Can relativistic mass or weakening of force be measured with a vacuum tube?

Jorma Jormakka and Wolfgang Sturm
Contact address:jorma.o.jormakka@gmail.com

Abstract: Most physicists do not actually believe that mass depends on velocity as in the relativistic mass formula, but still keep this concept, for instance because an identical formula, energy-momentum relation, is seen as useful. However, there is no sense in keeping false concepts and valid results that have been derived by using false arguments can also be derived in other ways, even in a correct way. Yet, if an alternative explanation is proposed, it would be good to get a proposal for an experiment that can support the new explanation. The presented article discusses experiments that could add strength to the explanation that relativistic mass is actually a wrong interpretation of force becoming weaker if the object of the force is escaping the force with nearly the propagation speed of the force. The article considers if such an experiment could be done with a vacuum tube but no experimental results are given yet in this paper.

1. Introduction

Today it is not recommended to use the term relativistic mass $m = \gamma m_0$. Instead, one should say energy-momentum relation $E^2 = (pc)^2 + (m_0 c^2)^2$. These equations are identical. Physicists apparently do not think that the mass actually grows because of velocity but at the same time think that the relativistic mass formula is correct and proven by many experiments. They also think that Einstein proved $E = mc^2$ correctly though his proof is based on the relativistic mass formula and the mass in that proof grows in the rest frame.

We will see that those many experiments do not verify the relativistic mass formula, they refute it. There is an alternative explanation for the experiments, weakening of force. A force from a field, like a static electric field, has a propagation speed and if the object that it tries to accelerate moves away with a speed close to this propagation speed, the effect of the force should be much weaker. This mechanism does give the relativistic mass formula, but only for force acting on a mass perpendicular to the velocity of the mass.

Section 2 proves that the relativistic mass formula is wrong and explains how force weakening can give the relativistic mass formula for apparent mass if the force is perpendicular to the velocity of the mass.

Section 3 reviews literature experiments that are claimed to verify the relativistic mass formula (or the energy-momentum relation, if it sounds any better, it is the same thing).

The three later sections of the presented paper investigate approaches for measuring relativistic mass or weakening of force with a vacuum tube.

Section 4 modifies the derivation of Child-Langmoir equation for relativistic mass or weakening of force. There is an effect, but it is small.

Section 5 briefly discusses if the phenomenon of relativistic mass or weakening of force can be detected in the brightness plot of a vacuum tube. The section explains why vacuum tube anode current or brightness relation from voltage does not show relativistic mass or weakening of force: other effects that influence the anode current cover the small effect related to $m = \gamma m_0$.

Section 6 looks at an experiment that can detect between classical physics, relativistic mass and weakening of force though the effect may be too small to be measured with a vacuum tube.As Section 3 explains, this experiment has been made before many times and it supports the weakening of force explanation.

Results of experiments with the vacuum tube will hopefully be given in a forthcoming paper by the autors.

2. Explanation of the relativistic mass with a force weakening

Paper [1] proves that the relativistic mass formula $m = \gamma m_0$ is false. The proof is simply that the force coming from kinetic energy is given by the second term in the Euler-Lagrange equations and the force doing work is given by the definition of work. Setting the forces equal

$$F = \frac{d}{dt}\frac{\partial}{\partial v}E_k(v) = \frac{d}{ds}W \tag{1}$$

and setting $E_k = W$ as it must be for conservation of energy gives the equation (the partial derivative of $E_k(v)$ with respect to v is derivative with respect to v)

$$\frac{d}{dt}\frac{d}{dv}E_k(v) = \frac{dt}{ds}\frac{d}{dt}E_k(v). \tag{2}$$

Solving this equation proves that the mass m must be constant. For convenience of the reader the proof in [1] is repeated here. Denoting

$$y(v) = \frac{d}{dv}E_k(v) \tag{3}$$

yields Equation (2) can be written as

$$\frac{dt}{ds}\frac{d}{dt}E_k(v) = \frac{dv}{dt}\frac{d}{dv}y(v) \tag{4}$$

$$\frac{1}{v}\frac{dv}{dt}\frac{d}{dv}E_k(v) = \frac{dv}{dt}\frac{d}{dv}y(v) \tag{5}$$

$$\frac{1}{v}y(v) = \frac{d}{dv}y(v) \tag{6}$$

$$\frac{1}{v}dv = \frac{1}{y}dy \tag{7}$$

$$y(v) = cv \tag{8}$$

where c is constant. Then

$$E_k(v) = \int y dv = \frac{1}{2} c v^2 \tag{9}$$

and as (9) holds in the limit when v approaches zero, we can identify c as constant mass m. Thus, for work to equal energy that can do this work (as it must equal), mass cannot depend on velocity.

