# Permutative Algebra

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**Abstract**: : This document is entitled (Permutation Algebra) Explores the relationships between the permuted of natural integers .

In this article I define the higher and lower permutations of integers.

The behaviour of sequences of integers by involving their permuted.

The research is only at is beginnings, I hope that young researchers will be interested in it, and why not draw mathematical theory's from it.

**Key words:**

Permutations of number, Natural integers

## Permutative Algebra

**Summary:** this document explores the relationships between natural integer and his permutations, to establish mathematical conjectures.

**Introduction**

- I define first the permuted of a number.
- There are two cases: the upper permuted and the lower permuted.
- The document presents integers in the form $N=(a+bn)c+1$;

  $a,b,n,c$ integers.

- Using the permutation of these numbers, we find sequence linked by mathematical relations..

**Examples**

- Several examples illustrate conjectures with these categories of numbers..
- The results show interesting relationships between the numbers.
- The document is a preliminary study on a permuted of numbers, encouraging young researchers to deepen these ideas.

**(I) Permutation of a number :**

14 His first higher permutation is 41

41 His first lower permutation is 14

35 His first higher permutation is 53

53 His first lower permutation is 35

**(II) Sequence of higher permuted of number 13:**

$$13 \cup 31 \cup 112 \cup 130 \cup 211 \cup 229 \cup 310 \cup 328 \ldots\ldots etc,$$

31 is the first higher permutation of number 13

112 is the secund higher permutation of number 13 : (13+99)

130 is the third higher permutation of number 13 : (31+99)

328 is the seventh higher permutation of number 13 ; (229+99)…..etc

**(III) Sequence of lower permuted of number 13:**

$$13 \cup -68 \cup -86 \cup -167 \cup -185 \cup -266 \ldots . etc$$

- 68 is the first lower permutation of number 13 ; (31-99)

- 86 is the secund lower permutation of number 13 ; (13-99)

- 167 is the third lower permutation of number 13 ; (-68-99)…..etc

**Example 1:**

$$N = (5 + 10n) * 17 + 1$$

$$5*17+1 = 86 \rightarrow 68$$
$$15*17+1 = 256 \rightarrow 85$$
$$25*17+1 = 426 \rightarrow 102$$
$$35*17+1 = 596 \rightarrow 119$$
$$45*17+1 = 766 \rightarrow 136$$
$$55*17+1 = 936 \rightarrow 153$$

$$86-68=18$$
$$256-85=171$$
$$426-102=324$$
$$596-119=477$$
$$766-136=630$$
$$936-153=783$$

$$65*17+1 = 1106 \rightarrow 170 = 10*17$$
$$75*17+1 = 1276 \rightarrow 187 = 11*17$$
$$85*17+1 = 1446 \rightarrow 204 = 12*17$$
$$95*17+1 = 1616 \rightarrow 221 = 13*17$$
$$105*17+1 = 1786 \rightarrow 238 = 14*17$$

$$1106-170=936$$
$$1276-187=1089$$
$$1446-204=1242$$
$$1616-221=1395$$
$$1786-238=1548$$

$171 \cup 324 \cup 477 \cup 630 \cup 783 \cup 936 \cup 1089 \cup 1242 \cup 1395$

.....153....153....153....153....153.....153.....153.....153

The permutations of 256

$58;(85);157;184;256.....$

The permutations of 426

$3;30;(102);129;201....$

The permutations of 596

$2;20;101;(119);200;218....$

The permutations of 1446

$6;60;105;159;(204);258;305....$

The permutations of 1616

$23;32;122;131;(221);230;320.....$

The permutations of 1786

$4;40;103;139;202;(238);301;337...$

**Example 2:**

$N = (17+10n)19+1$

$27*19+1 = 514 \rightarrow 1*19$
$37*19+1 = 704 \rightarrow 209 = 11*19$
$47*19+1 = 894 \rightarrow 399 = 21*19$
$57*19+1 = 1084 \rightarrow 589 = 31*19$
$67*19+1 = 1274 \rightarrow 779 = 41*19$
$77*19+1 = 1464 \rightarrow 969 = 51*19$
$87*19+1 = 1654 \rightarrow 1159 = 61*19$
$97*19+1 = 1844 \rightarrow 1349 = 71*19$

$514-19 = 495$
$704-209 = 495$
$894-399 = 495$
$1084-589 = 495$
$1274-779 = 495$
$1464-969 = 495$
$1654-1159 = 495$
$1844-1349 = 495$

The permutations of 704

$11 \cup 110 \cup (209) \cup 308 \cup 407$

The permutations of 894

$3 \cup 30 \cup 102 \cup 129 \cup 201 \cup 228 \cup 300 \cup (399)..$

The permutations of 1084

$49 \cup 94 \cup 148 \cup 193 \cup 247 \cup 292 \cup 346 \cup 391 \cup 445 \cup 490 \cup 544 \cup (589) \cup 643...$

