The Non-Relativistic Bi-Level Electron Model

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Abstract

This work presents the bi-level electron model, a quantitative model of a point-like spinning electron, which was inspired by similar classical models of spinning electrons that interpret the Zitterbewegung of Dirac electrons as a spin motion of point-like electrons. The new model is consistent with de Broglie's internal clock hypothesis and other features of a modified Born-Infeld model of electrons, as well as with Larmor precession of the electron spin in a uniform magnetic field. The equations of motion are discussed in non-relativistic approximation, while the relativistic case is left for future work.

1 Introduction

The main purpose of the bi-level electron model is to provide a simplified, but still quantitative version of a modified Born-Infeld model of electrons [Kra23]. While numerical computations with the latter model are currently limited to non-interacting electrons, the bi-level electron model may be used to numerically simulate electromagnetically interacting electrons in many interesting physical scenarios, for example, the precession of an electron's spin in a uniform magnetic field (which is discussed in Section 4.3).

The equations of motion of the bi-level electron model (including the employed notation) have been inspired by Rivas's kinematical theory of spinning particles [Riv01] and, in particular, by the separation of a fourth-order equation of motion into two second-order differential equations describing the motion of the particle's center of mass (CM) and the motion of the particle's center of charge (CC). These two equations represent the two levels of the bi-level electron model, i.e., the CM level and the CC level. The distinctive (and eponymous) feature of the bi-level electron model is that an external electromagnetic force on the electron enters both equations, i.e., both levels of the model. This is in loose analogy to a bi-level house with an entry between the two levels of the house and two short flights of stairs leading to the two floors.

Section 2 provides an overview of the modified Born-Infeld model of electrons and Rivas's equations of motion for spinning Galilei particles. Furthermore, some notable features of the electron model by Van Belle [Bel20] are mentioned. Section 3 defines the equations of motion of the bi-level electron model, while some of its features are described in Section 4. Section 5 discusses the model, and Section 6 concludes this paper.

2 Previous Work

2.1 Modified Born-Infeld Model of Electrons

The Lagrangian density $\mathscr{L}$ of the relevant modified Born-Infeld field theory [Kra24b] is defined as

$$\mathscr{L} \stackrel{\text{def}}{=} \frac{b^2}{\mu_0}\left(1 - \sqrt{1 - \frac{1}{b^2}(\partial^\mu A^\nu)(\partial_\mu A_\nu)}\right) \tag{1}$$

with the Born-Infeld parameter b specifying the maximum magnetic field strength, the vacuum permeability μ_0, and the electromagnetic four-potential A.

The corresponding Euler-Lagrange equations were solved numerically in previous work [Kra23] resulting in a rotating field solution with a peak moving at the speed of light c on a circular orbit with radius $\lambda\!\!\!^{-}_{\mathrm{C}} \stackrel{\text{def}}{=} \hbar/(m_{\mathrm{CM}}\, c)$, i.e., the reduced Compton wavelength of an electron of mass m_{CM}. Thus, the angular frequency of this circular motion is $\omega_{\mathrm{C}} \stackrel{\text{def}}{=} m_{\mathrm{CM}}\, c^2/\hbar = c/\lambda\!\!\!^{-}_{\mathrm{C}}$, i.e., the Compton angular frequency, as required by de Broglie's internal clock hypothesis [dB25]. While most features of the solution (electric charge, magnetic moment, radius of orbit, angular frequency) were imposed on it, the total field energy of the solution was matched to the rest mass energy of electrons by adjusting the Born-Infeld parameter b.

At large distances, the solution appears to show the same Lorentz-type interaction with electromagnetic fields as relativistic electrons [Kra24a].

The total field energy as well as the total momentum of the field solution were computed by numerically evaluating elements of the field's canonical stress-energy tensor [Kra24b]. While the total field energy of a free electron is conserved, the total momentum of a free electron is a rotating vector pointing in the direction of the velocity of the peak of the rotating field solution with an absolute value close to $m_{\mathrm{CM}}\, c/2$. The intrinsic angular momentum of the solution was evaluated similarly and appears to match the spin of electrons $\hbar/2$.

