The Dirac equation is not Lorentz covariant and it is not dependent on Special Relativity

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Abstract: Papers [1] and [2] prove beyond any doubt that Special Relativity Theory (SRT) and the relativistic mass equation are false. A natural question is what happens to the Dirac equation that certainly one would like to save. Some people even go so far as to suggest that since the Dirac equation works in many applications, it in some way proves that SRT is correct. They point to the Dirac equation being Lorentz invariant and derived to fullfil the energy-momentum relation of SRT. These kind of claims are false: the Dirac equation is not Lorentz invariant (or covariant) and it does not essentially depend on the energy-momentum relation in SRT and can be used in many applications. This short paper proves these two assertions.

1. The Dirac equation is not Lorentz invariant

The Dirac equation is

$$(i\hbar\gamma^\mu\partial_\mu - m)\,\psi = 0. \tag{1}$$

The Dirac matrices are

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{2}$$

$$\gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \tag{3}$$

$$\gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \tag{4}$$

$$\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \tag{5}$$

The partial differentials are

$$(\partial_0, \partial_1, \partial_2, \partial_3) = \left(\frac{1}{c}\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \tag{6}$$

Let us notice that m in (1) is constant, it is the rest mass. This is seen by looking at the derivation of the Dirac equation. It starts from the energy-momentum relation that is given in the following form

$$E^2 = (pc)^2 + (mc^2)^2 \tag{7}$$

and the construction goal of the Diract equation is that it is fully compatible with SRT. In SRT

$$E = mc^2 = \gamma m_0 c^2 \tag{8}$$

Notice that here γ is the Lorentz factor, not to be confused with Dirac matrices that unfortunately have upper indices that may get confused with powers of the Lorentz factor. So,

$$\gamma^{-2} E^2 = m_0^2 c^4 + \left(m_0^2 c^2 v^2 - m_0^2 c^2 v^2\right) \tag{9}$$

$$\gamma^{-2} E^2 = (m_0 v)^2 c^2 + m_0^2 c^4 \left(1 - \frac{v^2}{c^2}\right) \tag{10}$$

$$\gamma^{-2} E^2 = \gamma^{-2} (mv)^2 c^2 + (m_0 c^2)^2 \gamma^{-2} \tag{11}$$

$$E^2 = (pc)^2 + (m_0 c^2)^2. \tag{12}$$

This shows that m in (7) and therefore in (1) must be m_0.

In (1) we can name our coordinates any way we want, and let us call the coordinates of the inertial coordinate system of the laboratory on the Earth (well, the Earth rotates, but slowly) with the names (x', y', z', t') and let us imagine another inerial coordinate system (x, y, z, t) that moves with some speed $v \geq 0$ to the direction of the negative x'-axis. There is the Lorentz transform relating these coordinate systems

$$x' = \gamma(x - vt) \qquad t' = \gamma(t - (v/c^2)x) \qquad y' = y \qquad z' = z \tag{13}$$

$$x = \gamma(x' + vt') \qquad t = \gamma(t' + (v/c^2)x') \qquad y = y' \qquad z = z'. \tag{14}$$

As the coordinate system (x, y, z, t) is imagined, choosing a different v cannot possibly change the wave function ψ in (1) and we can use the Dirac matrices (2)-(5) for any v we choose. Especially, if we choose $v = 0$, the frame (x, y, z, t) is the same as (x', y', z', t'). For any choice of v, ψ in (1) is the same function, it is only expressed in different coordinates.

Notice next that for any function $f(x, t) = f(x(x', t'), t(x', t'))$ holds

$$\frac{\partial}{\partial x'} f(x(x', t'), t(x', t')) = \left(\frac{\partial x}{\partial x'}\frac{\partial}{\partial x} + \frac{\partial t}{\partial x'}\frac{\partial}{\partial t}\right) f(x, t)|_{(x,t)=(x(x',t'),t(x',t'))}. \tag{15}$$

