The Dirac equation is not Lorentz invariant

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Abstract: A favorite claim that editors of journals make when rejecting a manuscript proving serious errors in the Relativity Theory is that the Relativity Theory has been verified by countless experiments. The number of experiments that actually verify relativity may be much smaller than countless, as there are alternative explanations to of these all experiments. One claim related to the relativity theory is that the Dirac equation is Lorentz covariant. This claim has never been verified by any experiments. This article gives three different proofs that the Dirac equation is not Lorentz invariant. This is a simple mathematical question, all three proofs are simple mathematics.

Keywords: Lorentz invariance, Dirac equation, Lorentz transform, covariance.

1. The Dirac equation is not proven to be Lorentz covariant

The Dirac equation is

$$(i\hbar\gamma^{\mu}\partial_{\mu} - mc)\,\psi(x) = 0. \tag{1}$$

The Dirac matrices are

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{2}$$

$$\gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \tag{3}$$

$$\gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \tag{4}$$

$$\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \tag{5}$$

The partial differentials are

$$(\partial_0, \partial_1, \partial_2, \partial_3) = \left(\frac{1}{c}\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \tag{6}$$

The mass m in (1) is constant.

The Lorentz transform is

$$x' = \gamma(x - vt) \quad t' = \gamma(t - (v/c^2)x) \quad y' = y \quad z' = z \tag{7}$$

with the inverse transform

$$x = \gamma(x' + vt') \quad t = \gamma(t' + (v/c^2)x') \quad y = y' \quad z = z'. \tag{8}$$

In coordinates (t', x', y', z') the Dirac equation is

$$(i\hbar\gamma^\mu \partial_{\mu'} - mc)\, \psi'(t', x', y', z') = 0 \tag{9}$$

and in coordinates (t, x, y, z) the equation is

$$(i\hbar\gamma^\mu \partial_\mu - mc)\, \psi(t, x, y, z) = 0. \tag{10}$$

Equations (9) and (10) only mean that we can rename the coordinates (t, x, y, z) as (t', x', y', z') and still have the same equation with same solutions, only with new variable names. They have nothing to do with invariance of an equation in a coordinate transform, but we continue.

For any function $f(x, t) = f(x(x', t'), t(x', t'))$ holds

$$\frac{\partial}{\partial x'} f(x(x', t'), t(x', t')) = \left(\frac{\partial x}{\partial x'}\frac{\partial}{\partial x} + \frac{\partial t}{\partial x'}\frac{\partial}{\partial t} \right) f(x, t)|_{(x,t)=(x(x',t'),t(x',t'))}. \tag{11}$$

Let us write an operator

$$D' = \gamma^\mu \partial_{\mu'} = \gamma^0 \frac{1}{c}\frac{\partial}{\partial t'} + \gamma^1 \frac{\partial}{\partial x'} + \gamma^2 \frac{\partial}{\partial y'} + \gamma^3 \frac{\partial}{\partial z'} \tag{12}$$

then

$$D'\psi(x(x', t'), y(y'), z(z'), t(x', t')) = D'\psi(x', y', z', t') \tag{13}$$

$$= \left(\gamma^0 \frac{1}{c} \left(\frac{\partial t}{\partial t'}\frac{\partial}{\partial t} + \frac{\partial x}{\partial t'}\frac{\partial}{\partial x} \right) + \gamma^1 \left(\frac{\partial t}{\partial x'}\frac{\partial}{\partial t} + \frac{\partial x}{\partial x'}\frac{\partial}{\partial x} \right) + \gamma^2 \frac{\partial}{\partial y} + \gamma^3 \frac{\partial}{\partial z} \right) \tag{14}$$

$$\cdot\, \psi(x, y, z, t)|_{(x,y,z,t)=(x(x',t'),y(y'),z(z'),t(x',t'))} \tag{15}$$

Inserting the Lorentz transform (7) and simplifying gives

$$D'\psi(x', y', z', t') = \gamma^\mu \partial_{\mu'} \psi(x', y', z', t') \tag{16}$$

$$= \left(\left(\gamma^0\gamma + \gamma^1\gamma\frac{v}{c} \right) \frac{1}{c}\frac{\partial}{\partial t} + \left(\gamma^0\gamma\frac{v}{c} + \gamma^1\gamma \right) \frac{\partial}{\partial x} + \gamma^2 \frac{\partial}{\partial y} + \gamma^3 \frac{\partial}{\partial z} \right) \tag{17}$$

$$\cdot\, \psi(x, y, z, t)|_{(x,y,z,t)=(x(x',t'),y(y'),z(z'),t(x',t'))} \tag{18}$$

