A Note on Time Dilation and the 'Moving Clock'

Per Hokstad

phokstad@gmail.com

Abstract. This note discusses time dilation and the frequently referred statement that the effect of this phenomenon is that the '*moving clock runs slower'*. We point out that this effect is entirely caused by the experimental set-up. So, we cannot (of course) claim that the clock on one of the two reference frames (RFs) - moving relative to each other - runs more slowly than the clocks on the other RF. We follow up this by a detailed analysis of the 'travelling twin paradox'.

Key words: Time dilation; Moving clock; Lorentz transformation; Observational principle; Travelling twin.

1 Introduction

It is surprising that *time dilation* in the theory of special relativity (TSR) is still often referred to as a phenomenon where the 'moving clock runs slower'. We elaborate on this question, utilizing the Lorentz transformation, also discussing the travelling twin (TT) paradox. We clearly demonstrate that it is the way we perform the experiment (make the time measurements) that determines which clock(s) apparently run slower/faster.

We start by introducing some basic notation.

2 Notation

We have an RF, denoted K, which for simplicity has just one space coordinate, x. This RF has synchronized, stationary clocks located at virtually any position. Further, there is an object moving at velocity, v relative to K along the x-axis. The object starts out from the origin of K when the clock at this position on K reads 0. Further, (we imagine that) this moving object brings with it a clock, and when the object passes the origin at time 0, this clock is synchronized with the clock on K at this position.

Three fundamental parameters are related to the movement of this object. First:

τ = Clock reading of the clock following the moving object. We would say that this is the 'internal time' of the object/event, but usually, this is referred to as the 'proper time'.

This proper time, τ is independent of which RF we choose as the basis for our observations. But we also have two parameters (t, x), which specify events on the chosen RF, K:

x = Position of the moving object relative to K, (when the passing specified clock reads τ);

t = Clock reading of the clock permanently located at position x on K, when the moving clock reads τ; (this t is usually referred to as 'calendar time').

Further,

$v = x/t$, *i.e.* the velocity of the object relative to the RF, K.

c = speed of light.

3 The Lorentz Transformation (LT)

The Lorentz Transformation (LT) is the fundamental relation of the TSR. To present this we introduce a second RF, K_v, moving along K, at constant velocity, v. The parameters on K_v are denoted (t_v, x_v), (all clocks on K_v also being calibrated). Further, at time $t = 0$ on K, the clock at $x = 0$ and the clock at $x_v = 0$ on K_v are at the same location and are also calibrated. We restrict to consider single space parameters,

(x and x_v), and the LT then states that an event, (t, x) on K and the 'corresponding' event (t_v, x_v) on K_v are related through the following equations:

$$t_v = t_v(t,x) = \frac{t-(vx)/c^2}{\sqrt{1-(v/c)^2}} \tag{1}$$

$$x_v = x_v(t,x) = \frac{x-vt}{\sqrt{1-(v/c)^2}} \tag{2}$$

The single clock introduced in Section 2 above - showing time τ - now equals the clock located at $x_v = 0$ on K_v. And if we insert $x_v = 0$ in Eq. (2), this gives $x = vt$, and according to Eq. (1)

$$t_v = \tau = t\sqrt{1-(v/c)^2} \quad (\text{for } x_v = 0;\ x = vt) \tag{3}$$

We note that it is actually a consequence of the LT, Eqs. (1), (2), that

$$\sqrt{t^2-(x/c)^2} = \sqrt{t_v^2-(x_v/c)^2}\ (= \tau). \tag{4}$$

Thus, the clock reading, τ is the same, independent of RF, whilst the (t_v, x_v) – parameters of course depend on which RF we apply.

4 Time Dilation and Observational Principle

The above Eq. (3) gives the main result regarding the so-called time dilation in TSR. When the moving clock located at $x_v = 0$ - after time t on K - arrives at $x = vt$, it will read

$$t_v = \tau = t\sqrt{1-(v/c)^2};$$

Thus, the clock moving at speed, v, actually runs slower than the stationary clocks on K at a rate $\sqrt{1-(v/c)^2}$.

