho) = \frac{E_\rho}{R}\cancel{\frac{\partial \rho}{\partial \rho}} = \frac{mc^2}{eR^2}e^{-iwt} \tag{117}$$

Then, the resulting charge density is given by:

$$\rho_e = \varepsilon_0 \frac{mc^2}{eR^2}e^{-iwt} \tag{118}$$

It may seem contradictory to obtain a charge density while evaluating the motion of the center of charge as a point-like particle. This indicates that the model must be extended to define a volume over which such a charge density becomes meaningful.

4.13 Ampere's Law

Ampere's law is the most complex and consists of two parts.

$$\nabla \times \vec{B} = \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J} \tag{119}$$

We first calculate the curl of the magnetic field, which has only components in the z-direction.

$$\nabla \times \vec{B} = \frac{1}{R}\frac{\partial B_z}{\partial \phi}\hat{\rho} - \cancel{\frac{\partial B_z}{\partial \rho}}\hat{\phi} = -i\frac{B_s}{R}e^{-iwt}\hat{\rho} \tag{120}$$

To calculate the derivative of E with respect to time, we must apply the derivative using the product rule.

$$\frac{\partial \vec{E}}{\partial t} = \frac{\partial (E_\rho\, \hat{\rho})}{\partial t} = \frac{\partial E_\rho}{\partial t}\,\hat{\rho} + E_\rho \frac{\partial \hat{\rho}}{\partial t} \tag{121}$$

$$\frac{\partial \vec{E}}{\partial t} = -iwE_s e^{-iwt}\,\hat{\rho} - E_s e^{-iwt} w\,\hat{\phi} \tag{122}$$

By equating the curl of B with the radial component of the time variation of E, we verify that Ampere's law is also satisfied in this case.

$$\frac{1}{c^2}(-iwE_s e^{-iwt})\,\hat{\rho} = -\frac{iB_s}{R}e^{-iwt}\hat{\rho} \tag{123}$$

$$\frac{E_s}{B_s} = \frac{c^2}{Rw} = c \tag{124}$$

The tangential component of the time variation of E corresponds with the electric current density.

$$\vec{J} = \frac{mc}{\mu_0 eR^2}e^{-iwt}\hat{\phi} \tag{125}$$

As the current density vector is perpendicular to the electric field, the energy dissipation is zero ($\vec{J} \cdot \vec{E} = 0$).

4.14 Charge continuity equation

Merging the equations (118) and (125), the relationship between current and charge density is:

$$\vec{J} = \rho_e\, c\, \hat{\phi} \tag{126}$$

The relationship between charge density and current density must also satisfy the charge continuity equation.

$$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0 \tag{127}$$

We compute the divergence of the current density and the time derivative of the charge density.

$$\nabla \cdot \vec{J} = \frac{1}{R}\frac{\partial J_\phi}{\partial \phi} = -i\frac{mc}{\mu_0 e R^3}e^{-iwt} \tag{128}$$

$$\frac{\partial \rho}{\partial t} = \varepsilon_0 \frac{mc^2}{eR^2}(-iw)e^{-iwt} \tag{129}$$

By equating both terms, we verify that condition is also satisfied.

$$-\frac{mc}{\mu_0 e R^3} i e^{-iwt} = -\varepsilon_0 \frac{mc^2}{eR^2} i w e^{-iwt} \tag{130}$$

$$\frac{1}{\mu_0 \varepsilon_0} = c\, Rw = c^2 \tag{131}$$

4.15 Poisson equation

The Poisson equation relates the electrostatic potential and the charge density through the relation:

$$\nabla^2 V = \frac{\rho_e}{\epsilon_0} \tag{132}$$

We calculate the Laplacian of the scalar potential as the divergence of the gradient of the voltage.

$$\nabla^2 V = \nabla \cdot \nabla V = \frac{1}{R}\frac{\partial}{\partial \phi}\left(i\frac{mc^2}{eR}e^{-iwt}\right) = \frac{mc^2}{eR^2}e^{-iwt} \tag{133}$$

We obtain the same charge density as in (118)

$$\rho_e = \varepsilon_0 \frac{mc^2}{eR^2}e^{-iwt} \tag{134}$$

5 Toroidal Solenoid Electron Model

5.1 Secondary helical motion

Our Zitter electron model is based solely on the following four assumptions:

- Einstein's energy equations ($E = mc^2$ and $E = hf$)
- The electron's mass and electric charge
- The equations of Uniform Circular Motion
- The speed of light as the rotational velocity

From these four simple premises, we have derived a wide range of results consistent with established quantum phenomena and electron properties. However, with only these four premises, it is not possible to derive the electron's Anomalous Magnetic Moment (AMM). Likewise, it has not been possible to obtain the 1/2 spin value.