Notice that for $v = 0$ we have $(x, y, z, t) = (x', y', z', t')$, $\gamma = 1$, and

$$D\psi(x, y, z, t) = \gamma^\mu \partial_\mu \psi(x, y, z, t) \tag{19}$$

$$= \left(\gamma^0 \frac{1}{c}\frac{\partial}{\partial t} + \gamma^1 \frac{\partial}{\partial x} + \gamma^2 \frac{\partial}{\partial y} + \gamma^3 \frac{\partial}{\partial z}\right)\psi(x, y, z, t). \tag{20}$$

Thus, inserting the Lorentz transform shows that (9) can be written as

$$\left(i\hbar\gamma'^{\mu}\partial_\mu - mc\right)\psi(x'(x)) = 0 \tag{21}$$

where

$$\gamma'^0 = \gamma\left(\gamma^0 + \frac{v}{c}\gamma^1\right) \tag{22}$$

$$\gamma'^1 = \gamma\left(\frac{v}{c}\gamma^0 + \gamma^1\right) \tag{23}$$

$$\gamma'^2 = \gamma^2 \quad \gamma'^3 = \gamma^3. \tag{24}$$

The Dirac equation is Lorentz invariant if it can be shown that (21) can be put to form (10) by valid manipulations of (21).

The "proof" that the Dirac equation is invariant in a Lorentz transform (Lorentz covariant) tries to do just that. The "proof" inserts a transform S, a four-times-four constant matrix S, that depends on (constant) v, as follows

$$\left(i\hbar S\gamma'^{\mu}S^{-1}S\partial_\mu - Smc\right)\psi(x'(x)) = 0. \tag{25}$$

The scalar partial derivative ∂_μ and the constant mc commute with the constant valued matrix S, thus

$$\left(i\hbar S\gamma'^{\mu}S^{-1}\partial_\mu - mc\right)S\psi(x'(x)) = 0. \tag{26}$$

As $S\psi(x'(x))$ is just the unknown function to be solved, we can write it as $S\psi(x'(x)) = \psi(x)$. The equation is then almost as (1)

$$\left(i\hbar S\gamma'^{\mu}S^{-1}\partial_\mu - mc\right)\psi(x) = 0. \tag{27}$$

except for one problem: we should find S such that

$$S\gamma'^{\mu}S^{-1} = \gamma^{\mu}. \tag{28}$$

It is easy to prove that (28) is impossible. We should have

$$S\gamma\left(\gamma^0 + \frac{v}{c}\gamma^1\right)S^{-1} = \gamma^0 \tag{29}$$

$$S\gamma\left(\frac{v}{c}\gamma^0 + \gamma^1\right)S^{-1} = \gamma^1. \tag{30}$$

Solving $S\gamma^0 S^{-1}$ gives (γ^{-1} is the inverse of the Lorentz factor)

$$\frac{c}{v}S\gamma^0 S^{-1} + S\gamma^1 S^{-1} = \gamma^{-1}\frac{c}{v}\gamma^0 \tag{31}$$

$$\frac{v}{c}S\gamma^0 S^{-1} + S\gamma^1 S^{-1} = \gamma^{-1}\gamma^1 \tag{32}$$

Subtracting the first equation from the second gives

$$\left(\frac{v}{c}-\frac{c}{v}\right) S\gamma^0 S^{-1} = \gamma^{-1}\left(\gamma^1 - \frac{c}{v}\gamma^0\right). \tag{33}$$

In a similar manner we solve $S\gamma^1 S^{-1}$

$$S\gamma^0 S^{-1} + \frac{v}{c} S\gamma^1 S^{-1} = \gamma^{-1}\gamma^0 \tag{34}$$

$$S\gamma^0 S^{-1} + \frac{c}{v} S\gamma^1 S^{-1} = \gamma^{-1}\frac{c}{v}\gamma^1 \tag{35}$$

Subtracting the second equation from the first gives

$$\left(\frac{v}{c}-\frac{c}{v}\right) S\gamma^1 S^{-1} = \gamma^{-1}\left(\gamma^0 - \frac{c}{v}\gamma^1\right). \tag{36}$$

Summing (33) and (36) gives

$$\gamma\left(\frac{v}{c}-\frac{c}{v}\right) S\left(\gamma^0+\gamma^1\right) S^{-1} = \left(1-\frac{c}{v}\right)\left(\gamma^0+\gamma^1\right). \tag{37}$$