2.2 Rivas's Spinning Galilei Particle

Rivas's equations are presented here in the notation of the present work, which is, however, similar to Rivas's notation, in particular regarding the use of subscripts "CM" (center of mass) and "CC" (center of charge). Table 1 summarizes the differences to Rivas's original notation [Riv01].

Based on his kinematical theory of spinning particles, Rivas derived a fourth-order differential equation for the time-dependent position $\mathbf{r}_{\mathrm{CC}}$ of a spinning Galilei particle of mass m_{CM} and charge q_{CC} in an external electric field $\mathbf{E}(t, \mathbf{r}_{\mathrm{CC}})$ and an external magnetic field $\mathbf{B}(t, \mathbf{r}_{\mathrm{CC}})$ [Riv01, page 68]:

$$\frac{1}{4\omega_{\mathrm{C}}^2} \frac{\mathrm{d}^4\mathbf{r}_{\mathrm{CC}}}{\mathrm{d}t^4} + \frac{\mathrm{d}^2\mathbf{r}_{\mathrm{CC}}}{\mathrm{d}t^2} = \frac{q_{\mathrm{CC}}}{m_{\mathrm{CM}}} \left(\mathbf{E}(t, \mathbf{r}_{\mathrm{CC}}) + \mathbf{v}_{\mathrm{CC}} \times \mathbf{B}(t, \mathbf{r}_{\mathrm{CC}})\right) \tag{2}$$

with the velocity $\mathbf{v}_{\mathrm{CC}} \stackrel{\text{def}}{=} \mathrm{d}\mathbf{r}_{\mathrm{CC}}/\mathrm{d}t$ and the Compton angular frequency $\omega_{\mathrm{C}} \stackrel{\text{def}}{=} m_{\mathrm{CM}} c^2/\hbar$ in the case of an electron.

This fourth-order differential equation may be separated into two second-order differential equations by introducing the center-of-mass position $\mathbf{r}_{\mathrm{CM}}$. The equation of motion for the time-dependent position $\mathbf{r}_{\mathrm{CM}}$ is [Riv01, page 68]:

$$\frac{\mathrm{d}^2\mathbf{r}_{\mathrm{CM}}}{\mathrm{d}t^2} = \frac{q_{\mathrm{CC}}}{m_{\mathrm{CM}}} \left(\mathbf{E}(t, \mathbf{r}_{\mathrm{CC}}) + \mathbf{v}_{\mathrm{CC}} \times \mathbf{B}(t, \mathbf{r}_{\mathrm{CC}})\right) \tag{3}$$

while the equation of motion for the center of charge $\mathbf{r}_{\mathrm{CC}}$ is [Riv01, page 68]:

$$\frac{\mathrm{d}^2\mathbf{r}_{\mathrm{CC}}}{\mathrm{d}t^2} = 4\omega_{\mathrm{C}}^2 \left(\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}\right). \tag{4}$$

In the case of an electron, $\mathbf{v}_{\mathrm{CC}}$ is constrained by

$$|\mathbf{v}_{\mathrm{CC}}| = c. \tag{5}$$

Table 1: Symbols differing from Rivas's notation [Riv01].

Symbol here	Rivas's notation	Description
m_{CM}	m	mass resisting acceleration of center of mass
q_{CC}	e	charge of particle (located at center of charge)
$\mathbf{r}_{\mathrm{CC}}$	$\mathbf{r}$	position of center of charge
$\mathbf{r}_{\mathrm{CM}}$	$\mathbf{q}$	position of center of mass
$\mathbf{v}_{\mathrm{CC}}$	$\mathbf{u}$	velocity of center of charge
$\mathbf{v}_{\mathrm{CM}}$	$\mathbf{v}$	velocity of center of mass
$\lambda\!\!\!^{-}_{\mathrm{C}}$	$2R_0$	reduced Compton wavelength $\hbar/(m_{\mathrm{CM}}c)$
ω_{C}	$\omega/2$	Compton angular frequency $m_{\mathrm{CM}}c^2/\hbar$