To address these limitations, we introduced an additional postulate in our 2018 work "The Helical Solenoid Electron Model" [25], which adds a new degree of freedom to the system. This postulate assumes a second rotational motion with a radius much smaller than the Compton wavelength, resulting in a secondary helical trajectory. When applied to a stationary electron, this internal motion transforms the geometry from a simple ring into a toroidal solenoid. For a moving electron, the structure evolves from a helical path into a helical solenoid.

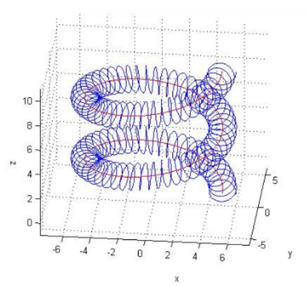

Figure 4: Helical Solenoid Model

5.2 Toroidal Solenoid Geometry

A toroidal solenoid provides two additional degrees of freedom compared to the ring geometry. In addition to the radius (R) of the torus, two new parameters appear: the thickness of the torus (r) and the number of turns around the torus (N).

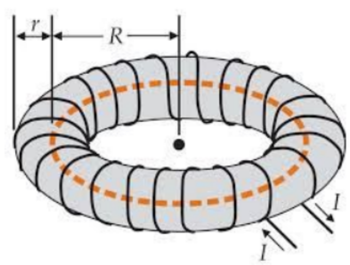

Figure 5: Toroidal Solenoid, N integer

The position of the toroidal solenoid can be parameterized as:

$$\begin{cases} x(t) = (R + r\cos Nwt)\cos wt \\ y(t) = (R + r\cos Nwt)\sin wt \\ z(t) = r\sin Nwt \end{cases} \tag{135}$$

The major radius R is equal to the reduced Compton wavelength ($R = \lambda\!\!\!^{-}_c$). It seems reasonable to assume that the minor radius r is just the classical electron radius, since $r_{cl} = \alpha \lambda\!\!\!^{-}_c$, although at this stage it remains only an intuition that requires rigorous geometric demonstration.

In a toroidal solenoid, any magnetic flux is confined within the torus. The storage of electromagnetic energy in a superconducting toroidal solenoid without energy loss is known as Superconducting Magnetic Energy Storage (SMES). In this case, the electron can be interpreted as a microscopic version of an SMES system.

5.3 Helical g-factor

Velocity is simply the time derivative of position:

$$\begin{cases} x'(t) = -(R + r\cos Nwt)\, w \sin wt - rNw \sin Nwt \cos wt \\ y'(t) = (R + r\cos Nwt)\, w \cos wt - rNw \sin Nwt \sin wt \\ z'(t) = rNw \cos Nwt \end{cases} \tag{136}$$

Where the velocity module is:

$$|r'(t)|^2 = (R + r\cos Nwt)^2 w^2 + (rNw)^2 \tag{137}$$

We postulate that the tangential velocity is always equal to the speed of light ($|r'(t)| = c$). For $R >> r$, the rotational velocity can be obtained as:

$$c^2 = (Rw)^2 + (rNw)^2 \tag{138}$$

$$\frac{c}{v_r} = \sqrt{1 + \left(\frac{rN}{R}\right)^2} \tag{139}$$

The second factor depends only on the geometry of electron. We call this value the helical g-factor. If $R >> rN$, the helical g-factor is slightly greater than 1,

$$g = \sqrt{1 + \left(\frac{rN}{R}\right)^2} \tag{140}$$

As a result, the rotational velocity is dependent on the helical g-factor and slightly lower than the speed of light:

$$v_r = c/g \tag{141}$$

With this new value of the rotational velocity, the frequency is defined by:

$$f_e = \frac{v_r}{2\pi R} = \frac{mc^2}{gh} \tag{142}$$

5.4 Toroidal arc length

When completing a full turn around a toroidal solenoid, the path length is greater than just $2\pi R$. The total length traveled is known as the arc length, and it can be calculated as follows:

$$\begin{aligned} l &= \int \sqrt{|r'(t)|^2} dt \\ &= \int \sqrt{(R + r\cos Nwt)^2 w^2 + (rNw)^2} dt. \end{aligned} \tag{143}$$