How can we object to this conclusion? First, it is a fact that the motion is relative. So, we could equally well say that the clocks on K are moving (at velocity $-v$) relative to the clock at $x_v = 0$; which would give the 'opposite' conclusion. Further, we should note that there are *three* clocks involved in the experiment. This result in an asymmetry; the 'moving clock' at $x_v = 0$ is first compared with one clock at $x = 0$ (on K); next with another clock at $x = vt$ (also on K).

However, we can easily change this. If we let the *second* clock comparison (at time t) take place at the location $x = 0$; then Eq. (1) directly gives

$$t_v = t/\sqrt{1-(v/c)^2} \tag{5}$$

i.e. the 'opposite' result of Eq. (3).

Therefore, we conclude that there is no reason to claim that the clock(s) on one RF move faster/slower than the clock(s) on the other RF. It is entirely the experimental set-up ('observational principle'; *cf* Hokstad (2016)), that gives this apparent effect. It is always the RF where we apply two clocks, which apparently run faster than the single clock on the other RF. And we cannot claim that the single clock is moving, while the other two are not; we could equally well say that it is the two clocks on K that are moving.

Remark. Actually, it is not required that we restrict to apply three clocks. The essential point is how we perform the second clock comparison: we can generalize the above approach by performing the second clock comparison at position $x = ut$, giving $t_v = \frac{1-(uv)/c^2}{\sqrt{1-(v/c)^2}}t$.

Various choices of u may now give all kinds of time dilations; *e.g.* choosing $u = 0$ results in Eq. (5), and letting $u = v$ gives Eq. (3). In particular, by choosing

$$u = v/(1+\sqrt{1-(v/c)^2})$$

there will not be observed any time dilation, (*i.e.* $t = t_v$); *cf.* Hokstad (2016).

5 The Travelling Twin Paradox

Despite the facts presented above, the so-called travelling twin (TT) example[1] has been presented as a proof that the travelling twin ages less than the brother staying at the earth. And, as explained above, the TT's clock by his arrival to the 'star' will actually read $t_v = \tau = t\sqrt{1-(v/c)^2}$; whilst the clocks of the earth's RF will read t. And exactly the same argument applies for the return to the earth. Apparently, this contradicts the conclusion of the previous Section, and a more thorough discussion is required.

A crucial point here is that in the present case we do not have a complete symmetry between two RFs. As the TT reverses his movement at the star; the velocity relative to earth changes from v to $-v$. Thus, it is only the RF of the earth that apply for the whole experiment. In order to describe this within the framework of the TSR, we should actually apply three RFs. First, the earth's RF, secondly, the RF of the TT from the earth; thirdly, the TT's RF on his return[2].

To investigate this situation in detail, we first present graphs of Eq. (1); see Fig. 1. Here we give both t and t_v as a function of x/c; i.e. the number of years required for the light to travel the distance x on K. We choose two values for t; *i.e.* $t = 0$ and $t = t^*$, (see the two black horizontal lines in Fig. 1). The corresponding t_v values (for various x-values) are presented as blue lines (for $v > 0$), and red lines (for $v < 0$). Thus, the red (blue) lines simply give the graphical presentation of Eq. (1), inserted $t = 0$ and $t = t^*$, respectively. The x- values run from $x = 0$ to $x = vt^*$.

First, for $t = 0$ we get $t_v = 0$ for $x = 0$. And for $x = vt^*$ we get $t_v = -\frac{(vt^*/c)^2}{\sqrt{1-(v/c)^2}}$ for, *cf.* the point A in the diagrams.

Next for $t = t^*$ we get $t_v = t^*/\sqrt{1-(v/c)^2}$ for $x = 0$, (Eq. (5)); *cf.* point B in the diagrams. Next for $x = vt^*$ we get $t_v = t^*\sqrt{1-(v/c)^2}$ for (Eq. (3)); *cf.* point C in the diagrams.