For a resting center of mass, i.e., $\mathbf{v}_{\mathrm{CM}} = \mathbf{0}$, without external electromagnetic field, i.e., $\mathbf{E}(t, \mathbf{r}_{\mathrm{CC}}) = \mathbf{B}(t, \mathbf{r}_{\mathrm{CC}}) = \mathbf{0}$, these equations of motion are solved by $\mathbf{r}_{\mathrm{CC}}$ spinning with angular frequency $2\omega_{\mathrm{C}}$ around $\mathbf{r}_{\mathrm{CM}}$ on a circular orbit of radius $\lambdabar_{\mathrm{C}}/2$. This angular frequency matches the angular frequency of the Zitterbewegung of Dirac electrons.

In this case, the particle's spin is $\hbar/2$ since the anti-orbital spin $\mathbf{S}$ relative to the center of mass is [Riv01, page 66]:

$$\mathbf{S} \stackrel{\text{def}}{=} -m_{\mathrm{CM}} \left(\mathbf{r}_{\mathrm{CC}} - \mathbf{r}_{\mathrm{CM}}\right) \times \left(\mathbf{v}_{\mathrm{CC}} - \mathbf{v}_{\mathrm{CM}}\right), \tag{6}$$

thus,

$$|\mathbf{S}| = m_{\mathrm{CM}} \left|\mathbf{r}_{\mathrm{CC}} - \mathbf{r}_{\mathrm{CM}}\right| \left|\mathbf{v}_{\mathrm{CC}} - \mathbf{0}\right| = m_{\mathrm{CM}} \frac{\lambdabar_{\mathrm{C}}}{2} c = \frac{\hbar}{2}. \tag{7}$$

It should be noted that Rivas's theory requires a spin contribution of a rotating orientation to explain the electron spin g-factor $g \approx 2$ [Riv01, pages 86–97 and pages 264–266]. This appears to imply a considerable complication for any electron model based on Rivas's theory, which the present work attempts to avoid.

2.3 Van Belle's Spinning Electron with Effective Mass

Van Belle's model of a spinning electron [Bel20] differs considerably from Rivas's spinning Galilei particle. It is mentioned here because some of its features are very close to the modified Born-Infeld model of electrons and the bi-level electron model.

Focusing again on the case of a resting center of mass, i.e., $\mathbf{v}_{\mathrm{CM}} = \mathbf{0}$, without external electromagnetic field and ignoring the anomalous magnetic moment of electrons, Van Belle's model states that $\mathbf{r}_{\mathrm{CC}}$ spins with angular frequency ω_{C} around $\mathbf{r}_{\mathrm{CM}}$ on a circular orbit of radius $\lambdabar_{\mathrm{C}}$. While this angular frequency is not consistent with the Zitterbewegung of Dirac electrons, it is consistent with de Broglie's hypothetical internal clock.

The electron's spin in Van Belle's model is approximately $\hbar/2$ because the "effective" mass m_{CC} at the center of charge (m_γ in Van Belle's notation) is assumed to be $m_{\mathrm{CM}}/2$. Thus, the spin is $m_{\mathrm{CC}}\, \lambdabar_{\mathrm{C}}\, c \approx \hbar/2$. The electron's spin g-factor $g \approx 2$ is explained via the Ampèrian loop model for the magnetic moment of an electron's charge q_{CC} moving at the speed of light c on a circular loop of radius $\lambdabar_{\mathrm{C}}$.

These features are shared (at least in some sense) by the modified Born-Infeld model (see Section 2.1) as well as the bi-level electron model, which is described next.

3 Description of the Non-Relativistic Bi-Level Electron Model

As in Rivas's model of a spinning Galilei particle (see Section 2.2), the motion of an electron in the non-relativistic bi-level electron model is determined by two second-order differential equations for the positions $\mathbf{r}_{\mathrm{CM}}$ and $\mathbf{r}_{\mathrm{CC}}$ with the constraint that the speed $|\mathbf{v}_{\mathrm{CC}}|$ of the position $\mathbf{r}_{\mathrm{CC}}$ is equal to the speed of light c; see Eq. (5).