Let us write an operator

$$D' = \gamma^\mu \partial_{\mu'} = \gamma^0 \frac{1}{c}\frac{\partial}{\partial t'} + \gamma^1 \frac{\partial}{\partial x'} + \gamma^2 \frac{\partial}{\partial y'} + \gamma^3 \frac{\partial}{\partial z'} \tag{16}$$

then

$$D'\psi(x(x',t'),y(y'),z(z'),t(x',t')) = D'\psi(x',y',z',t')$$
$$= \left(\gamma^0 \frac{1}{c}\left(\frac{\partial t}{\partial t'}\frac{\partial}{\partial t} + \frac{\partial x}{\partial t'}\frac{\partial}{\partial x}\right) + \gamma^1\left(\frac{\partial t}{\partial x'}\frac{\partial}{\partial t} + \frac{\partial x}{\partial x'}\frac{\partial}{\partial x}\right) + \gamma^2\frac{\partial}{\partial y} + \gamma^3\frac{\partial}{\partial z}\right)$$
$$\cdot\psi(x,y,z,t)|_{(x,y,z,t)=(x(x',t'),y(y'),z(z'),t(x',t'))} \tag{17}$$

Inserting the Lorentz transform (14) and simplifying gives

$$D'\psi(x',y',z',t') = \gamma^\mu \partial_{\mu'}\psi(x',y',z',t')$$
$$= \left(\left(\gamma^0\gamma v + \gamma^1\gamma\right)\frac{1}{c}\frac{\partial}{\partial t} + \left(\gamma^0\gamma\frac{v}{c} + \gamma^1\gamma\frac{v}{c^2}\right)\frac{\partial}{\partial x} + \gamma^2\frac{\partial}{\partial y} + \gamma^3\frac{\partial}{\partial z}\right)$$
$$\cdot\psi(x,y,z,t)|_{(x,y,z,t)=(x(x',t'),y(y'),z(z'),t(x',t'))} \tag{18}$$

Notice that for $v=0$ we have $(x,y,z,t)=(x',y',z',t')$, $\gamma=1$, and

$$D\psi(x,y,z,t) = \gamma^\mu \partial_\mu \psi(x,y,z,t)$$
$$= \left(\gamma^1\frac{1}{c}\frac{\partial}{\partial t} + \gamma^1\frac{v}{c^2}\frac{\partial}{\partial x} + \gamma^2\frac{\partial}{\partial y} + \gamma^3\frac{\partial}{\partial z}\right)\psi(x,y,z,t). \tag{19}$$

Equation (1) for (19) is

$$i\hbar D\psi(x,y,z,t) = m_0\psi(x,y,z,t). \tag{20}$$

For (18) the equation is

$$i\hbar D'\psi(x,y,z,t)|_{(x,y,z,t)=(x(x',t'),y(y'),z(z'),t(x',t'))}$$
$$= m_0\psi(x,y,z,t)|_{(x,y,z,t)=(x(x',t'),y(y'),z(z'),t(x',t'))}. \tag{21}$$

The wavefunction $\psi(x,y,z,t)$ is the same in (20) and (21) before (x,y,z,t) are expressed as functions of (x',y',z',t') in (21). ψ is all the time the same function as imagining another inertial frame cannot change ψ. Equation (21) is not possible to fullfil for a freely chosen v, $0 \leq v < c$, because it depends on v, directly and through the Lorentz factor γ. We conclude that the Diract equation is not Lorentz invariant: equation (21) is not fulfilled, for (x',y',z',t') the Dirac equation has some different form than for (x,y,z,t) if $v>0$.

The "proof" that the Dirac equation is Lorentz covariant has a fundamental thinking error, deriving from raising and lowering indices in tensor calculus. But raising and lowering indices is not the case here. In that "proof" it is claimed that the Dirac matrices are transformed as

$$\gamma^{\mu'} = U^{-1}\gamma^\mu U \tag{22}$$

where U is some unitary matrix. There is no need at all to change the Dirac matrices because we are imagining a frame (x,y,z,t) that moves fast with respect to our frame (x', y', z', t'). It cannot change anything in our frame. Secondly, it is suggested that the wave function would transform as

$$\psi' = U\psi. \tag{23}$$

This is an even more absurd idea. If we imagine a frame (x, y, z, t), then it does not change the wavefunction that we have. It is not transformed by any unitary matrix. The only thing that is done when changing coordinates is expressing the old coordinates as functions of the new coordinates and inserting these expressions to the wavefunction. It is just like changing any function that is expressed as $f(x, y, z, t)$ into a function of (x', y', z', t'), that is, express it as $f(x(x', y'z', t'), y(x', y'z', t'), z(x', y'z', t'), t(x', y'z', t'))$ and then it is in the coordinates (x', y', z', t'):

$$f(x(x', y'z', t'), y(x', y'z', t'), z(x', y'z', t'), t(x', y'z', t')) = f(x', y', z', t'). \tag{24}$$

2. Discarding $m = \gamma m_0$ does not discard Dirac's equation

Among several objections why SRT cannot be wrong is the claim that if SRT is wrong and Lorentz invariance is not required, then much of quantum physics, including the Dirac equation, also go down the drain and those so beautiful equations do work.