Simplifying

$$S\left(\gamma^0+\gamma^1\right) S^{-1} = \sqrt{\frac{c-v}{c+v}}\left(\gamma^0+\gamma^1\right). \tag{38}$$

Taking the determinant from both sides shows

$$\det\left(S\left(\gamma^0+\gamma^1\right)S^{-1}\right) = \det(S)\det\left(\gamma^0+\gamma^1\right)(\det S)^{-1} = \det\left(\gamma^0+\gamma^1\right) \tag{39}$$

while

$$\det\left(\sqrt{\frac{c-v}{c+v}}\left(\gamma^0+\gamma^1\right)\right) = \sqrt{\frac{c-v}{c+v}}\det\left(\gamma^0+\gamma^1\right). \tag{40}$$

There cannot be S satisfying (28). In some explanations of this "proof", the equation (28) is changed to $S\gamma'^{\mu}S^{-1} = \gamma^{\nu}$. This makes no difference: all steps (29) to (40) can be made, only the right side has γ^{ν_0} and γ^{ν_1} instead of γ^0 and γ^1. The result is the same: in (40) the left side does not depend on v while the right side does. The step (28) in the "proof" is impossible anyway it is tried, this is because (21) is not (10) in a different appearance and cannot be transformed to (10) by any valid transform or other manipulation as will be shown in the next section with a concrete example. The "proof" that the Dirac equation is Lorentz covariant is false. In the next two sections we give two proofs that the Dirac equation is not invariant under the Lorentz transform.

2. Proof by an example: the Dirac equation is not Lorentz invariant

First let us look at an example to understand what means for an equation to be invariant under a coordinate transform. Force $F = ma$ in Newtonian mechanics is invariant under the Galileo transform

$$x' = x - vt \quad t' = t. \tag{41}$$

We can verify that the force is invariant by calculating

$$t' = t \ \rightarrow \ dt' = dt \tag{42}$$

$$x' = x - vt \ \rightarrow \ dx' = dx - vdt \tag{43}$$

$$\frac{dx'}{dt'} = \frac{dx - vdt}{dt} = \frac{dx}{dt} - v \tag{44}$$

$$d\left(\frac{dx'}{dt'}\right) = d\left(\frac{dx}{dt}\right) \tag{45}$$

$$\frac{d^2x'}{dt'^2} = \frac{d\left(\frac{dx'}{dt'}\right)}{dt'} = \frac{d\left(\frac{dx}{dt}\right)}{dt} = \frac{d^2x}{dt^2}. \tag{46}$$

Thus

$$F'(x') = m\frac{d^2x'}{dt'^2} = m\frac{d^2x}{dt^2} = F(x). \tag{47}$$

This is invariance of an equation of motion under a coordinate transform: $F'(x') = F(x)$.

Next we continue to the Dirac equation. A simple solution to Dirac's equation, one of the free particle solutions, is

$$\psi_1(t, x, y, z) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} e^{-imc^2t/\hbar}. \tag{48}$$

This wave function solves (10) and if we replace t with t'

$$\psi(t', x', y', z') = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} e^{-imc^2t'/\hbar}. \tag{49}$$

it solves (9) but notice that $\psi_1(t, x, y, z)$ in (48) is not the same function as $\psi(t', x', y', z')$ in (49), unlike with the force in (47). Especially

$$\psi(t'(t, x)) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} e^{-imc^2(\gamma(t-(v/c^2)x)/\hbar} \tag{50}$$

Is not a solution to (10) as is see by trying

$$i\hbar\gamma^\mu\partial_\mu\psi(t'(t, x)) = mc \begin{pmatrix} \gamma \\ 0 \\ 0 \\ \gamma\frac{v}{c^2} \end{pmatrix} e^{-imc^2(\gamma(t-(v/c^2)x)/\hbar} \tag{51}$$

while

$$mc\psi(t'(t,x)) = mc \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} e^{-imc^2(\gamma(t-(v/c^2)x)/\hbar} \tag{52}$$

Let us reflect for a while what is the difference between (21) and (47). In both calculations we have started with an equation with x', t', for the Dirac equation this starting point is equation (9), inserted the coordinate transform and got an equation with coordinates x, t. The equation we got in (47) is the original equation of motion written with coordinates x, t, so it is like (10) for the Dirac equation. But unlike in (47), the result (21) is not the original Dirac equation in (10), the result is only a different form of (9). Equation (21) has the solution (49) while (10) has the solution (48). Consider what would be the result if the Dirac equation was not Lorentz invariant. Still we could write (9) and (10) and they would have the same form because we only renamed the coordinates. Then we would insert the Lorentz transfrom to get to an equation like (21). What would that equation look like if the Diract equation is not Lorentz invariant? Yes, it would not be (10), it would be a different equation, it would be (9) in a different form. This is exactly what we get.

