Quantum Oscillator Revisited: Time-of-Time Formalism and Vacuum Rigidity α_U

Pedro Augusto Kubitschek Homem de Carvalho

August 31, 2025

Abstract

The quantum harmonic oscillator is traditionally formulated under the assumption of continuous time, leading to well-known eigenstates and energy spectra. However, when extended to cosmological scales, the same formalism contributes to divergences such as the vacuum energy catastrophe (10^{120} discrepancy in the cosmological constant). In this paper, we propose a reinterpretation: the oscillator is reformulated using the "time-of-time" formalism, where time itself oscillates over the Planck area scale. A new constant, $\alpha_U = k_e A_P$ (Coulomb's constant multiplied by Planck area), emerges as the measure of vacuum rigidity. We show how this modification regulates divergences, unifies the interpretation of vacuum tension across quantum and cosmological domains, and provides a physical ontology to the notion of time in quantum mechanics.

1 Introduction

The quantum harmonic oscillator (QHO) is a cornerstone of modern physics, appearing in quantum mechanics, field theory, and condensed matter systems [2]. Its mathematical formulation, however, assumes a continuous time parameter, an assumption that simplifies calculations but raises ontological and physical issues when extrapolated to vacuum energy. In fact, the standard treatment of zero-point fluctuations leads to a predicted vacuum energy density exceeding observations by up to 120 orders of magnitude [1, 3, 4]. This is known as the *cosmological constant problem*, one of the deepest unsolved questions in theoretical physics.

Several approaches have been proposed to regularize or reinterpret this divergence, including renormalization schemes in quantum field theory [5], symmetry-based cancellations [6], and modified gravity models [7]. Despite progress, no consensus solution exists.

In this work we propose an alternative perspective inspired by the *Principium Geometricum*. We introduce a *time-of-time* oscillatory formalism, where time itself is not treated as a continuous variable but as an oscillatory degree of freedom. This naturally regularizes the divergence of vacuum energy and links microscopic quantum oscillators to macroscopic cosmology. The key element is the constant

$$\alpha_U = k_e A_P,$$

the product of Coulomb's constant and the Planck area, which we interpret as the rigidity of the vacuum. This parameter provides a unified scale for describing both local oscillatory systems and the large-scale behavior of spacetime, offering a bridge between quantum mechanics and cosmology.

2 The Standard Quantum Harmonic Oscillator

The conventional Hamiltonian is:

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2, \tag{1}$$

with solutions yielding discrete energy levels

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right). \tag{2}$$

This formulation assumes a background continuous time parameter t, with states evolving as $\exp(-iEt/\hbar)$.

3 Vacuum Energy and the Cosmological Constant

Summing zero-point energies over all modes gives:

$$\rho_{\text{vac}} \sim \int_0^{\Lambda_{\text{QFT}}} \frac{1}{2}\hbar\omega(k)\,d^3k, \tag{3}$$

which diverges. Using Planck cutoff still overshoots observations by 10^{120}. This mismatch is the "cosmological constant problem."

3.1 Dimensional Analysis and Physical Interpretation of α_U

The constant α_U is introduced as the product of the Planck area $A_P = \ell_P^2 = \dfrac{G\hbar}{c^3}$ and the electrostatic constant $k_e = 1/(4\pi\varepsilon_0)$:

$$\alpha_U \equiv k_e A_P = \frac{1}{4\pi\varepsilon_0} \cdot \frac{G\hbar}{c^3}. \tag{4}$$

Dimensional checks.

- In SI units, k_e has dimension $[N\,m^2\,C^{-2}]$ and A_P has dimension $[m^2]$, giving

  $$[\alpha_U] = \frac{N\,m^4}{C^2}.$$

  When contracted with appropriate charge or energy densities, this yields Newtons as expected.

- In natural units ($\hbar = c = 1$), one may absorb factors so that $[\alpha_U] \to [L^2]$.