The mechanism of weakening of a force is explained in [2]. We repeat the essential part of this explanation here for the convenience of the reader.

Let us take a mass m moving to the positive x-axis with the speed v in the rest frame, the (x, y) coordinate system. The mass does not change with velocity. A field stationed in the rest frame exerts force to the mass. The propagation speed of the field is c in the rest frame. The field makes an interaction with the mass changing the momentum of the mass from some angle. This angle can be anything, the interaction can come from the direction parallel to the velocity v, from a perpendicular direction, or from any other direction.

To make the scenario clear: the mass is moving on the x-axis to the right. The mass is at $(0, 0)$ at the time $t = 0$. The source of the interaction is at the point (x, y), $y < 0$, in the negative y-half-plane, The distance of (x, t) from $(0, 0)$ is L. The line from (x, y) through $(0, 0)$ makes the angle ϕ with the x-axis. The angle ϕ can have any value from $-\pi$ to π.

The interaction changes both the mass's momentum and the field's. From this we conclude that the interaction is made with a two-way protocol of exchanging momentum. Let us propose the most simple two-way protocol for the interaction of the field with the mass with only two messages.

First the field sends a message to the mass, it takes time T_1. Then the mass sends a message to the field, it takes time T_2. When the mass receives the first message it is at the point $x = vT_1$. It sends the second message immediately, from the same point. The second message comes to the field at the time T_2. The total interaction time is $T = T_1 + T_2$.

From geometry we get the equations:

$$(cT_1)^2 = L^2 + (vT_1)^2 + 2LvT_1 \cos\phi \tag{10}$$

$$(cT_2)^2 = L^2 + (vT_2)^2 - 2LvT_2 \cos\phi. \tag{11}$$

Solving the equations gives

$$T_1 = \frac{Lv\cos\phi}{c^2 - v^2} + \frac{L\sqrt{c^2 - v^2 + v^2\cos^2\phi}}{c^2 - v^2} \tag{12}$$

$$T_2 = -\frac{Lv\cos\phi}{c^2 - v^2} + \frac{L\sqrt{c^2 - v^2 + v^2\cos^2\phi}}{c^2 - v^2} \tag{13}$$

and therefore

$$T = \frac{2L\sqrt{c^2 - v^2 + v^2\cos^2\phi}}{c^2 - v^2}. \tag{14}$$

Derivating T with respect to ϕ and setting the derivative to zero gives the equation for the extremal values of T as

$$\cos\phi\sin\phi = 0. \tag{15}$$

This means that the extremal points are at the values $\phi = n\pi$, $n = 0, 1, 2, ...$ for the maximum

$$T_{max} = \frac{2L}{c}\gamma^2 \tag{16}$$

and at the values $\phi = \frac{\pi}{2} + n\pi$, $n = 0, 1, 2, ...$ for the minimum

$$T_{min} = \frac{2L}{c}\gamma. \tag{17}$$

Let us now see that we get the relativistic mass formula as the apparent mass if the force acts perpendicularly to the velocity of the mass. Then the interaction by the field deviates the mass from its orbit to the y-direction. It adds the speed Δu to the mass m in one interaction of time T. The acceleration in the y-direction is

$$a_v = \frac{\Delta u}{T} \tag{18}$$

and as the interaction is perpendicular to the velocity of the mass, we use the formula (17) for T. The force is

$$F_v = ma_v = m\frac{\Delta u}{T} = m\frac{\Delta uc}{2L}\gamma^{-1}. \tag{19}$$

If the mass is at rest, $v = 0$ and the force is

$$F_0 = m\frac{\Delta uc}{2L}. \tag{20}$$

We see that

$$\frac{F_v}{F_0} = \gamma^{-1} \tag{21}$$

That is, if the rest mass is m_0, then the dynamic equation is $\gamma^{-1}F = m_0 a$, the force is weakened. We can understand the equation also as $F = (\gamma m_0)a = ma$ for $m = \gamma m_0$, the mass has grown.