No valid transform by a matrix S can change (21) to become (10), because to become (10), the solution for (21) must be a solution for (10).

In order to see it clearly, we will calculate a simple example for the free particle solution (48). It solves (10) because

$$\gamma^0\partial_0\psi_1(t) = \gamma^0 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \frac{1}{c}\frac{\partial}{\partial t} e^{-imc^2 t/\hbar} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \frac{1}{c}\frac{mc^2}{i\hbar} e^{-imc^2 t'/\hbar} = \frac{mc}{i\hbar}\psi_1(t) \tag{53}$$

$$\partial_1\psi_1(t) = \partial_2\psi_1(t) = \partial_3\psi_1(t) = 0. \tag{54}$$

Next we check if (50) is a solution to (21). First we calculate the terms of γ'^0

$$\gamma^0\partial_0\psi(t'(t,x)) = \gamma\frac{mc}{i\hbar}\psi_1(t'(t,x)) \tag{55}$$

$$\gamma^1\partial\psi_1(t) = \gamma^1 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \frac{\partial}{\partial x} e^{-imc^2 t'(t,x)/\hbar} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1 \end{pmatrix} \frac{mc^2}{i\hbar} e^{-imc^2 t'(t,x)/\hbar} \left(-\gamma\frac{v}{c^2}\right) \tag{56}$$

$$\partial_2\psi(t'(t,x)) = \partial_3\psi_1(t'(t,x)) = 0 \tag{57}$$

and with these we get

$$\gamma'^0\partial_0\psi(t'(t,x)) = \frac{mc}{i\hbar}e^{-imc^2t'(t,x)/\hbar}\begin{pmatrix}1\\0\\0\\-\frac{v}{c}\end{pmatrix}(\gamma)^2 \tag{58}$$

$$\gamma'^1\partial_1\psi(t'(t,x)) = \frac{mc}{i\hbar}e^{-imc^2t'(t,x)/\hbar}\begin{pmatrix}-\frac{v^2}{c^2}\\0\\0\\\frac{v}{c}\end{pmatrix}(\gamma)^2 \tag{59}$$

$$\gamma'^2\partial_3\psi(t'(t,x)) = \gamma'^3\partial_3\psi(t'(t,x)) = 0. \tag{60}$$

Summing (58), (59) and (60) shows that (50) is a solution to (21)

$$\gamma'^\mu\partial_\mu\psi(t'(t,x)) = \frac{m_0c}{i\hbar}\psi(t'(t,x)). \tag{61}$$

This is natural because (21) is only a rewriting of (9). But (50) is not a solution to (10) as is shown in (51) and (52). Equation (21) is not a rewriting of (10) and it cannot be modified to (10) with any valid transform S or otherwise. This proves that the Dirac equation is not Lorentz invariant. The next section gives another proof.

3. About invariance of an equation under a coordinate transform

Let us consider a coordinate transform by matrix M with $\det M = ad - cb = 1$

$$\begin{pmatrix}x'\\t'\end{pmatrix} = M\begin{pmatrix}x\\t\end{pmatrix} \quad M = \begin{pmatrix}a & b\\c & d\end{pmatrix} \quad M^{-1} = \begin{pmatrix}d & -b\\-c & a\end{pmatrix} \tag{62}$$

The requirement for a first order operator to be invariant under the coordinate transform M is

$$\alpha\frac{d}{dx'} + \beta\frac{d}{dt'} = \alpha\frac{d}{dx} + \beta\frac{d}{dt} \tag{63}$$