In contrast to Rivas's model, an external Lorentz force $q_{\mathrm{CC}} \left(\mathbf{E}(t, \mathbf{r}_{\mathrm{CC}}) + \mathbf{v}_{\mathrm{CC}} \times \mathbf{B}(t, \mathbf{r}_{\mathrm{CC}})\right)$ enters both equations. However, the external force is considerably modified in both cases.

In the equation of motion for $\mathbf{r}_{\mathrm{CM}}$, a time average of the external force accelerates the mass m_{CM} at $\mathbf{r}_{\mathrm{CM}}$:

$$\frac{\mathrm{d}^2 \mathbf{r}_{\mathrm{CM}}}{\mathrm{d}t^2} = \frac{q_{\mathrm{CC}}}{m_{\mathrm{CM}}} \frac{\omega_{\mathrm{C}}}{2\pi} \int_{t-2\pi/\omega_C}^{t} \left(\mathbf{E}(t', \mathbf{r}_{\mathrm{CC}}(t')) + \mathbf{v}_{\mathrm{CC}}(t') \times \mathbf{B}(t', \mathbf{r}_{\mathrm{CC}}(t'))\right) \mathrm{d}t' \tag{8}$$

At the time of writing, the physical processes causing the time average in Eq. (8) (in terms of the modified Born-Infeld model; see Section 2.1) are not well understood; thus, the specific time integral over exactly one period $T = 2\pi/\omega_{\mathrm{C}}$ of the electron's internal clock (ticking at the Compton angular frequency ω_{C}) is a model assumption. The reason for the specific form of the time integral is the approximation

$$\frac{\omega_{\mathrm{C}}}{2\pi} \int_{t-2\pi/\omega_C}^{t} \left(\mathbf{E}(t', \mathbf{r}_{\mathrm{CC}}(t')) + \mathbf{v}_{\mathrm{CC}}(t') \times \mathbf{B}(t', \mathbf{r}_{\mathrm{CC}}(t'))\right) \mathrm{d}t' \approx \mathbf{E} + \mathbf{v}_{\mathrm{CM}}(t) \times \mathbf{B} \tag{9}$$

for slowly varying $\mathbf{v}_{\mathrm{CM}}(t)$ and (approximately) uniform fields $\mathbf{E}$ and $\mathbf{B}$, which shows that the external Lorentz force on $\mathbf{r}_{\mathrm{CC}}$ may appear as an external Lorentz force on $\mathbf{r}_{\mathrm{CM}}$.

The external Lorentz force on $\mathbf{r}_{\mathrm{CC}}$ also enters the equation of motion for $\mathbf{r}_{\mathrm{CC}}$:

$$\frac{\mathrm{d}^2\mathbf{r}_{\mathrm{CC}}}{\mathrm{d}t^2} = \omega_{\mathrm{C}}^2 \left(\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}\right) + \frac{q_{\mathrm{CC}}}{m_{\mathrm{CC}}} \mathrm{proj}_{\mathbf{v}_{\mathrm{CC}} \times (\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}})} \left(\mathbf{E}(t, \mathbf{r}_{\mathrm{CC}}) + \mathbf{v}_{\mathrm{CC}} \times \mathbf{B}(t, \mathbf{r}_{\mathrm{CC}})\right) \tag{10}$$

In comparison to Rivas's model (see Eq. (4)), a factor $4 = 2^2$ is missing since electrons in the bi-level electron model are spinning with the angular frequency ω_{C} instead of $2\omega_{\mathrm{C}}$ in Rivas's model (see also the comment in Section 2.3).

More importantly, an external force term is included that is missing in Rivas's model. One interpretation of the right-hand side of Eq. (10) is that it represents a sum of an internal constraint force represented by $\omega_{\mathrm{C}}^2 \left(\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}\right)$, which tries to keep $\mathbf{r}_{\mathrm{CC}}$ at a distance of $\lambda\!\!\!^{-}_{\mathrm{C}} = c/\omega_{\mathrm{C}}$ from $\mathbf{r}_{\mathrm{CM}}$, and an external Lorentz force that is projected onto the direction $\mathbf{v}_{\mathrm{CC}} \times (\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}})$ by a vector projection of the form

$$\mathrm{proj}_{\mathbf{b}}\mathbf{a} \stackrel{\mathrm{def}}{=} \frac{\mathbf{a} \cdot \mathbf{b}}{\mathbf{b} \cdot \mathbf{b}}\mathbf{b}. \tag{11}$$

A vector projection onto a direction orthogonal to $\mathbf{v}_{\mathrm{CC}}$ prevents the external Lorentz force from changing the speed $|\mathbf{v}_{\mathrm{CC}}|$, and, therefore, is consistent with the constraint $|\mathbf{v}_{\mathrm{CC}}| = c$.