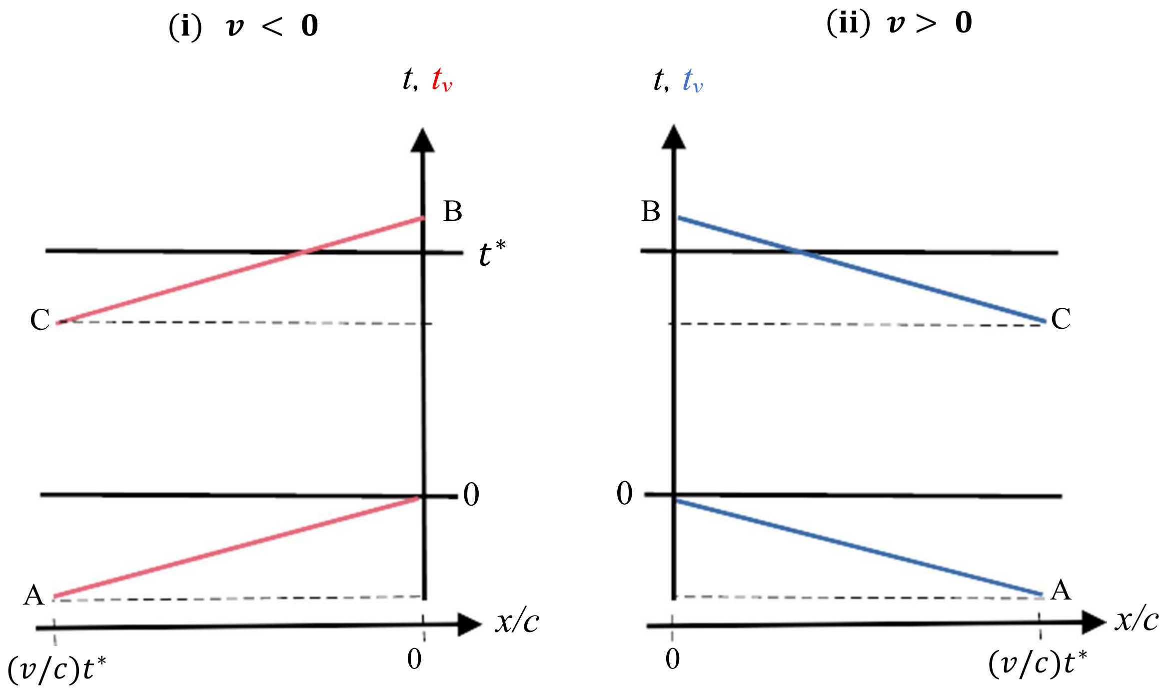

Figure 1. Graphical presentation of the Lorentz Transformation.

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[1] This paradox is described *e.g.* by Debs and Redhead (1996) and Schuler and Robert (2014).

[2] This corresponds to the *three brothers' approach*, suggested by Lords Halsbury, being referred in Debs and Redhead (1996).

By combining these results, ($v > 0$ and $v < 0$) we can now resolve the TT problem. We restrict to consider a standard numerical example, (*e.g.* Mermin (2005), Hokstad (2023 a, b)). In this case the TT leaves the earth in a rocket of velocity, $v = 0.6c$. This gives $\sqrt{1-(v/c)^2} = 0.8$. He travels to a 'star' at a distance of 3 light years from the earth. *i.e.* $x/c = 3$. Further, the earth's RF has a clock located at the star, and by the arrival of the TT, this clock will read

$$t^* = x/v = 3c/0.6c = 5 \text{ years.}$$

At this instant the TT's clock will – see *Eq*. (3) - read

$$\tau = t_v = t^* \sqrt{1-(v/c)^2} = 5 \cdot 0.8 = 4 \text{ years.}$$

In summary, the arrival of the rocket to the star represents the following event (in the earth's RF):

$$t_v = 4 \text{ years}, \quad x/c = 3 \text{ years}, \qquad t^* = 5 \text{ years}$$

Using these numbers; Fig. 2 presents the graph of the total travel. The *black* lines represent the clock readings of the earth's RF by the departure ($t = 0$), by the arrival to the star ($t = 5$), and finally by the twin's return to the earth ($t = 10$).

The *blue* lines represent the clock readings of the TT's RF by the departure and the arrival to the star, respectiely. In particular, note that when he arrives at the star this clock there reads $t_v = 4$; whilst the clock of his RF then located at the earth reads $t_v = 6.25$ years

Further, the two *red* lines represent the clock readings of the TT's RF by the departure from the star, and by the return to the earth. We can consider the TT to be located at the origin also of this RF. And by the departure from the star, the clock at this origin is calibrated with the RF used *to* the star. Thus, when $t = 5$, the red and blue lines have the common value $t_v = 4$.