**Example 3:**

$N=(3+6n)19+1$

$3*19+1=58\rightarrow 85=5*17$
$9*19+1=172\rightarrow 136=8*17$
$15*19+1=286\rightarrow 187=11*17$
$21*19+1=400\rightarrow 238=14*17$
$27*19+1=514\rightarrow 289=17*17$
$33*19+1=628\rightarrow 340=20*17$
$39*19+1=742\rightarrow 391=23*17$
$45*19+1=856\rightarrow 442=26*17$
$51*19+1=970\rightarrow 493=29*17$

$58-85=-27$
$172-136=36$
$286-187=99$
$400-238=162$
$514-289=225$
$628-340=288$
$742-391=351$
$856-442=414$
$970-493=477$

$-27\cup 36\cup 99\cup 162\cup 225\cup 288\cup 351\cup 414\cup 477$
$.....63....63.....63.....63.....63.....63.....63.....63$

The permutations of 136

$37;73;(136);172;235;271...$

The permutations of 391

$49;94;148;193;247;292;346;(391);445....$

The permutations of 442

$46;64;145;163;244;262;343;361;(442)..$

The permutations of 187

$88;(187);286;385....$

The permutations of 289

$19;91;118;190;217;(289);316....$

**Example 4:**

$N=(9+10n)13+1$

$9*13+1=118\rightarrow 91=7*13$ $\quad 118-91=27$

$19*13+1=248\rightarrow 104=8*13$ $\quad 248-104=144$

$29*13+1=378\rightarrow 117=9*13$ $\quad 378-117=261$

$39*13+1=508\rightarrow 130=10*13$ $\quad 508-130=378$

$49*13+1=638\rightarrow 143=11*13$ $\quad 638-143=495$

$59*13+1=768\rightarrow 156=12*13$ $\quad 768-156=612$

$69*13+1=898\rightarrow 169=13*13$ $\quad 898-169=729$

$79*13+1=1028\rightarrow 182=14*13$ $\quad 1028-182=846$

$89*13+1=1158\rightarrow 195=15*13$ $\quad 1158-195=963$

$99*13+1=1288\rightarrow 208=16*13$ $\quad 1288-208=1080$

$27\cup 144\cup 261\cup 378\cup 495\cup 612\cup 729\cup 846\cup 963\cup 1080$

...117....117.....117....117....117.....117....117...117...117

The permutations of 118

$19:(91);118;190....$

The permutations of 1158

$69;96;168;(195);267;294;366;393.....$

The permutations of 248

$5;50;(104);149;203...$

The permutations of 1288

$1;10;100;109;199;(208);298;307.....$

The permutations of 1028

$38;83;137;(182);236;281.....$

**Example 5:**

$N=(5+8n)23+1$

$5*23+1=116\rightarrow 1*17$
$13*23+1=300\rightarrow 102=6*17$
$21*23+1=484\rightarrow 187=11*17$
$29*23+1=668\rightarrow 272=16*17$
$37*23+1=852\rightarrow 357=21*17$
$45*23+1=1036\rightarrow 442=26*17$
$53*23+1=1220\rightarrow 527=31*17$
$61*23+1=1404\rightarrow 612=36*17$
$69*23+1=1588\rightarrow 697=41*17$
$77*23+1=1772\rightarrow 782=46*17$
$85*23+1=1956\rightarrow 867=51*17$

$116-17=99$
$300-102=198$
$484-187=292$
$668-272=396$
$852-357=495$
$1036-442=594$
$1220-527=693$
$1404-612=792$
$1588-697=891$
$1772-782=990$
$1956-867=1089$

$99\cup 198\cup 292\cup 396\cup 495\cup 594\cup 693\cup 792\cup 891\cup 990\cup 1089$

**Example 6:**

$N=(25+n)*23+1$

$25*23+1=576\rightarrow 414=18*23$
$26*23+1=599\rightarrow 644=28*23$
$27*23+1=622\rightarrow 874=38*23$
$28*23+1=645\rightarrow 1104=48*23$
.
*.etc*

**Exemple 7:**

$N = (9+3n)37+1$

$9*37+1 = 334 \rightarrow 37 = 1*37$
$12*37+1 = 445 \rightarrow 148 = 4*37$
$15*37+1 = 556 \rightarrow 259 = 7*37$
$18*37+1 = 667 \rightarrow 370 = 10*37$
$21*37+1 = 778 \rightarrow 481 = 13*37$
$24*37+1 = 889 \rightarrow 592 = 16*37$
$27*37+1 = 1000 \rightarrow 703 = 19*37$
$30*37+1 = 1111 \rightarrow 814 = 22*37$

**References :**


1- http://sites.mathdoc.fr/PMO/PDF/D_DJEBBAR_85_01.pdf

2- https://fr.wikipedia.org/wiki/Srinivasa_Ramanujan

3- https://fr.wikipedia.org/wiki/Math%C3%A9matiques_arabes