Furthermore, a vector projection onto a direction orthogonal to $\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}$ prevents the external Lorentz force from acting in the same direction as the internal force represented by $\omega_{\mathrm{C}}^2 \left(\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}\right)$. The physical motivation for this projection is that an internal constraint force could instantly adapt to compensate for any external force in the same direction, effectively removing the projection of the external force onto the direction of the constraint force. Thus, the vector projection of the external Lorentz force onto $\mathbf{v}_{\mathrm{CC}} \times (\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}})$ appears to be physically plausible.

The mass m_{CC} of the mass point at $\mathbf{r}_{\mathrm{CC}}$ in Eq. (10) is set to $m_{\mathrm{CM}}/2$ as suggested by the modified Born-Infeld model of electrons (see Section 2.1). Thus, the system of Eqs. (8) and (10) might remind some readers of a rigid body with an external force applied to a single charged mass point that is part of the rigid body (as in Eq. (10)), which in turn affects the center-of-mass motion of the rigid body (as in Eq. (8)).

4 Features of the Non-Relativistic Bi-Level Electron Model

This section describes some of the emergent features of the bi-level electron model on the center-of-mass (CM) level as well as the center-of-charge (CC) level.

4.1 CM Level with Uniform Force Fields: A Classical Electron

As mentioned in the previous section, the approximation in Eq. (9) is justified under certain conditions (slowly varying $\mathbf{v}_{\mathrm{CM}}$ and (approximately) uniform fields $\mathbf{E}$ and $\mathbf{B}$). With this approximation, Eq. (8) becomes:

$$\frac{\mathrm{d}^2\mathbf{r}_{\mathrm{CM}}}{\mathrm{d}t^2} \approx \frac{q_{\mathrm{CC}}}{m_{\mathrm{CM}}} \left(\mathbf{E} + \mathbf{v}_{\mathrm{CM}} \times \mathbf{B}\right), \tag{12}$$

which is just the acceleration of a classical electron of charge q_{CC} and mass m_{CM} at $\mathbf{r}_{\mathrm{CM}}$ in an (approximately) uniform electromagnetic field.

4.2 CC Level without External Force: A Spinning Electron

Without external electromagnetic field, i.e., $\mathbf{E} = \mathbf{B} = \mathbf{0}$, Eq. (10) becomes

$$\frac{\mathrm{d}^2\mathbf{r}_{\mathrm{CC}}}{\mathrm{d}t^2} = \omega_{\mathrm{C}}^2 \left(\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}\right). \tag{13}$$

Due to the constraint $|\mathbf{v}_{\mathrm{CC}}| = c$, all non-trivial solutions for $\mathbf{r}_{\mathrm{CC}}(t)$ are circular orbits of radius $\lambda\!\!\!^{-}_{\mathrm{C}} = c/\omega_{\mathrm{C}} = \hbar/(m_{\mathrm{CM}}c)$ (the reduced Compton wavelength) around the center of mass $\mathbf{r}_{\mathrm{CM}}$. The angular frequency of all orbits is the Compton angular frequency ω_{C}, as suggested by de Broglie's internal clock hypothesis.