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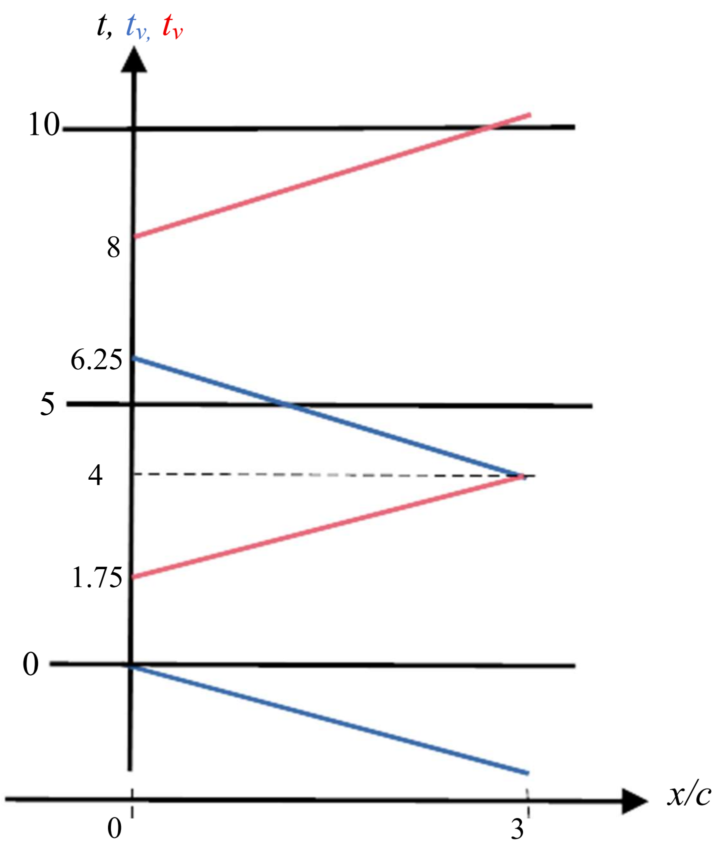

Figure 2. Graphical presentation of the Lorentz Transformation; TT paradox; Three RFs: *Black* (earth); *Blue* (TT from the earth); *Red* (TT back towards the earth). All units in years.

It is noteworthy to observe that the clock *on the earth* of the TT's RFs at this instant changes dramatically from 6.25 years to 1.75 years; (blue and red line, respectively). When 5 more years have passed on the earth (and $t = 10$), the TT's RF has increased with 6.25 years, but reads only 8 years at this position. So this confirms that by the return to the earth, the TT's clock actually shows 8 years; (he has apparently aged 8 years). However, we realize that this reduction from 10 to 8 years is not caused by the velocity of the TT relative to earth. It is entirely an effect of his abrupt change of velocity at the star!

As we previously in this paper said that the degree of time dilation is given by the decision on where to perform the second clock comparison; we will in this case say that it is the decision on *how* to reunite the twins that determines the apparent age difference when they reunite; *cf*. Hokstad (2023b).

6 Conclusions

In summary

- The so-called time dilation is not real, as it is caused by the experimental set-up ('observational principle'). Thus, we should stop using the phrase 'moving clock runs slower'. Actually, when we use a 'three clock comparison' it is always the RF, where we apply *one* clock, that time appears to run slower.
- More generally, the 'degree of time dilation' is *entirely* given by the decision on how (where) to perform the second comparison of clocks! One specific choice actually results in no time dilation between the two RFs!
- The 'TT paradox' does not alter this conclusion. This is actually an experiment of three RFs; (in order to operate within the framework of the TSR); as there is one moving RF, *from* the earth; another *to* the earth. It is a fact that the clock of the returning twin (in the current numerical example) shows 8 years; whilst the clock of the earthbound twin shows 10 years, but this is *entirely* caused by the *turning* of the travelling twin (switching of RFs and clocks) at the 'star'. So, this effect is not caused by the travelling (relative movement of twins) itself.

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