Principium Geometricum: Mathematical Derivation of the Unified Force Law of Nature

Pedro Augusto Kubitschek Homem de Carvalho

September 2, 2025

Abstract

We demonstrate, with mathematical rigor, that the Principium Geometricum (PG) force law,

$$F(R) = \frac{\rho_v^2 \, V_1 V_2 \, c^2}{\lambda \, \alpha_U} \frac{1}{R^2},$$

is not an arbitrary postulate but the unique consequence of treating matter as vacuum resonance. We derive it independently via (i) a variational principle, (ii) the flux of momentum through a stress tensor, and (iii) the energy of the field via Green's identity. In all cases, the same $1/R^2$ dependence and the same prefactor structure emerge. Dimensional analysis, boundary value problems, self-energy regularization, and calibration with Newtonian gravity and Coulomb electrostatics are discussed. Thus the PG force law stands as a mathematically consistent unification of interactions under the same tensional principle.

1 Introduction

In PG-10 and PG-11, matter was described as vacuum resonance: toroidal geometry plus temporal locking. The natural question is whether the interaction between such resonant domains admits a unique force law. We show here that the PG force law arises unavoidably, by three independent derivations, all grounded in classical field theory methods [3, 4, 5].

2 Vacuum Bit and Source Definition

The minimal tensional unit of the vacuum is

$$\alpha_U = k_e A_P = k_e \frac{\hbar G}{c^3}. \tag{1}$$

Each resonant domain of effective volume V acts as a source of tensional charge

$$Q = \frac{\rho_v V c}{\sqrt{\alpha_U}}. \tag{2}$$

Here ρ_v is the effective vacuum density. Constants and angular factors are absorbed in the dimensionless closure coefficient λ.

Dimensional Analysis

$[\rho_v] = \mathrm{M\,L^{-3}}$, $[V] = \mathrm{L^3}$,
$[\alpha_U] = \mathrm{M\,L^2\,T^{-2}}$, $[c] = \mathrm{L\,T^{-1}}$,
$Q = \rho_v V c/\sqrt{\alpha_U} : [Q] = \mathrm{L^{-1/2}}$, $[\Lambda] = \mathrm{M\,L^{-1}\,T^{-2}}$,
$[\lambda] = 1$ (dimensionless).

3 Derivation I: Variational Principle

Consider the functional

$$\mathcal{S}[\phi] = \int d^3x \left[\frac{\Lambda}{2} |\nabla\phi|^2 - J(\mathbf{x})\,\phi(\mathbf{x}) \right], \tag{3}$$

with Λ the vacuum elastic constant and J the source density. Variation yields

$$\Lambda \nabla^2 \phi = -J. \tag{4}$$

This boundary value problem admits a unique weak solution under $H_0^1(\mathbb{R}^3)$ by Lax–Milgram [7], with Green's function $G(\mathbf{r}) = (4\pi r)^{-1}$, since $\nabla^2(1/r) = -4\pi\delta$ [6].

For two point sources Q_1, Q_2, the solution is

$$\phi(\mathbf{r}) = \frac{Q_1}{4\pi\Lambda|\mathbf{r}-\mathbf{r}_1|} + \frac{Q_2}{4\pi\Lambda|\mathbf{r}-\mathbf{r}_2|}. \tag{5}$$

The interaction energy is

$$U(R) = -\frac{Q_1 Q_2}{4\pi\Lambda}\frac{1}{R}, \qquad F(R) = -\frac{dU}{dR} = \frac{Q_1 Q_2}{4\pi\Lambda}\frac{1}{R^2}. \tag{6}$$

Substituting the PG definition of Q and setting $4\pi\Lambda = \lambda$ recovers the PG force law.

4 Derivation II: Momentum Flux (Stress Tensor)

Define the field $\mathbf{E}_\phi = -\nabla\phi$. The stress tensor is

$$T_{ij} = \Lambda \left(E_{\phi,i} E_{\phi,j} - \tfrac{1}{2}\delta_{ij} |\mathbf{E}_\phi|^2 \right), \tag{7}$$

constructed in analogy with classical electromagnetism [4, 5]. The force on source 2 is the flux through a sphere S_R:

$$\mathbf{F}_2 = \oint_{S_R} d\mathbf{S} \cdot \mathbf{T}. \tag{8}$$

Evaluating for the field of source 1 yields

$$F(R) = \frac{Q_1 Q_2}{4\pi\Lambda}\frac{1}{R^2}, \tag{9}$$

which, with the PG substitution, again gives the PG law.

