# Sigma-8 Anomaly as Potential Evidence for Fractal Spacetime (Part 1)

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Abstract

The $\sigma_8$ anomaly of cosmology refers to discrepancies in the amplitude of matter fluctuations, where low redshift observations favor *weaker matter clustering* than predicted by the standard $\Lambda CDM$ model. Elaborating from the hypothesis of *continuous spacetime dimensions*, this work offers an alternative explanation of the anomaly, based on scale-dependent corrections to the evolution of density fluctuations. In our interpretation, the $\sigma_8$ anomaly can be viewed as observational evidence for fractal spacetime effects at cosmological scales.

**Key words:** standard cosmology, sigma 8 anomaly**,** fractal spacetime, evolving spacetime dimensions, dimensional fluctuations.

## 1. Introduction and Motivation

The $\sigma_8$ parameter of cosmology measures the root-mean-square (rms) amplitude of matter density fluctuations on the scale of $R = 8h^{-1}\text{Mpc}$, where $h$ represents the normalized Hubble parameter. It is defined as

$$\sigma_8^2 = \left\langle \delta^2 \right\rangle_R \tag{1}$$

where $\delta = \delta\rho / \rho$ is the perturbation in matter density, referred also to as *over density*. Its definition and Fourier decomposition are, respectively [6],

$$\delta(\vec{x},t) = \frac{\rho(\vec{x},t) - \bar{\rho}(t)}{\bar{\rho}(t)} \tag{2}$$

$$\delta(\vec{x},t) = \int d^3k \, \delta(\vec{k},t) \exp(i\vec{k}\cdot\vec{x}) \tag{3}$$

where (2) quantifies the local density deviation normalized to the average value.

Cosmic microwave background (CMB) data from the **Planck satellite mission** [1], targeting the early Universe, predicts a higher value of $\sigma_8$. In particular, based on the $\Lambda CDM$ model, the Planck mission measures *primordial density fluctuations* at redshift $z \propto 1100$, from which cosmological parameters (like $\sigma_8$, matter density $\Omega_m$, Hubble constant $H_0$) are inferred. Planck finds a relatively high value of $\sigma_8$, around

$$\boxed{\sigma_8^{Planck} \approx 0.83 \pm 0.01}$$

By contrast, cosmological surveys from weak gravitational lensing and galaxy clustering report a lower value of $\sigma_8$ [2 – 3, 13]. In particular, the **KiDS** (Kilo-Degree Survey) and **DES** (Dark Energy Survey) measure how the large-scale distribution of matter distorts the shape of distant galaxies. These surveys probe the late-time clustering amplitude of matter, roughly at redshifts $z \propto 0.3 - 0.7$. KiDs and DES find a systematically lower $\sigma_8$ than Planck, typically,

$$\boxed{\sigma_8^{KiDS/DES} \propto 0.75 - 0.78}$$

which is about $5 - 10\,\%$ lower than Planck's value.

In a nutshell, the disparity is at the $2-3\sigma$ level, it is called the "$\sigma_8$ tension" or "anomaly" and it boils down to the puzzle: *Why do density fluctuations appear to grow less efficiently than expected from the standard model of cosmology (the $\Lambda CDM$ model)?*.

Starting from the hypothesis of *continuous spacetime dimensions,* we offer here an alternative explanation of the anomaly, based on scale-dependent corrections to the evolution of density fluctuations. We speculate that the $\sigma_8$ anomaly can be viewed as *observational evidence for fractal spacetime effects at cosmological scales.*

A defining feature of *complex dynamics* [4 – 6, 17 - 19] is that spacetime dimensionality is no longer a fixed integer but a continuous variable running with the energy scale $\mu$ and fluctuating near $4$,

$$D(\mu)=4+\varepsilon(\mu),\ \ \left|\varepsilon(\mu)\right|<<1 \tag{4}$$

In this context, evolving fractality follows from *critical behavior in continuous dimensions* and emerges in two scenarios:

a) either far above the Standard Model scale and near the UV cutoff, or

b) near the crossover point to the IR limit of low-energy physics [14].

Breaking the smooth geometry of classical spacetime implies that e*ffective field theories* - such as the Standard Model, General Relativity and the $\Lambda CDM$ cosmology - are **limiting cases** of dynamics in an ever-changing multifractal background. As a result, cosmological observables (such as power spectrum, entropy, transport coefficients) are expected to acquire anomalous scaling exponents driven by the flow of dimensional deviation $\varepsilon(\mu)$.

The paper is divided into two parts and organized in the following way: next section maps the anomaly to the onset of evolving spacetime dimensions; section 3 ties the anomaly to generalized entropy concepts suitable for the description of complex dynamics processes. The second part of the paper elaborates on model building, numerical analysis and comparison of predictions with actual data.

## 2. Mapping the Anomaly to Evolving Spacetime Dimensions

The amplitude of density fluctuations in standard cosmology depends on the growth rate of perturbations,

$$f(a) \approx \Omega_m(a)^{\gamma} \tag{5}$$

in which $a$ is the expansion parameter, $\Omega_m$ the normalized matter density and $\gamma \approx 0.55$ [7 - 9]. Likewise, the amplitude of density fluctuations depends on the linear growth factor $\Delta(z)$, where $z$ stands for the redshift [7 - 9], that is,

$$\sigma_8(z) \propto \Delta(z) \tag{6}$$

A spacetime background with evolving continuous dimensions is associated with unconventional dynamic properties, such as,

a) **Fractional generalization** of the Poisson equation (FPE),

$$\nabla^2\Phi \to \nabla^{2+\varepsilon}\Phi \tag{7}$$

which changes the way density fluctuations affect gravitational potentials. Along with other similar equations deploying fractional derivatives and integrals, FPE is a powerful analytic tool for probing the Dark Matter sector of cosmology and for model building in Modified Newtonian Dynamics (MOND)-like theories [12, 15 - 17].

b) **Correction to the growth index** as in

$$\gamma \to \gamma + \Delta\gamma(\varepsilon) \tag{8}$$

with a potentially slower growth in structure for appropriate observation scales $\varepsilon(\mu) \propto \varepsilon(z)$.

c) **Fractal scaling**: while in ordinary 3D space the variance of density perturbations scales with the volume as

$$\left\langle \delta^2 \right\rangle \propto R^{-3} \tag{9a}$$

in fractal dimensions $D = 3 + \varepsilon$, (9a) becomes

$$\left\langle \delta^2 \right\rangle \propto R^{-D} \tag{9b}$$

which directly modifies $\sigma_8$.

By a) – c), if the effective dimension of spacetime deviates from 4, Planck's early-universe observations (assuming 4D standard cosmology) *overestimate* late-time clustering. By the same token and as previously mentioned, weak lensing surveys probe the critical regime of small redshifts $z$, measuring a smaller $\sigma_8$.

**3. Entropy production and multifractals**

A fractal spacetime background associated with the onset of complex dynamics is characterized by *non-extensive entropies*, such as Tsallis or Rényi entropies [6, 10 - 11]. They take the following form, respectively:

$$S_q^{(T)} = \frac{1 - \sum_i p_i^q}{q-1} \quad \text{(Tsallis)} \tag{10}$$

$$S_q^{(R)} = \frac{1}{1-q} \log(\textstyle\sum_i p_i^q) \quad \text{(Rényi)} \tag{11}$$

in which $q \in R$ is the non-extensivity parameter. Since $q = q(\varepsilon)$, (10) and (11) are expected to change the thermodynamics of the perturbation growth. As a result, over densities $\delta = \delta(\varepsilon)$ dissipate differently, which leads to suppressed clustering amplitudes compared to standard entropy scaling.

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