3. What did the experiments verifying $m = \gamma m_0$ really show?

Albert Einstein was not the first to propose the relativistic mass formula. In his book *The meaning of relativity* he credits Hendrik A. Lorentz as being the first to have written the equation $m = \gamma m_0$. In 1904, Lorentz proposed that the mass in the direction of movement (longitudinal mass) is different from the mass in the prependicular direction of movement (transverse mass). He suggested that the longitudal mass is $m = \gamma^3 m_0$ while the transverse mass is $m = \gamma m_0$.

This idea of Lorentz is almost what [2] proposes: if a force accelerates a mass m_0 in a direction that is parallel to the constant velocity of m_0, the apparent mass is $m = \gamma^2 m_0$, while if the force accelerates a mass m_0 in the perpendicular direction to the constant velocity of m_0, then the apparent mass is $m = \gamma m_0$. In [2] (see Section 2), the mass is not increasing. It is the force that is decreasing. The apparent mass is caused by the force F being felt by a moving mass as $\gamma^{-2}F$ in the parallel direction and as $\gamma^{-1}F$ in the perpendicular direction. Then $\gamma^{-j}F = m_0 a$ $(j = 1, 2)$ can be interpreted as apparent mass $F = \gamma^j m_0 a$.

The Lorentz force of a magnetic field on a moving charged particle is perpendicular to the velocity of the particle. Therefore when deviating a mass moving on a straight line by a magnetic field, the apparent mass of the particle in the direction of acceleration caused by the deviating magnetic field is $m = \gamma m_0$.

In a particle accelerator, a charged particle with (rest) mass m_0 is accelerated with a magnetic field, i.e., with a force that is prependicular to the velocity of the particle. Therefore, if a charged particle is accelerated with a constant Lorentz force F over a distance s, using energy $E = Fs$, the mass gets the energy $\gamma^{-1}E$. This energy equals the kinetic energy of the particle $(1/2)m_0 v^2$ after it is accelerated. We write this equation with the apparent mass $m = \gamma m_0$

$$\gamma^{-1}E = \frac{1}{2}m_0 v^2 \qquad E = \frac{1}{2}\gamma m_0 v^2 = \frac{1}{2}mv^2. \tag{22}$$

If a charged particle is accelerated with a static electic field, then the force is parallel to the velocity of the particle. The force is seen as $\gamma^{-2}F$ by the mass and the apparent mass is $\gamma^2 m_0$.

There were early tests by J. J. Thomson (1881, 1893) and George F. C. Searle (1897) suggesting that a charged mass moving in a magnetic field depends on the field energy. In 1901 Walter Kaufmann published his analysis of beta particles that were created by a radiactive process and accelerated by a cathode tube, i.e., a static electic field. When the particles reached speed v, he deviated them with a magnetic field. What he should have found if the relativistic mass formula is the following. The force of the electric field is parallel to the velocity. Therefore he gives the particle of mass m_0 energy E and the kinetic energy of the particle is $E = (1/2)mv^2$ where $m = \gamma m_0$. The particle moves distance s with the constant speed $v = \sqrt{2E/m}$ in time $t = s/v = s\sqrt{m/2E}$. Then he deviates the particle with a constant Lorentz force F that causes the acceleration $a = F/m$. In constant acceleration a, the particle deviates the distance

$$h = \frac{1}{2}at^2 = \frac{1}{2}\frac{F}{m}\frac{s^2 m}{2E} = \frac{1}{2}\frac{s^2 F}{E}. \tag{23}$$

The mass cancels out. But Kaufmann did measure relativistic mass. We get relativistic mass from this experiment if we assume that the electric field gives the particle apparent mass $m_1 = \gamma^2 m_0$ and the magnetic field gives the particle the apparent mass $m = \gamma m_0$. Then

$$h = \frac{1}{2}at^2 = \frac{1}{2}\frac{F}{m_1}\frac{s^2 m}{2E} = \frac{1}{2}\frac{F}{\gamma m_0}\frac{s^2 m_0}{2E} \tag{24}$$

and Kaufmann could interpret the result in the following way: the electric field does not change the mass but as Thomson found, the magnetic field does increase the mass as $m = \gamma m_0$. But notice that in (24) γ is at denominator while in (23) it is a multiplicator. Kaufman measured that h grows when v grows, so (24) is wrong. It is also possible that Kaufmann accepted the idea by Lorentz that the longitudinal mass grows faster than transverse mass. Kaufmann considered the mass increase as apparent mass. Hermann Starke (1903) made an experiment similar to Kaufmann's and obtained the same result.