Approximating for $R \gg Nr$ and replacing the helical g-factor results in:

$$\begin{aligned} l &= \int \sqrt{(Rw)^2 + (rNw)^2} dt \\ &= \int Rw \sqrt{1 + (rN/R)^2} dt = gR \int w dt = 2\pi g R. \end{aligned} \tag{144}$$

This means that the arc length of a toroidal solenoid is equivalent to the length of the circumference of a ring of radius $R' = gR$:

$$l = 2\pi g R = 2\pi R'. \tag{145}$$

In calculating the electron's angular momentum, we must take into consideration the helical g-factor. The value of the rotational velocity is reduced in proportion to the equivalent radius, so that the angular momentum remains constant:

$$L = mR' v_r = m\,(gR)\left(\frac{c}{g}\right) = \hbar. \tag{146}$$

5.5 Magnetic Moment

The electric current flowing through a toroidal solenoid has two components, a toroidal component (red) and a poloidal component (blue).

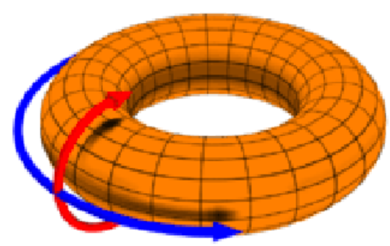

Figure 6: Toroidal and Poloidal currents.

The two components of the electric current give rise to two distinct magnetic moments: the poloidal component (blue) produces an axial magnetic moment, while the toroidal component (red) generates a secondary magnetic moment tangential to the ring.

$$\vec{\mu_e} = \mu_r\, \hat{\phi} + \mu_m\, \hat{z} \tag{147}$$

By symmetry, the secondary magnetic moment, tangential to the ring, averages out over each complete turn and therefore does not produce any external magnetic moment.

The axial component of the magnetic moment of the toroidal solenoid is slightly greater than that of the Ring Model and can be calculated analytically [26] [27].

$$\mu_m = \frac{1}{2} \int \vec{r} \times \vec{j} dV = \frac{I}{2} \int \vec{r} \times \vec{v} dt \tag{148}$$

$$\mu_m = \frac{I}{2} \int_0^{2\pi} (R + r\cos N\theta)^2\, d\theta \tag{149}$$

$$\mu_m = \frac{I}{2}\left[2\pi R^2 + \pi r^2\right] \quad (150)$$

The result coincides with that of a simple current loop, but includes an additional term proportional to the ratio between the minor and major radii of the torus, r/R. The exact value of the axial magnetic moment is:

$$\mu_m = I\pi R^2\left[1 + \frac{1}{2}\left(\frac{r}{R}\right)^2\right] \quad (151)$$

This toroidal solenoid geometry is well known in electronics where it is used to design inductors and antennas. In many practical configurations, this additional axial component is undesirable, and several techniques are available to compensate for it. For instance, one may employ a symmetric counter-winding with opposite chirality or introduce a series capacitor with a properly tuned value. In both cases, the axial term proportional to r/R is effectively canceled.

We refer to this geometry-dependent correction as the "toroidal g-factor", which quantifies the anomalous contribution arising from the solenoidal topology.

$$g_T = \left[1 + \frac{1}{2}\left(\frac{r}{R}\right)^2\right] \quad (152)$$

$$\mu_m = I\,\pi R^2 g_T \quad (153)$$

Substituting the expression for the electric current derived from the toroidal solenoid model of the electron yields a formula for the magnetic moment that depends on both the "helical g-factor" and the "toroidal g-factor".

$$I = ef = \frac{emc^2}{gh} \quad (154)$$

$$\mu_m = I\,\pi R^2 g_T = \mu_B \frac{g_T}{g} \quad (155)$$

If the relation $g_T = g^2$ holds, the resulting magnetic moment of the electron corresponds to one Bohr magneton multiplied by the electron's anomalous magnetic moment.

$$g_T = g^2 \quad \Rightarrow \quad \mu_m = g\,\mu_B \quad (156)$$

In this case, we obtain a value slightly greater than one Bohr magneton, consistent with experimental measurements of the electron's AMM. Specifically, the value we have defined as the 'helical g-factor' corresponds to the quantity commonly expressed as $(g-2)/2$ in AMM calculations.