Writing

$$\alpha\frac{d}{dx'} + \beta\frac{d}{dt'} = \alpha\left(\frac{dt}{dx'}\frac{d}{dx} + \frac{dt}{dx'}\frac{d}{dt}\right) + \beta\left(\frac{dt}{dt'}\frac{d}{dx} + \frac{dt}{dt'}\frac{d}{dt}\right) \tag{64}$$

$$= \alpha\left(d\frac{d}{dx} - c\frac{d}{dt}\right) + \beta\left(-b\frac{d}{dx} + a\frac{d}{dt}\right) = (\alpha d - \beta b)\frac{d}{dx} + (\beta a - \alpha c)\frac{d}{dt} \tag{65}$$

shows that the following equations must hold

$$\alpha d - \beta b = \alpha \quad \beta a - \alpha c = \beta \tag{66}$$

Solving

$$\beta = \alpha \frac{d-1}{b} = \alpha \frac{c}{a-1} \tag{67}$$

we see that α cancels and the only condition is that $d = 2 - a$

$$(d-1)(a-1) = bc \quad \rightarrow \quad da - bc + 1 = a + d \quad \rightarrow \quad d = 2 - a \tag{68}$$

The coefficients α and β can be scalars, vectors, matrices, anything as long as they are constants. As the Dirac equation is of the first order and this calculation is so elementary, it seems strange that anybody even suggested that the Dirac equation could be invariant under the Lorentz transform. The Lorentz transform

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = M \begin{pmatrix} x \\ t \end{pmatrix} \qquad M = \begin{pmatrix} \gamma & -v\gamma \\ -\frac{v}{c^2}\gamma & \gamma \end{pmatrix} \qquad \det M = 1 \tag{69}$$

obviously does not satisfy $d = 2 - a$ except in the case that $v = 0$. The Galileo transform satisfies this condition

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = M \begin{pmatrix} x \\ t \end{pmatrix} \qquad M = \begin{pmatrix} 1 & -v \\ 0 & 1 \end{pmatrix} \qquad \det M = 1. \tag{70}$$

Let us als look at the requirement for a second order operator to be invariant under the coordinate transform M

$$\alpha \frac{d^2}{dx'^2} + \beta \frac{d^2}{dt'^2} = \alpha \frac{d^2}{dx^2} + \beta \frac{d^2}{dt^2} \tag{71}$$

Writing

$$\alpha \frac{d}{dx'} + \beta \frac{d}{dt'} = \alpha \left(\frac{dt}{dx'} \frac{d}{dx} + \frac{dt}{dx'} \frac{d}{dt} \right) \left(\frac{dt}{dx'} \frac{d}{dx} + \frac{dt}{dx'} \frac{d}{dt} \right) \tag{72}$$

$$+ \beta \left(\frac{dt}{dt'} \frac{d}{dx} + \frac{dt}{dt'} \frac{d}{dt} \right) \left(\frac{dt}{dt'} \frac{d}{dx} + \frac{dt}{dt'} \frac{d}{dt} \right) \tag{73}$$

$$= \alpha \left(d\frac{d}{dx} - c\frac{d}{dt} \right) \left(d\frac{d}{dx} - c\frac{d}{dt} \right) + \beta \left(-b\frac{d}{dx} + a\frac{d}{dt} \right) \left(-b\frac{d}{dx} + a\frac{d}{dt} \right) \tag{74}$$

$$= \left(d^2\alpha + b^2\beta \right) \frac{d^2}{dx^2} + \left(c^2\alpha + a^2\beta \right) \frac{d^2}{dt^2} - 2\left(cd\alpha + ab\beta \right) \frac{d}{dx}\frac{d}{dt}. \tag{75}$$

We get the two equations

$$d^2\alpha + b^2\beta = \alpha \tag{76}$$

$$c^2\alpha + a^2\beta = \beta \tag{77}$$

and a third equation which may be needed depending on the function on which this operator acts

$$cd\alpha + ab\beta = 0 \tag{78}$$

Solving equations (76) and (77) gives

$$-\beta = \alpha\frac{c^2}{a^2-1} = \alpha\frac{d^2-1}{b^2} \tag{79}$$

Coefficient α cancels and

$$b^2c^2 = (1-ad)^2 = a^2d^2 - a^2 - d^2 + 1 \tag{80}$$

$$-2ad = -a^2 - d^2 \quad \rightarrow \quad d = a. \tag{81}$$

Thus, $d = a$. The condition that $\det M = 1$ gives another condition

$$c = \frac{a^2-1}{b}. \tag{82}$$

The transform M must be of the form

$$M = \begin{pmatrix} a & b \\ (a^2-1)/b & a \end{pmatrix}. \tag{83}$$

The Galileo transform has this form since in it $a = 1$ and $b = v$ (but $a = \pm 1$ causes divide by zero and it allowed only as a limit). The Lorentz transform also does have the correct form.