The magnetic moment μ of the orbiting (i.e., spinning) charge $q_{\rm CC}$ at $\mathbf{r}_{\rm CC}$ is (according to the Ampèrian loop model)

$$\mu = \frac{|q_{\rm CC}|c\lambda\!\!\!^{-}_{\rm C}}{2} = \frac{|q_{\rm CC}|\hbar}{2m_{\rm CM}}. \tag{14}$$

The angular momentum (i.e., spin) of the orbiting (i.e., spinning) mass $m_{\rm CC} = m_{\rm CM}/2$ at $\mathbf{r}_{\rm CC}$ relative to $\mathbf{r}_{\rm CM}$ is

$$\mathbf{s}_{\rm CM} \stackrel{\rm def}{=} m_{\rm CC}(\mathbf{r}_{\rm CC} - \mathbf{r}_{\rm CM}) \times (\mathbf{v}_{\rm CC} - \mathbf{v}_{\rm CM}). \tag{15}$$

Thus,

$$|\mathbf{s}_{\rm CM}| = m_{\rm CC}c\lambda\!\!\!^{-}_{\rm C} = \frac{m_{\rm CM}}{2}c\frac{\hbar}{m_{\rm CM}c} = \frac{\hbar}{2}. \tag{16}$$

Therefore, the spin g-factor of this model is 2.

4.3 CC Level with External Force: An Electron with Precessing Spin

In general, the external force term in Eq. (10) results in a torque $\boldsymbol{\tau}_{\rm CM}$ relative to $\mathbf{r}_{\rm CM}$, which leads to a changing spin $\mathbf{s}_{\rm CM}$ according to

$$\frac{\mathrm{d}\mathbf{s}_{\rm CM}}{\mathrm{d}t} = \underbrace{m_{\rm CC}(\mathbf{v}_{\rm CC} - \mathbf{v}_{\rm CM}) \times (\mathbf{v}_{\rm CC} - \mathbf{v}_{\rm CM})}_{\mathbf{0}} + \underbrace{m_{\rm CC}(\mathbf{r}_{\rm CC} - \mathbf{r}_{\rm CM}) \times \left(\frac{\mathrm{d}^2\mathbf{r}_{\rm CC}}{\mathrm{d}t^2} - \frac{\mathrm{d}^2\mathbf{r}_{\rm CM}}{\mathrm{d}t^2}\right)}_{\boldsymbol{\tau}_{\rm CM}}. \tag{17}$$

An important example is the precession of spin $\mathbf{s}_{\rm CM}$ in a uniform magnetic field $\mathbf{B}$, which is discussed in some detail in this section. The experimentally confirmed angular frequency of spin precession in this case is the Larmor frequency $|q_{\rm CC}g|B/(2m_{\rm CM})$ with $B \stackrel{\rm def}{=} |\mathbf{B}|$.

For the calculation, let $\mathbf{E} = \mathbf{0}$, $\mathbf{B} = (0, 0, B)^{\rm T}$, $\mathbf{r}_{\rm CC}(t \approx 0) \approx (\lambda\!\!\!^{-}_{\rm C}\cos(\omega_{\rm C}t), 0, -\lambda\!\!\!^{-}_{\rm C}\sin(\omega_{\rm C}t))^{\rm T}$ (neglecting the effect of spin precession, which is small compared to the spin motion), and $\mathbf{r}_{\rm CM}(0) = \mathbf{0}$. It follows: $\mathbf{v}_{\rm CC}(t \approx 0) \approx (-c\sin(\omega_{\rm C}t), 0, -c\cos(\omega_{\rm C}t))^{\rm T}$ and $\mathbf{s}_{\rm CM}(t = 0) = (0, \hbar/2, 0)^{\rm T}$.

In this scenario, the right-hand side of Eq. (8) is always approximately $\mathbf{0}$ since the integral over the external force over exactly one period of the spin motion is always approximately $\mathbf{0}$, thus: $\mathrm{d}^2\mathbf{r}_{\rm CM}(t)/\mathrm{d}t^2 \approx \mathbf{0}$, $\mathbf{v}_{\rm CM}(t) \approx \mathbf{0}$, and $\mathbf{r}_{\rm CM}(t) \approx \mathbf{0}$.