5.6 Spin 1/2

The quantity N corresponds to the ratio between the angular frequency of rotation around the ring (w) and the angular frequency of rotation around the toroidal axis (w_i).

$$w_i = Nw \quad (157)$$

The parameter N influences only the toroidal component of the magnetic moment, whose time-averaged value vanishes due to symmetry. In contrast, the axial component, which is the physically meaningful contribution, remains independent of N. In a previous paper [25], we incorrectly stated that the equality $g_T = g^2$ holds for all values of N, whereas it is evident that the relation is satisfied only for a specific value of N.

$$\left[1 + \frac{1}{2}\left(\frac{r}{R}\right)^2\right] = \left(\sqrt{1 + \left(\frac{rN}{R}\right)^2}\right)^2 \quad (158)$$

$$N^2 = \frac{1}{2} \quad (159)$$

If N were an integer, the motion would be stationary, and N represented the number of complete revolutions around the ring per toroidal cycle. If N were a rational number, $N = n/m$, the motion would become periodic and repeat after $n \times m$ revolutions.

Finally, if N is irrational, as in this case, the motion becomes non-periodic, and the charge trajectory eventually approach each point on the toroidal surface arbitrarily, resulting in uniform coverage of the entire structure.

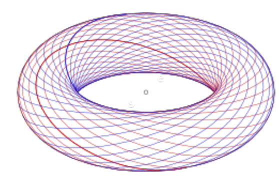

Figure 7: Toroidal Solenoid, N irrational

In this model, we interpret the value of N^2 as the quantum number corresponding to the electron's spin-1/2. We assume that the experimental value of the electron's spin-1/2 is related to its internal angular momentum, but does not correspond directly to it. We acknowledge that this identification of N^2 with the electron's spin-1/2 is not sufficiently well justified, and we regard providing a proper justification as an open question for future refinements of the present model.

5.7 Stern–Gerlach Experiment

The Stern–Gerlach (SG) experiment demonstrates the quantization of angular momentum by sending a beam of neutral atoms—typically silver (Ag) or sodium (Na), each containing a single unpaired electron—through an inhomogeneous magnetic field. The magnetic field gradient exerts a force on the magnetic moment associated with the electron's spin, resulting in the spatial separation of the atomic beam into two discrete components on the detector screen.

This outcome is interpreted as evidence that the electron possesses an intrinsic magnetic moment of approximately one Bohr magneton and that this magnetic moment can only take on two discrete orientations with respect to the field, commonly referred to as "spin up" and "spin down".

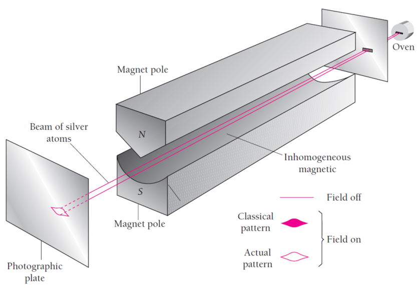

Figure 8: Stern-Gerlach experiment

It is commonly argued that classical particles would not exhibit discrete splitting: a classical ensemble with randomly oriented magnetic moments should produce a continuous distribution on the detector, since such moments would undergo Larmor precession rather than align with the magnetic field. However, this reasoning overlooks the possibility of internal damping mechanisms that could cause a classical magnetic moment to eventually align with the field. A familiar example is a magnetic compass, which initially precesses when disturbed, but internal friction causes it to settle in alignment with Earth's magnetic field instead of precessing indefinitely.

This opens the door to a classical reinterpretation of the SG experiment. In our toroidal solenoid model of the electron, there exists a secondary non-axial component of the magnetic moment that effectively acts as an internal frictional mechanism facilitating alignment. Such effects are well described by the Landau–Lifshitz equation, which introduces a phenomenological damping term into the dynamics of the magnetic moment.

The key distinction between the classical and quantum interpretations lies in the timescale of alignment. In the quantum view, alignment with the magnetic field occurs instantaneously as a result of measurement-induced projection. In contrast, the classical view predicts a finite (though potentially extremely short) relaxation time before the moment aligns with the field. Therefore, if the internal damping mechanism is sufficiently fast, the SG experiment can be interpreted consistently within both classical and quantum frameworks.

Furthermore, if a sufficiently precise SG experiment could detect a measurable delay in the alignment process, revealing a nonzero relaxation time, it would directly challenge the standard quantum interpretation, which assumes instantaneous state projection.