George F. C. Searle made a related measurement in 1897 and proposed a different electromagnetic mass formula for longitudinal mass. Max Abraham proposed a formula for transverse mass. Abraham's equation was later compared to the relativistic mass equation by Lorentz (and adopted by Einstein). Adolf Bestelmeyer set the electric field and the magnetic field in orthogonal positions and obtained a different apparent mass, but as in his experiment the electric force and magnetic force are orthogonal to velocity, he should have got the relativistic mass formula. Alfred Beuchener repeated the experiment in 1908 and the result supported the relativistic mass formula for transverse mass. Karl E. Hupka's, Günther Neumann's and Gharles-Eugene Guye's and Charles Lavanchy's results agreed with Kaufmann's result.

Modern measurement, notably by William Bertozzi (1962, 1964), have confirmed that in a particle accelerator the energy that a charged particle gets when accelerated with a magnetic field (i.e., force is perpendicular to the velocity) corresponds to apparent mass of $m = \gamma m_0$. This is what is expected, it does not verify the relativistic mass formula, it only verifies force weakening when the force is perpendicular to the direction of velocity.

Let us still mention particle accelerator experiments where two high velocity particles are collided and they produce a heavy particle. The mass of the heavy particle agrees with what the theory predicts, and consequenty the colliding particles must have increased mass. Or is it so? No. The theory is built to include the energy-momentum relation (it is e.g. in the derivation of the Dirac equation) and therefore the theory predicts an apparent mass for the heavy particle. This apparent mass agrees with apparent masses of the colliding particles. Yet, the real masses of these particles need not be their apparent masses. The experiment can also be explained with weakening of force.

We see that all early researchers concluded that the apparent mass in the transverse direction is $m = \gamma m_0$ and many of them concluded that the apparent mass growth is not the same in longitudinal and transverse directions. After Einstein's Special Relativity Theory the second observation was forgotten and the idea that it is only apparent mass was dropped.The formula $m = \gamma m_0$ in the form $E^2 = (pc)^2 + (m_0 c^2)^2$ was raised up to be a fundamental truth. Modern measurements have not added anything to the old experiments. They have only checked that the apparent mass in the transverse direction is $m = \gamma m_0$.

There have been many undergraduate experiments that verify the relativistic mass formula when the force is in the transverse direction to the velocity. These experiments are used to convince students that the energy-momentum relation is true. This is a bit curious because this experiment refutes the energy-momentum relation and it is a repetition of Kaufmann's experiment where Kaufmann, and the others, did realize that the mass is only apparent and that the transverse mass does not equal longitudinal mass as the energy-momentum relation claims.

The information about earlier measurements is from Wikipedia pages "Tests of relativistic energy and momentum" and "Kaufmann-Bucherer-Neumann experiments". References to the original articles are in those pages and there is no reason to clutter the references section of this presented paper with those articles. An original article should not have more than absolutely necessary references.

We continue to discuss experiments with a cathode ray tube.

4. Anode current as a function of voltage

Anode current I_a as a function of voltage U_a in the vacuum tube [3] roughly follows the $U_a^{3/2}$ law, but not exactly as Table 2 in Section 3 shows. Yet, from this table we can conclude that the operational area of the tube is at least partially in the space-charge-limited area and the anode current is to some degree modelled by the Child-Langmoir equation. We will proceed to modify the Child-Langmoir equation to include relativistic mass and weakening of force, and see if these possible effects are large enough to be detected.

The velocity v of electrons in a vacuum tube is classically calculated from energy as

$$\frac{1}{2} m_e v^2 = eU \tag{25}$$

where U voltage, d is the distance between cathode and anode, e is the charge of an electron and m_e is the mass of an electron. Solving v from this equation gives

$$v = v_0 = \sqrt{\frac{2eU}{m_e}} \tag{26}$$

and in (4) we named this velocity as v_0.