This is not the situation at all. Discarding false claims does not throw out correct results, the only thing that happens is that they must be justified, that is derived, differently.

In the particular case of the Dirac equations, we do acknowledge that Dirac was inspired by SRT and wanted to formulate an equation that was fully relativistic. But he did not write a Lorentz covariant equation, as we saw already. He also did not use anything else than the energy-momentum relations in the derivation of the Dirac equation.

Dirac started like Klein and Gordon from

$$E^2 = p^2c^2 + m^2c^4. \tag{25}$$

Let us again remind that as the Dirac equation intentionally fulfills SRT, $m = m_0$ in (25). Then, due to several problems in the Klein-Gordon equations, Dirac wrote a matrix equation

$$E^2 1_4 = (\gamma^0 pc + \gamma^1 pc + \gamma^2 pc + \gamma^3 pc)^2 \tag{26}$$

and managed to find four-times-four matrices γ^μ that satisfy (26). Then he followed Klein and Gordon replacing

$$E \to -i\hbar\frac{\partial}{\partial t} \qquad p \to -i\hbar\nabla$$

and the Dirac equation follows. The only place where he needed SRT is (25). This equation was not originally invented by Einstein. It was empirically found by many researchers in the beginning of the 20th century. Lorentz probably was the first to write down the relativistic mass formula.

However, the findings did not prove that $m = \gamma m_0$ is true. They proved that $m = \gamma m_0$ is false, only the experiments were not correctly interpreted. In these experiments an electron was speeded up to a high speed v with some energy E, thus

$$\frac{1}{2}mv^2 = E. \tag{27}$$

You are free to think of the mass in (27) as classical mass m_0 or as relativistic mass $m = \gamma m_0$. When the electron reached the speed v, it continued with constant speed and in time t it made the distance s

$$s = vt \qquad t^2 = \frac{s^2}{v^2} = \frac{s^2 m}{2E}. \tag{28}$$

Then the electron was deviated from its straight orbit with the Lorentz force, i.e., magnetic force that acts perpendicular to the velocity of the electron. This force was constant as the electron has a constant charge and the velocity was constant. A constant force causes constant acceleration to the direction perpendicular to the velocity. The speed that the electron reaches because of this acceleration in the perpendicular direction to v is small, so a classical treatment is quite sufficient. The acceleration is according to a classical formula

$$a = \frac{F}{m}. \tag{29}$$

Because of this perpendicular acceleration the electron deviates from its orbit and we can measure this deviation. In constant acceleration the electron moved distance h

$$h = \frac{1}{2}at^2 = \frac{1}{2}\frac{F}{m}\frac{s^2 m}{2E} = \frac{1}{4}\frac{s^2 F}{E}. \tag{30}$$

We see that m cancels out. If the relativistic mass formula $m = \gamma m_0$ is true, then none of these experiments could have found any relativistic mass, but all of them did. The solution proposed in [2] gives just this result: the mass growth is only apparent, actually force weakens, a force of a field that has the propagation speed c cannot effectively give this force to a particle that moves with a speed close to c. If the force acts from a direction that is perpendicular to the velocity of the particle, as it does in (29), then we get the apparent mass that follows the equation $m = \gamma m_0$. But if the force comes from the direction that is parallel to the velocity, as it does in (27), then the apparent mass is $m = \gamma^2 m_0$. This difference of apparent mass results into γ not cancelling out in the calculation that we made. All these old experiments support the concept of weakening of force and refute the relativistic mass.

What this means for the Dirac equation, which uses (25), an equivalent formula to $m = \gamma m_0$, is that the equation is correct in many situations: if the force is perpendicular to the velocity or nearly perpendicular. But the equation is not correctly derived for situations when the force is parallel to the velocity. In any case, there is no reason to discard Dirac's equation because of Einstein's errors. All that is correct can be saved even if Einstein's errors are rejected. It only requires looking at the derivations and modifying them where needed.

5. References

1. Jormakka J. Three fatal errors in the Relativity Theory, 2025. Available at the ResearchGate. Also in vixra.

2. Jormakka J. Refutation that experiments verify the relativity theory and a more reasonable proof of $E = mc^2$. 2025. Available at the ResearchGate. Also in vixra.