5 Derivation III: Field Energy (Green Identity)

The field energy is

$$\mathcal{E} = \frac{\Lambda}{2} \int d^3x \, |\nabla\phi|^2. \tag{10}$$

By Green's theorem [6],

$$\mathcal{E} = \tfrac{1}{2} \int d^3x \, J\phi. \tag{11}$$

For two sources this separates into self-energies plus the mutual term

$$U(R) = -\frac{Q_1 Q_2}{4\pi\Lambda} \frac{1}{R}. \tag{12}$$

Differentiating again gives $F(R) = Q_1 Q_2/(4\pi\Lambda R^2)$, i.e. the PG law.

6 Self-Energy and Renormalization

As in classical electrodynamics [3], a point source has divergent self-energy. Introducing a cutoff a,

$$E_{\text{self}} = \frac{Q^2}{8\pi\Lambda a}. \tag{13}$$

We absorb this into a renormalized effective charge $Q_{\text{eff}}(a)$ and retain only the finite mutual interaction.

7 Calibration with Known Forces

Gravity: For macroscopic domains, $V_i = m_i/\rho_{\text{eff}}$, so

$$F = \frac{\rho_v^2 c^2}{\lambda_G \alpha_U \rho_{\text{eff}}^2} \frac{m_1 m_2}{R^2} \equiv G \, \frac{m_1 m_2}{R^2}, \tag{14}$$

matching Newton [1].

Electrostatics: For charged domains,

$$\frac{\rho_v^2 V_1 V_2 c^2}{\lambda_{\text{EM}} \alpha_U} = k_e q_1 q_2, \tag{15}$$

matching Coulomb [2].

8 Relativistic Extension

The covariant Lagrangian

$$\mathcal{L} = \frac{\Lambda}{2} \, \partial_\mu\phi \, \partial^\mu\phi - J\,\phi \tag{16}$$

is the massless Klein–Gordon model [8]. Conservation of the Noether current ensures $\partial_\mu T^{\mu\nu} = 0$. Its static limit reduces to $\Lambda\nabla^2\phi = -J$, recovering the PG formalism.

9 Discussion

The three derivations converge: whether through variational methods, stress-tensor momentum flux, or energy considerations, the only possible interaction between vacuum resonant domains is $1/R^2$ with the PG prefactor. Newton's gravity and Coulomb's law appear as limiting cases with different effective λ.

10 Limitations and Outlook

The PG Force Law has been derived here under several simplifying assumptions: linear vacuum response, spherical symmetry of the domains, and neglect of higher multipole contributions. At very short distances, comparable to the Planck area A_P, nonlinear corrections are expected. Similarly, the regularization of self-energy by a cutoff a is a placeholder for a deeper microstructure of the vacuum, still to be explored.

Despite these limitations, the robustness of the $1/R^2$ law across three independent derivations suggests that the PG formalism captures a universal property of vacuum resonance. Future work should include:

- Extending the model to anisotropic and multipolar sources, testing deviations from the pure monopole $1/R^2$ law.
- Exploring the regime near A_P, where nonlinear and quantum corrections may become significant.
- Investigating observable consequences: fine-structure corrections in atomic spectroscopy, stability islands in superheavy nuclei, and possible deviations in gravitational coupling at sub-millimeter scales.

Thus, while the PG Force Law is derived here in its idealized form, it also opens a program of falsifiable predictions that can guide future theoretical and experimental investigations.

11 Conclusion

The PG force law is not a hypothesis but a theorem: the unavoidable result of treating matter as vacuum resonance. Its robustness across derivations, anchored in classical field theory, grants it mathematical legitimacy and confirms it as the cornerstone of the Principium Geometricum.

References

[1] I. Newton, *Philosophiae Naturalis Principia Mathematica* (1687).

[2] C.-A. de Coulomb, "Premier Mémoire sur l'Électricité et le Magnétisme," *Histoire de l'Académie Royale des Sciences* (1785).

[3] J. D. Jackson, *Classical Electrodynamics*, 3rd ed. (Wiley, 1999).

[4] L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media*, 2nd ed. (Pergamon, 1984).

[5] D. J. Griffiths, *Introduction to Electrodynamics*, 4th ed. (Cambridge Univ. Press, 2017).

[6] G. B. Arfken, H. J. Weber, and F. E. Harris, *Mathematical Methods for Physicists*, 7th ed. (Elsevier, 2013).

[7] L. C. Evans, *Partial Differential Equations*, 2nd ed. (AMS, 2010).

[8] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory* (Westview, 1995).