In order to include relativistic mass or weakening of force, we will derive v from force and not from energy as the equations are given for a force. For relativistic mass

$$\gamma m_e a = eE = e\frac{U}{d} \tag{27}$$

where a is acceleration and E is the electric field, and for weakening of force in direction parallel to velocity

$$m_e a = \gamma^{-2} eE \tag{28}$$

while for the perpendicular direction is it

$$m_e a = \gamma^{-1} eE \tag{29}$$

identical with the relativistic mass. In this section the force is parallel to the velocity v. We solve the equation (27)

$$a = \frac{dv}{dt} = \left(1 - \frac{v^2}{c^2}\right) \frac{eU}{m_e d} = A - \frac{A}{c^2} v^2 \tag{30}$$

in power series form

$$v = \sum_{i=0}^{\infty} c_{2i+1} t^{2i+1} \tag{31}$$

$$c_1 = A = \frac{eU}{m_0 d} = \frac{v_0^2}{2d}$$

$$c_{i+2i} = \frac{A}{c^2} \frac{-1}{2i+1} \sum_{j=0}^{i-1} c_{2j+1} c_{2i-2j+1}. \tag{32}$$

In this section we will use an approximation from the first two terms:

$$v = At - \frac{1}{3c^2}(At)^3. \tag{33}$$

This approximation is sufficient for seeing if the relativistic mass or weakening of force can be detected by measurements.

The distance d from cathode to anode is

$$d = \int v dt = \frac{1}{2} A t^2 - \frac{1}{12c^2} t^4 \tag{34}$$

Solving t^2 from this second order polynomial gives

$$t^2 = 3 \frac{c}{A}^2 \left(1 - \sqrt{1 - \frac{4}{3} \frac{A}{c^2} d}\right). \tag{35}$$

It is necessary to take the minus sign in the second order polynomial solution above, the plus sign gives v higher than c. Inserting A, the first order approximation gives

$$t^2 = 3 \left(\frac{c}{A}\right)^2 \frac{2}{3} \frac{A}{c^2} = \left(\frac{2d}{v_0^2}\right), \tag{36}$$

i.e.,

$$\frac{d}{t} = \frac{1}{2} v_0 \tag{37}$$

as when the acceleration is constant. Using this approximation for t, we can check that the power series (31) converges: A is on the range c^2, t is on the range c^{-1} and $c_i t^i$ is on the range At/c for every (odd) i. The sum in (31) gives an alternating finite sum which has low powers of 2 as numerators and factorials (or sort of, containing only odd numbers) as denominators, calculation of some first terms shows the pattern.

Inserting the first order approximation of t and the formula of A to v gives

$$v = v_0 \left(1 - \frac{1}{3}\frac{v_0^2}{c^2}\right). \tag{38}$$

We will make a similar calculation as in the derivation of the classical Child-Langmoir equation. The starting point is the Poisson equation

$$\nabla^2 U = -\frac{\rho}{\epsilon_0} \tag{39}$$

where

$$\rho = \frac{J}{v}. \tag{40}$$

Here J is the current density, we will later comment on how to get the anode current I_a.

The classical Child-Langmoir equation is derived in a 1-dimensional space and ∇ is the second derivative of x. By a standard trick

$$\frac{d}{dx}\left(\frac{dU}{dx}\right)^2 = 2\frac{d^2U}{dx^2}\frac{dU}{dx} \tag{41}$$

we get equation (39) to the easier form

$$\frac{d}{dx}\left(\frac{dU}{dx}\right)^2 = \frac{d^2U}{dx^2}\frac{dU}{dx} = -2\frac{J}{\epsilon_0 v}\frac{dU}{dx}. \tag{42}$$

Inserting v and v_0, and multiplying by dx gives

$$d\left(\frac{dU}{dx}\right)^2 = -\frac{2}{\epsilon_0}J\sqrt{\frac{m_e}{2e}}U^{-\frac{1}{2}}\left(1 - \frac{2}{3}\frac{eU}{m_e c^2}\right)^{-1} dU. \tag{43}$$