The torque $\boldsymbol{\tau}_{\rm CM}$ may be computed with the help of Eqs. (8) and (10), noting that $\mathrm{d}^2\mathbf{r}_{\rm CM}(t)/\mathrm{d}t^2 \approx \mathbf{0}$ and $m_{\rm CC}(\mathbf{r}_{\rm CC} - \mathbf{r}_{\rm CM}) \times \omega_{\rm C}^2(\mathbf{r}_{\rm CM} - \mathbf{r}_{\rm CC}) = \mathbf{0}$:

$$\begin{aligned}
\boldsymbol{\tau}_{\rm CM} &\approx (\mathbf{r}_{\rm CC} - \mathbf{r}_{\rm CM}) \times q_{\rm CC}\,\mathrm{proj}_{\mathbf{v}_{\rm CC}\times(\mathbf{r}_{\rm CM}-\mathbf{r}_{\rm CC})}\left(\mathbf{E}(t, \mathbf{r}_{\rm CC}) + \mathbf{v}_{\rm CC} \times \mathbf{B}(t, \mathbf{r}_{\rm CC})\right) && (18)\\
&\approx q_{\rm CC}\,\mathbf{r}_{\rm CC} \times \mathrm{proj}_{\mathbf{r}_{\rm CC}\times\mathbf{v}_{\rm CC}}\mathbf{v}_{\rm CC} \times \mathbf{B} && (19)\\
&= q_{\rm CC}\,\mathbf{r}_{\rm CC} \times \frac{(\mathbf{v}_{\rm CC} \times \mathbf{B}) \cdot (\mathbf{r}_{\rm CC} \times \mathbf{v}_{\rm CC})}{(\mathbf{r}_{\rm CC} \times \mathbf{v}_{\rm CC}) \cdot (\mathbf{r}_{\rm CC} \times \mathbf{v}_{\rm CC})}(\mathbf{r}_{\rm CC} \times \mathbf{v}_{\rm CC}) && (20)\\
&\approx -q_{\rm CC}\frac{(\mathbf{v}_{\rm CC} \times \mathbf{B}) \cdot (\mathbf{r}_{\rm CC} \times \mathbf{v}_{\rm CC})}{\lambda\!\!\!^{-2}_{\rm C}c^2}\lambda\!\!\!^{-2}_{\rm C}\mathbf{v}_{\rm CC} && (21)\\
&= q_{\rm CC}(\mathbf{B} \cdot \mathbf{r}_{\rm CC})\mathbf{v}_{\rm CC}. && (22)
\end{aligned}$$

Apparently, $\boldsymbol{\tau}_{\rm CM}$ is rapidly oscillating (with angular frequency $2\,\omega_{\rm C}$) as $\mathbf{r}_{\rm CC}$ and $\mathbf{v}_{\rm CC}$ are spinning with angular frequency $\omega_{\rm C}$. At least for small B, $\omega_{\rm C}$ is expected to be much greater than the angular frequency of the spin precession; thus, it is reasonable to work with the time-averaged torque $\langle\boldsymbol{\tau}_{\rm CM}\rangle$ over one period of the spin motion. Furthermore, it is sufficient to compute this time-averaged torque for $t \approx 0$ since the precession continues at the same rate afterwards due to the symmetry of the scenario. Thus:

$$\begin{aligned}
\langle\boldsymbol{\tau}_{\rm CM}(t \approx 0)\rangle &= \frac{\omega_{\rm C}}{2\pi}\int_0^{2\pi/\omega_{\rm C}} \boldsymbol{\tau}_{\rm CM}(t')\mathrm{d}t' && (23)\\
&\approx \frac{\omega_{\rm C}q_{\rm CC}}{2\pi}\int_0^{2\pi/\omega_{\rm C}} (\mathbf{B} \cdot \mathbf{r}_{\rm CC}(t'))\mathbf{v}_{\rm CC}(t')\mathrm{d}t' && (24)\\
&\approx \frac{\omega_{\rm C}q_{\rm CC}}{2\pi}\int_0^{2\pi/\omega_{\rm C}} (-B\lambda\!\!\!^{-}_{\rm C}\sin(\omega_{\rm C}t'))(-c\sin(\omega_{\rm C}t'), 0, -c\cos(\omega_{\rm C}t'))^{\rm T}\,\mathrm{d}t' && (25)\\
&= \frac{\omega_{\rm C}q_{\rm CC}B\lambda\!\!\!^{-}_{\rm C}c}{2\pi}\int_0^{2\pi/\omega_{\rm C}} \left(\sin^2(\omega_{\rm C}t'), 0, \sin(\omega_{\rm C}t')\cos(\omega_{\rm C}t')\right)^{\rm T}\mathrm{d}t' && (26)
\end{aligned}$$