6 Anomalous magnetic moment

6.1 Toroidal Geometry

In the toroidal solenoid model, we have calculated the trajectory of the Center of Charge (CC) around the torus. Since the result is an irrational value, the trajectory is non-periodic and passes uniformly through all points on the toroidal surface. Given the extremely high traversal frequency, it is not possible to determine the exact position of the charge at any given moment. However, we can adopt an equivalent model in which, on average, the charge is considered to be distributed throughout the volume of the torus.

The torus is characterized by two radii: a major radius ($R = \lambda_c$) and a minor radius (r).

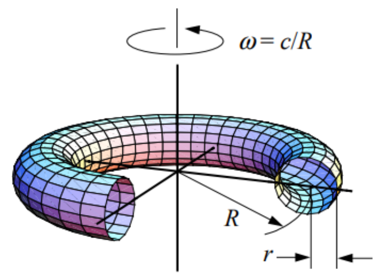

Figure 9: Toroidal Electron Model.

The volume of the torus is calculated as the area of the minor circle multiplied by the circumference with radius R:

$$A = \pi r^2, \qquad V = 2\pi R\, \pi r^2 \tag{160}$$

6.2 The minor radius

Using the volume of the torus along with the values of the electric and magnetic fields, the minor radius r of the toroid can be determined in five distinct ways: From the (i) charge density, (ii) current density, (iii) energy density, (iv) energy flux density and (v) momentum density.

6.2.1 Charge density

We can compute the charge density simply by dividing the total charge by the volume of the torus:

$$\rho_e = \frac{e}{V} = \frac{e}{2\pi R \pi r^2} \tag{161}$$

By equating this result to the charge density calculated in (118), we can calculate the minor radius of the torus.

$$\rho_e = \frac{e}{2\pi R \pi r^2} = \varepsilon_0 \frac{mc^2}{eR^2} \tag{162}$$

Surprisingly, we do not obtain the expected value of the classical electron radius, but rather a length approximately nine times greater (26.31 fm).

$$r = R\sqrt{\frac{2\alpha}{\pi}} \tag{163}$$

6.2.2 Current density

Similarly, we can calculate the current density simply by dividing the total electric current by the area of the toroid's circular cross-section.

$$J = \frac{I}{A} = \frac{ef}{\pi r^2} = \frac{ec}{2\pi R \pi r^2} \tag{164}$$

By equating this result to the current density calculated in (125), we can calculate the minor radius of the torus.

$$J = \frac{ec}{2\pi R \pi r^2} = \frac{mc}{\mu_0 e R^2} \tag{165}$$

We obtain the same value.

$$r = R\sqrt{\frac{2\alpha}{\pi}} \tag{166}$$

6.2.3 Energy density

Using Maxwell's equations, we can compute the electromagnetic energy density with the following formula:

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0}B^2 \tag{167}$$

$$u = \frac{mc^2}{4\pi\alpha R^3} \tag{168}$$

We can also calculate the energy density by dividing the electron's total energy by the volume of the torus.

$$u = \frac{E}{V} = \frac{mc^2}{2\pi R\, \pi r^2} \tag{169}$$

By equating this result two values, we can calculate the minor radius of the torus.

$$u = \frac{mc^2}{4\pi\alpha R^3} = \frac{mc^2}{2\pi R\, \pi r^2} \tag{170}$$

Obtaining the same value for the third time.

$$r = R\sqrt{\frac{2\alpha}{\pi}} \tag{171}$$

6.2.4 Energy flux density

We can calculate the power per unit area by dividing the electron's energy by the rotation period and by the circular cross-sectional area of the torus.

$$\frac{E}{T\pi r^2} = \frac{mc^2}{\pi\, r^2}\frac{1}{T} = \frac{mc^2}{\pi\, r^2}\frac{mc^2}{h} = \frac{m^2c^4}{h\,\pi\, r^2} \tag{172}$$

The Poynting vector represents the power flux of an electromagnetic field, that is, the power per unit area. By equating the magnitude of the Poynting vector obtained in (108) with this energy density, the minor radius of the torus can be determined.

$$|\vec{S}| = \frac{mc^3}{4\pi\alpha R^3} = \frac{m^2c^4}{h\,\pi\, r^2} \tag{173}$$

Obtaining exactly the same value as before.