When integrationg this equation, the right side has an integral of the type

$$\int \frac{dx}{\sqrt{x}(1-bx)} = -\frac{1}{\sqrt{b}}\ln(1-\sqrt{bx}) + \frac{1}{\sqrt{b}}\ln(1+\sqrt{bx}) \tag{44}$$

which has an approximation by first two nonzero terms as

$$\int \frac{dx}{\sqrt{x}(1-bx)} = 2\sqrt{x} + \frac{2}{3}bx^{\frac{3}{2}} \tag{45}$$

Integrating (43), using the approximation (45), yields

$$\left(\frac{dU}{dx}\right)^2 = \frac{4}{\epsilon_0}(-J)\sqrt{\frac{m_e}{2e}}\left(U^{\frac{1}{2}} + \frac{2}{9}\frac{e}{m_e c^2}U^{\frac{3}{2}}\right). \tag{46}$$

Taking square roots (J is negative) and integrating

$$\frac{dU}{dx} = \left(\frac{4}{\epsilon_0}(-J)\sqrt{\frac{m_e}{2e}}\right)^{\frac{1}{2}} U^{\frac{1}{4}}\left(1 + \frac{2}{9}\frac{e}{m_e c^2}U\right)^{\frac{1}{2}}. \tag{47}$$

$$U^{-\frac{1}{4}}\left(1 + \frac{2}{9}\frac{e}{m_e c^2}U\right)^{-\frac{1}{2}} dU = \left(\frac{4}{\epsilon_0}(-J)\sqrt{\frac{m_e}{2e}}\right)^{\frac{1}{2}} dx. \tag{48}$$

We again take a first order approximation in the left side and integrating

$$\frac{4}{3}U^{\frac{3}{4}}\left(1 - \frac{e}{9m_e c^2}\frac{1}{7}U\right) = \left(\frac{4}{\epsilon_0}(-J)\sqrt{\frac{m_e}{2e}}\right)^{\frac{1}{2}} x. \tag{49}$$

Inserting boundary conditions $x = 0$ at $U = 0$, $x = d$ at $U = U_d$ and $dx/dt = 0$ at $U = 0$, and squaring we get a modified Child-Langmoir formula

$$J = -\frac{4}{9}\epsilon_0\sqrt{\frac{2e}{m_e}}U_d^{\frac{3}{2}}d^{-2}\left(1 - \frac{eU}{21m_e c^2}\right)^2. \tag{50}$$

This formula shows that weakening of force does show in anode current density J but for velocities of electrons that can be achieved in an ordinary vacuum tube the effect is too small. For the relativistic mass we get (by first order approximation)

$$v = v_0\left(1 - \frac{1}{6}\frac{v_0^2}{c^2}\right). \tag{51}$$

and the current density is

$$J = -\frac{4}{9}\epsilon_0\sqrt{\frac{2e}{m_e}}U_d^{\frac{3}{2}}d^{-2}\left(1 - \frac{eU}{42m_e c^2}\right)^2. \tag{52}$$

In principle an experiment that finds out if electrons behave classically, have relativistic mass, or if the force weakens with velocity, can be made, but detecting such small effects is challenging.

Anode current I_a is obtained from current density J by dividing by some area and it may be best to calibrate the equation with measured values instead of the constants in the formula. This appeared very clearly in the vacuum tube that was considered in experiments. In the literature one often sees the Child-Langmoir equation in the form

$$I_a = \frac{4}{9}\epsilon_0\sqrt{\frac{2e}{m_e}}U_d^{\frac{3}{2}}d^{-2} = KU_d^{\frac{3}{2}}d^{-2} \tag{53}$$

but when inserting $K = 0.002334\ mA/V^{\frac{3}{2}}$, $e/m = 0.1758819 * 10^{12}\ C/kg$ and values from the data sheet [3] page 5: if $U_a = 2500V$ then $I_a = 4.24mA$ gives d as c. eight meters, not as c. 14 inches what it is according to [3].