$$= \frac{\omega_C q_{CC} B \lambda\!\!\!^{-}_C c}{2\pi} \left[\left(\frac{t}{2} - \frac{\sin(2\omega_C t)}{4\omega_C}, 0, \frac{-1}{2\omega_C}\cos^2(\omega_C t) \right)^T \right]_0^{2\pi/\omega_C} \quad (27)$$

$$= \left(\frac{\omega_C q_{CC} B \lambda\!\!\!^{-}_C c}{2\pi} \frac{2\pi}{2\omega_C}, 0, 0 \right)^T \quad (28)$$

$$= \left(\frac{q_{CC} B \hbar}{2 m_{CM}}, 0, 0 \right)^T \quad (29)$$

Since $q_{CC} < 0$ for electrons (and $B > 0$ by definition), this means that spin $\mathbf{s}_{CM}$ precesses in the direction of the negative x axis for $t \approx 0$. With a positive angular frequency ω_p of the spin precession, this leads to

$$\frac{d\mathbf{s}_{CM}(t \approx 0)}{dt} \approx \langle \boldsymbol{\tau}_{CM}(t \approx 0) \rangle \quad (30)$$

$$\left(\frac{-\omega_p \hbar}{2}, 0, 0 \right)^T \approx \left(\frac{q_{CC} B \hbar}{2 m_{CM}}, 0, 0 \right)^T \quad (31)$$

$$\omega_p \approx \frac{|q_{CC}| B}{m_{CM}}. \quad (32)$$

Thus, ω_p in the bi-level electron model is approximately equal to the Larmor frequency for $g \approx 2$ as expected based on standard theory and experimental measurements. This result was derived without employing the electron's magnetic moment in order to show in detail how the external force term in Eq. (10) leads to spin precession.

5 Discussion and Future Work

As mentioned in Section 1, the main purpose of the bi-level electron model is to provide a simplified, but still quantitative version of a modified Born-Infeld model of electrons [Kra23]. The calculation of the spin precession in Section 4.3 shows that this objective has been achieved at least to some degree as calculations of trajectories of electrons in external electromagnetic fields appear to be significantly more complicated in the modified Born-Infeld model.

However, since the bi-level electron model is a simplified model, its explanatory power is limited; for example, it cannot explain why the mass m_{CC} of the point-like charge at $\mathbf{r}_{CC}$ is $m_{CM}/2$ nor why this charge can move at the speed of light. It is only in combination with the mentioned modified Born-Infeld model that the bi-level electron model can fulfill its purpose. The question whether the motion of the charge and mass at $\mathbf{r}_{CC}$ violates special relativity should, therefore, be discussed in the context of the modified Born-Infeld model. Within the bi-level electron model, however, one should note that the charge and mass at $\mathbf{r}_{CC}$ cannot move in a straight line nor can it move independently of the mass point $\mathbf{r}_{CM}$.

There are, of course, open questions about the bi-level electron model that should be addressed in future work. These include a relativistic version of the model and numerical simulations. Furthermore, the force term in Eq. (8) should be revised as the integral in Eq. (8) is neither physically justified nor fully explained by the modified Born-Infeld model. More generally, a proof that the bi-level electron model approximates (in some sense) the modified Born-Infeld model would be very useful.

6 Conclusion

This work presents the bi-level electron model, a new quantitative electron model, which was inspired by similar classical models of spinning electrons. In the new model, the calculation of the spin g-factor and the frequency of Larmor precession appears to be conceptually easier than in similar classical models of spinning electrons. Furthermore, the new model might serve as a simplified approximation to a modified Born-Infeld model of electrons, which might be an important step towards applying the latter model to a much wider range of problems. There are, however, several open questions about the new model that had to be left for future work.

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A Revisions

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