$$r = R\sqrt{\frac{2\alpha}{\pi}} \tag{174}$$

6.2.5 Momentum density

Finally, we can calculate the linear momentum density from the Poynting vector using the formula:

$$\vec{p}_{vol} = \frac{1}{c^2}\vec{S} = \frac{mc}{4\pi\alpha R^3}\hat{\phi} \tag{175}$$

On the other hand, the linear momentum density is given by the total linear momentum divided by the volume of the torus.

$$\vec{p}_{vol} = \frac{\vec{p}}{V} = \frac{mc}{2\pi R\, \pi r^2}\hat{\phi} \tag{176}$$

By equating both expressions, we once again obtain the same value for the minor radius for the fifth time.

$$\frac{mc}{4\pi\alpha R^3} = \frac{mc}{2\pi R\, \pi r^2} \tag{177}$$

$$r = R\sqrt{\frac{2\alpha}{\pi}} \tag{178}$$

We can also compute the angular momentum by integrating the linear momentum density, yielding the same value $\hbar$.

$$L = \vec{r} \times \vec{p}_{vol} = \frac{1}{c^2}\int \vec{r} \times \vec{S}\; dV \tag{179}$$

$$L = \frac{R}{c^2}\left(\frac{mc^3}{4\pi\alpha R^3}\right)\left(2\pi R \pi r^2\right) = \hbar \tag{180}$$

6.3 g-factor

Simply by assuming the existence of orthogonal electric (E) and magnetic (B) fields, associated with the motion of the center of charge and equal in magnitude to the Schwinger limits, we have obtained a value for the minor radius of the torus of 26.31 fm (about nine times the classical radius of the electron) calculated in five different ways.

By incorporating all the previously derived elements that contribute to the helical g-factor, we obtain an exact value for it.

$$g = \sqrt{1 + \left(\frac{rN}{R}\right)^2}, \quad \frac{r}{R} = \sqrt{\frac{2\alpha}{\pi}}, \quad N^2 = \frac{1}{2} \tag{181}$$

$$g = \sqrt{1 + \frac{\alpha}{\pi}} = 1.0011607 \tag{182}$$

This result has been derived solely from the fundamental energy relations $E = mc^2$ and $E = hf$, together with the known values of the electron's mass and charge, and the assumption of rotational motion at the speed of light.

In a previous paper [25], we obtained the same value for the g-factor through a numerological argument involving the Schwinger correction term $(\alpha/2\pi)$. By contrast, the current result emerges directly from geometric reasoning.

6.4 Quantum Electrodynamics (QED)

The Dirac equation does not allow for the calculation of the electron's anomalous magnetic moment (AMM). To account for this discrepancy, Quantum Electrodynamics (QED) was developed as a more complete theory of electromagnetic interactions. Within the QED framework, the AMM arises from radiative corrections—particularly quantum vacuum fluctuations—and its theoretical value can be calculated with remarkable precision, matching experimental results to over 12 decimal places.

Motivated by this, we aimed to provide a geometric reinterpretation of this theoretical result within our Zitter Electron Model framework. But what we discovered was so striking that it led us to seriously question the validity of QED as a physical theory. This critical historical analysis is presented in our paper titled "Something is wrong in the state of QED" [28]. From this investigation, we concluded that the theoretical calculation used to derive the AMM is not reliable, nor even mathematically legitimate.

From our perspective, the last experimental measurement of the electron's g-factor that can be regarded as fully independent and reliable was conducted in 1961 by Schupp, Pidd, and Crane [29], who reported a value of g=1.0011609. Subsequent measurements, while significantly more precise, have relied heavily on theoretical input from QED to interpret raw data, making it difficult to assess their independence from the theoretical framework they are intended to test.

Source / Model	g-factor	Error
Last trusted measurement (1961)	1.0011609	–
Schwinger correction $(1 + \alpha/2\pi)$	1.0011614	5 ppm
QED (currently accepted value)	1.0011596	13 ppm
This model $\left(\sqrt{1 + \alpha/\pi}\right)$	1.0011607	2 ppm

Table 2: Comparison of theoretical and experimental values of the electron g-factor

If we compare the currently accepted value of the g-factor, as calculated by QED, with the experimental value obtained in 1961, we observe that the discrepancy is significantly larger than what would result from simply adding Schwinger's correction factor $(\alpha/2\pi)$.

Furthermore, when we compare our theoretical value of the g-factor with the 1961 experimental result, we find that our result is actually more accurate. In addition to being more precise, our value has a clear geometric justification and a simpler mathematical derivation.

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