5. Can the brightness versus voltage show relativistic mass or weakening of force?

Let us consider what energy one electron transports from the cathode to the anode. Ignoring relativistic mass or weakening of the force, the electronic field gives the electron energy which is fully converted to kinetic energy according to the formula

$$E = eU = \frac{1}{2} m_e v_0^2 = W_k \tag{54}$$

The electron is accelerated with a constant force, thus acceleration is constant and the average velocity is $v_{ave} = v_0/2$. The time it takes for the electron to move from the cathode to the anode is

$$t = \frac{d}{v_{ave}} = \frac{2d}{v_0}. \tag{55}$$

The power is

$$P = \frac{W_k}{t} = \frac{\frac{1}{2} m_e v_0^2}{t} = \frac{eU}{t} = U\frac{e}{t} = UI_e. \tag{56}$$

There is no additional power coming from kinetic energy, the power comes from the electric field and it is converted to kinetic power which then is the electric power. From this equation we see what should happen if there is relativistic mass or if the force weakens: in both cases the same energy from the eletric field is accelerating the electron but as either the mass of the electron is larger or the force has less effect, the velocity of the electron will be smaller. There will be less anode current and less power.

For one electron we get the relations between anode current and power as functions of voltage as

$$I_a = \frac{e}{t} = \frac{ev_0}{2d} = \frac{e}{2d}\sqrt{\frac{2eU}{m_e}} = K_e U^{\frac{1}{2}} \tag{57}$$

$$P_e = UI_a = K_e U^{\frac{3}{2}}. \tag{58}$$

But these are not the anode current and power (i.e. brightness) that we can measure from a vacuum tube as functions of voltage or read from the data sheet [3]. Voltage releases electrons from the cathode, as does temperature (the data sheet [3] mentions heating voltage). In the area of the Child-Langmoir equation electrons are released as a linear function of voltage and we get the relations

$$I_a = KU^{\frac{3}{2}} \tag{59}$$

$$P = UI_a = KU^{\frac{5}{2}}. \tag{60}$$

The considered vacuum tube had a data sheet plotting brightness as a function of voltage. Brightness is, or should be, linearly dependent on power. The relation of brightness to voltage does not quite agree with the Child-Langmoir equation. The following figures are from the data sheet, we do not need more points to make a comparison:

Table 1

Voltage(V)	Brightness
1500	24
2000	54
2500	100

If power relates as $P = KU^\alpha$, then for any two voltages U_a, U_b, the brightness should linearly correspond to powers P_a and P_b, and

$$\alpha = \frac{\ln \frac{P_a}{P_b}}{\ln \frac{U_a}{U_b}}. \tag{61}$$

We get

$$\alpha = \frac{\ln \frac{54}{24}}{\ln \frac{2000}{1500}} = 2.819 \tag{62}$$

$$\alpha = \frac{\ln \frac{100}{24}}{\ln \frac{2500}{1500}} = 2.797 \tag{63}$$

$$\alpha = \frac{\ln \frac{100}{54}}{\ln \frac{2500}{2000}} = 2.761 \tag{64}$$

Notice that when we move to higher voltages, α seems to aproach the value 2.5 from the Child-Langmoir equation, but the operating area of the tube is not fully in the space-charge-limited area. We do not see any reduction of anode current or power, i.e., brightness, due to relativistic mass or weakening of force, and we could not see it: the effect would be too small. The anode current cannot be precisely given by equations. There are many different mechanisms that influence it and the Child-Langmoir equation is at most a limit equation for large voltages. Though it is relatively good for predicting anode current in the studied vacuum tube, see the table below, the equation does not accurately give the current. The values for Child-Langmoir are normalized to give correctly the anode current for 2500V as the equation is a limit for high voltage. For instance, for 1750V, $I_a = 4.24 * (1750/2500)\sqrt(1750/2500)$.

Table 2

Voltage(V)	Anode current(mA)	From Child-Langmoir(mA)
1000	1.11	1.07

1250	1.54	1.50
1500	2.02	1.97
1750	2.53	2.48
2000	3.03	3.03
2250	3.65	3.62
2500	4.24	4.24

We see from this table that the anode current from the data sheet is higher on smaller voltages than the one given by the Child-Langmoir equation when normalized to agree with the highest voltage. This is what should be expected: there are other mechanisms, like temperature, that release electrons.

We conclude that brightness and anode current plots in the data sheet [3] of the vacuum tube cannot answer the question of whether there is relativistic mass or weakening of force. The possible small effect of slightly smaller speed of electrons is covered by a much larger effect of how many electrons voltage releases from the cathode. We do see that the anode current was larger compared to voltage than what the Child-Langmoir equation gives in small voltages but the effect is too large to be caused by a minimal change in electron velocity.