If the function that the operator acts on is a linear function of x' and t':

$$f(x', t') = kx' + rt' \tag{84}$$

then the term

$$\frac{d}{dx}\frac{d}{dt}f(x'(x,t), t'(x',t')) = 0 \tag{85}$$

and we can forget equation (78). This is the case e.g. in (47), invariance of $F = ma$ under the Galileo transform. If the function f is not linear in x' and t', then because of (78)

$$\beta = -\frac{cd}{ab}\alpha \tag{86}$$

and only a second order differential operator of the type in (71) satisfying this condition is invariant under M.

4. Why should anything be invariant under the Lorentz transform?

Not much is invariant under the Lorentz transform, but we can always mention the ratio of the space difference to the time difference. Because of the construction of the Lorentz transform, for light-like world paths holds

$$c' = \frac{x_2' - x_1'}{t_2' - t_1'} = \frac{x_2 - x_1}{t_2 - t_1} = c. \tag{87}$$

Notice that while c is the speed of light in the (x,t) frame, c' is not the speed of light in the (x',t') frame. The coordinate system (x',t') has the relation

$$t' = \gamma^{-1}t - \frac{v}{c^2}x'. \tag{88}$$

The relation (88) is obtained by inserting $x = \gamma^{-1}x' + vt'$ into $t' = \gamma(t-(v/c^2)x)$. This relation means that (x',t') is a coordinate system with a local time t'. It is not the time on the coordinate axis, the projection of (x',t') on the t'-axis is

$$Pr_{t'}(x',t') = \gamma^{-1}t = t' + \frac{v}{c^2}x'. \tag{89}$$

Coordinate systems with a local time, like our familiar timezone time, do not satisfy the condition that the coordinate system has independent coordinates. This condition means the following: a coordinate system $(x_1, ..., x_n)$ has independent coordinates if the projection of the point $(x_1, ..., x_n)$ on the jth coordinate axis is x_j. This condition must not be confused with linear independence of the coordinate vectors. The coordinate vectors are linearly independent and the coordinate system has orthogonal coordinate axes. The condition speaks about the value on the coordinate axis. In a local time coordinate system the value of time depends on the place. In such a coordinate system it is necessary to take projections on coordinate axes, one cannot take the number from the n-tuple $(x_1, ..., x_n)$ and use it as the projection.

The speed of light in the (x',t') frame is

$$\frac{Pr_{x'}(x_2' - x_1')}{Pr_{t'}(T_2' - t_1')} = \frac{\gamma(x_2 - x_1)}{\gamma^{-1}(t_2 - t_1)} = \gamma^2(c \pm v). \tag{90}$$

The Lorentz transform does not make the speed of light equal c in all inertial coordinate systems. This is why it is totally unnecessary to demand Lorentz invariance from any equations of motion.

5. A note on the Dirac equation

A supporter of the relativity theory once claimed to me that if the relativity theory is not valid, then also the Dirac equation and all good that follows from it, including antimateria, fine-structure constant, quantum gauge field theories that use spinor fields and more, would also be wrong as the Dirac equation is relativistic, i.e., Dirac tried to make the equation to fully comply with Special Relativity. This supporter of the relativity theory also seemed to think that as these fine things seem to work, they in some way "prove" that the relativity theory is correct, or that at least the countless physical experiments that agree with the relativity theory "verify" it. None of these fine developments that have come from the Dirac equation need to be discarded because of pointing out to any error in the relativity theory, and no finite set of experiments where the theory predicts correctly can ever prove or verify a theory, experiments can only falsify a theory. There can always be alternative explanations and in the case of the relativity theory, there really are alternative explanations. They are not difficult to find.

Failure to be Lorentz covariant does no harm to the Dirac equation. This equation is not correctly derived, its derivation uses the same heuristic substitutions that the Schrödinger equation introduced. Therefore the Dirac equation is heuristic and its value is solely determined by its usefulness. The Dirac equation is heuristic and it remains heuristic after dropping the false claim that it is Lorentz covariant. It is certainly not, and it is a mathematical issue, not "proven" by experiments, or well described by standard textbooks, as another supporter of the relativity theory once claimed. The three proofs that the Dirac equation is not Lorentz invariant settle the case.

As for the fine-structure constant, it can be derived from the Dirac equation and it is the third term in the power series composition of mc^2, but exactly the same correction can be obtained if the kinetic energy is not $E_k = mc^2 - m_0c^2 = (\gamma - 1)m_0c^2$ but instead $E_k = \gamma^{1.5}m_0v^2$, where $\gamma^{1.5}m_0$ is the longitudinal (apparent) mass, see [1] and also [2][3].

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