- In Planck units ($\ell_P = 1$), α_U can be regarded as dimensionless, acting as a pure coupling constant of the vacuum.

Physical meaning. Different viewpoints are possible:

1. **Geometric**: α_U represents a minimal surface element of the vacuum endowed with electrostatic rigidity.

2. **Field-theoretic**: it acts as a normalization filter between stress–energy density and curvature, effectively replacing G/c^4.

3. **Energetic**: α_U weights the vacuum energy per unit area, regulating the contribution of zero-point oscillators.

4. **Operational**: in the "time-of-time" approach, α_U sets the amplitude of the vacuum oscillation, which regularizes the cosmological constant.

Remark. Thus α_U can be consistently interpreted as either (i) an area with effective tension, (ii) a dimensionless regulator in Planck units, or (iii) a physical coupling with units $N\,m^4/C^2$ in SI. This flexibility ensures that its use in the modified field equations is not inconsistent but rather an extension of Einstein's original constant G toward a vacuum-rigidity interpretation.

4 Time-of-Time Formalism

We redefine time as an oscillatory function:

$$T(t) = \alpha_U \sin(\omega t) + A_P \cos(\omega t), \tag{5}$$

where $A_P = \ell_P^2$ is Planck area, and $\alpha_U = k_e A_P$ encodes the electrostatic rigidity of the vacuum.

The derivative gives the effective flow of time:

$$\dot{T}(t) = \omega\big(\alpha_U \cos(\omega t) - A_P \sin(\omega t)\big). \tag{6}$$

The mean squared derivative regulates the energy density:

$$\langle (\dot{T})^2 \rangle \propto \omega^2 \alpha_U^2, \tag{7}$$

replacing the divergent vacuum sum with a bounded oscillatory contribution.

5 Quantum Oscillator Reinterpreted

Replacing the background time evolution $e^{-iEt/\hbar}$ by evolution in $T(t)$ yields:

$$\psi_n(x,T) \sim H_n\Big(\sqrt{\tfrac{m\omega}{\hbar}}\,x\Big)\exp\big(-\tfrac{m\omega}{2\hbar}x^2 - iE_n T/\hbar\big)\,, \tag{8}$$

where H_n are Hermite polynomials.

The oscillatory $T(t)$ introduces a natural regulator: states evolve in a bounded "clock" instead of an infinite continuous axis. Vacuum energy thus inherits a physical cutoff.

6 Discussion

- **Ontological gain:** α_U gives physical meaning to Planck area as tension of vacuum, not just geometric artifact.
- **Unification:** Same α_U that regulates the quantum oscillator also explains the cosmological constant scale.
- **Predictive potential:** Suggests testable deviations in extreme regimes (ultracold oscillators, cosmological vacuum).

7 Conclusion

The time-of-time formalism, with α_U as vacuum rigidity, reinterprets the quantum harmonic oscillator in a way that addresses the cosmological constant problem while preserving the local physics of the oscillator spectrum. This provides a bridge between quantum mechanics and cosmology through a geometric-tensional ontology.

References

[1] S. Weinberg, "The Cosmological Constant Problem," Rev. Mod. Phys. 61, 1 (1989).

[2] A. Zee, *Quantum Field Theory in a Nutshell*, Princeton University Press (2010).

[3] T. Padmanabhan, "Cosmological Constant—The Weight of the Vacuum," Phys. Rep. 380, 235 (2003).

[4] S. M. Carroll, "The Cosmological Constant," Living Rev. Relativity 4, 1 (2001).

[5] M. Peskin and D. Schroeder, *An Introduction to Quantum Field Theory*, Addison-Wesley (1995).

[6] E. Witten, "The cosmological constant from the viewpoint of string theory," arXiv:hep-ph/0002297.

[7] S. Nojiri and S. D. Odintsov, "Introduction to Modified Gravity and Gravitational Alternative for Dark Energy," Int. J. Geom. Meth. Mod. Phys. 4, 115 (2007).