6. Relativistic mass or weakening of force in perpendicular direction

In this section we discuss an experiment that does work. It is made already many times but not in the cathode ray tube that is considered for experiments. An electron is accelerated to a chosen speed and then deviated with a static electronic field that is in perpendicular direction to the velocity vector. The electron flies a distance s in the x-direction after having reached the intended speed v and we assume that it does not slow down. Whether the mass of the electron is constant $m = m_e$ or changes with velocity as in the relativistic mass formula, the speed that the accelerating electronic field $E = U/d$ gives is

$$v = \sqrt{\frac{2eU}{m}}. \tag{65}$$

The field E does not exist in the area where the electron moves, thus it moves with constant speed v to the x-direnction. The time it moves over the distance s is

$$t = \frac{s}{v} \quad (66).$$

The electron is deviated in the y-direction with another electrostatic field E_1 between two parallel plates. U_1 is the voltage difference between the plates and h is the distance of the plates. They cause an acceleration a of the electron to the y-direction and it moves distance l in the y-direction. Then

$$a = \frac{eE_1}{m} = \frac{eU_1}{mh} \tag{67}$$

$$l = \frac{1}{2}at^2 = \frac{1}{2}\frac{eU_1}{mh}\frac{s^2}{v^2} \tag{68}$$

Inserting v and simplifying

$$l = \frac{1}{2}\frac{eU_1}{mh}\frac{s^2 m}{2eU} = \frac{1}{4}\frac{s^2}{h}\frac{U_1}{U}. \tag{69}$$

We see that l in (69) does not depend on m. In the classical and relativistic mass cases the experiment gives the same result. However, let us look at the case of weakening of force.

Unlike in Section 4 where we needed v as a function of time, here we need v only at the end of the acceleration phase. We can interpret weakening of force in equation (17) as the mass m having the velocity dependent formula $m = \gamma_v^2 m_e$. That is,

$$v = \sqrt{\frac{2eU}{\gamma_v^2 m_e}}. \tag{70}$$

Notice that v is constant in this experiment and consequently $\gamma_v = (1-v^2/c^2)^{-1/2}$ is constant. We will write γ in formulas (17) and (18) with the name γ_v because there is also a velocity to the y-direction. This velocity in the y-direction in this experiment is very small, we do not need to consider the effect of force weakening on the velocity in y-direction.

The force that deviates the electron in the y-direction is given in (18) and we get

$$a = \frac{\gamma_v^{-1} eE_1}{m_e} = \frac{\gamma_v^{-1} eU_1}{m_e h} \tag{71}$$

The distance in y-direction is

$$l = \frac{1}{2}at^2 = \frac{1}{2}\frac{\gamma_v^{-1} eU_1}{m_e h}\frac{s^2}{v^2} \tag{72}$$

Inserting v

$$l = \frac{1}{2}\frac{\gamma_v^{-1} eU_1}{m_e h}\frac{s^2 \gamma_v^2 m_e}{2eU}. \tag{73}$$

Simplifying

$$l = \gamma_v \frac{1}{4}\frac{s^2}{h}\frac{U_1}{U}. \tag{74}$$

Because weakening of force depends on the direction (see section 2), it is different to a direction that is parallel or perpendicular to the velocity. This is why γ terms do not cancel as they do in the case of relativistic mass. We get a detectable difference in (73). The measured value can tell if the case is classical or Einsteinian, or if it is weakening of force.

The difference in (74) might be measurable by a vacuum tube like the one in [3]. In $U = 2500V$ γ is about $1 - 0.005 = 0.995$, half a percent difference in deviation distance. This experiment is essentially the same as Kaufmann's experiment, the difference being deviating an electron with a static electric field instead of a static magnetic field. The sense of repeating this experiment is that Kaufmann's experiment does not definitely tell if the measured deviation is as in (22). We know from modern experiments that the transverse mass is $m = \gamma m_0$, but the longitudial mass cannot be measured with a particle accelerator, those use a magnetic field to accelerate a charged particle. We have to use a cathode ray tube. The question is if the logitudinal mass agrees with $m = \gamma^2 m_0$ or e.g. as Lorentz suggested, $m = \gamma^3 m_0$. Kaufmann's experiment only tells that the longitudial mass grows faster than the transverse mass.

The plans for experiments in the continuation paper to this one focus on brightness. Possibly an empirical result is achievable.

7. References

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