

THE NFW PROFILE IS COMPATIBLE WITH THE DECAYING LAW OF VELOCITY WITH -0.25 AS POWER OF RADIUS

Author Manuel Abarca Hernandez **email:** mabarcaher1@gmail.com

1. INTRODUCTION

This work is a complement of the paper [3] Abarca,M.2025 that it is focused on the demonstration that a decaying law of velocity depending on -0.25 as power of radius into the halo region is compatible with the NFW profile published by [4] Huang et al. (2016), [5] Sofue (2020) and [6] karukes et al. (2020), regarding the MW galaxy and compatible with data published by [7] Sofue (2015) and [8] Zhang et al. (2024) regarding the M31 galaxy.

That law of velocity was got by the author [1] Abarca, M. (2024) in the framework of DMbQG theory, DM by Quantum Gravitation theory, using the data of rotation curve into the halo region provided by the papers of Sofue [5] and [7].

Throughout the explanation, the papers [1] and [3] are cited as the reader may find some details related to some specific calculus. The other five papers [4], [5], [6], [7] and [8] are the sources of data.

2. Calculus of the proportionality constant of the velocity decaying law

In the framework of DMbQG theory into the halo region, the velocity decays according to $v = a \cdot r^{-0.25}$, being the parameter **a** constant, whose value may be calculated by a couple of data: $a = v \cdot r^{0.25}$.

It is right to check that the dynamical mass $M_{DYN} = V^2 \cdot \text{Radius} / G$ is mathematically equivalent to direct mass, $M_{DIRECT} = a^2 \cdot (\text{Radius})^{0.5} / G$ if it is assumed the velocity law $v = a \cdot r^{-0.25}$. The Direct mass represents the total mass enclosed by the sphere with a specific radius, according the DMbQG theory.

The formula to calculate parameter **a** comes from [1] Abarca (2024), see epigraph 8.7 where is explained that according the DMbQG theory, into the halo region the velocity decays according the formula $V = a \cdot r^{-0.25}$.

As the rotation curve data have non negligible errors, it is calculated the parameter **a** for the whole point set and afterwards it is made its average value. In the epigraph 10.1 of the paper [1] it is explained why the halo begins at 30 kpc for the MW Galaxy.

As the total mass depend on the parameter a^2 , this value is highlighted even more than parameter **a**, because with the direct mass formula it is known the distribution of total mass into the whole halo region.

2.1 Calculus of parameter a using the data provided by [4] Huang et al. (2016)

Using the procedure explained before, it is got in the table 1 the parameter a, which is a 3 % lower than the one got by Sofue. See table 2. It is a very good matching because the error measures are bigger than 10 %.

Table 1 parameter a using the Huang (2016) data				
Velocity	Radius	Radius	Velocity	Parameter a
km/s	kpc	m	m/s	$m^{1.25} / s$
211,2	31,29	9,65516E+20	211200	3,7229E+10
217,93	33,73	1,04081E+21	217930	3,9143E+10
219,33	36,19	1,11671E+21	219330	4,0094E+10
213,31	38,73	1,19509E+21	213310	3,9661E+10
200,05	41,25	1,27285E+21	200050	3,7786E+10
190,15	43,93	1,35555E+21	190150	3,6486E+10
198,95	46,43	1,43269E+21	198950	3,8706E+10
192,91	48,71	1,50304E+21	192910	3,7984E+10
198,9	51,56	1,59099E+21	198900	3,9724E+10
185,88	57,03	1,75977E+21	185880	3,8071E+10

173,89	62,55	1,93011E+21	173890	3,6448E+10
196,36	69,47	2,14364E+21	196360	4,2251E+10
175,05	79,27	2,44603E+21	175050	3,8929E+10
			Total sum	5,0251E+11
			Average a	3,8655E+10
			Average a ²	1,4942E+21

2.2 Calculus of parameter a using the [5] Sofue (2020) MW data

The rotation curve data (radius and velocity) have been taken from table 6, pag 25 of [5] Sofue (2020).

radius	velocity	radius	velocity	parameter a
kpc	km/s	m	m/s	m ^{1.25} /s
30,448	229,60	9,40E+20	229600	4,01976E+10
33,493	222,50	1,03E+21	222500	3,98939E+10
36,842	215,00	1,14E+21	215000	3,94787E+10
40,527	207,10	1,25E+21	207100	3,89453E+10
44,579	200,30	1,38E+21	200300	3,85746E+10
49,037	194,70	1,51E+21	194700	3,84004E+10
53,941	189,80	1,66E+21	189800	3,83367E+10
59,335	186,20	1,83E+21	186200	3,85164E+10
65,268	184,70	2,01E+21	184700	3,91273E+10
71,795	183,90	2,22E+21	183900	3,98973E+10
78,975	181,40	2,44E+21	181400	4,03040E+10
86,872	175,50	2,68E+21	175500	3,99333E+10
95,560	167,70	2,95E+21	167700	3,90787E+10
			Total sum	5,10684E+11
			Average a	3,92834E+10
			Average a ²	1,543186E+21

3. Calculus of the parameter a² using the direct mass with the virial data calculated by the NFW method

The direct mass was got in the paper [1] Abarca (2024), where it is developed fully the DMbQG theory.

$M_{TOTAL}(< r) = \frac{a^2 \cdot \sqrt{r}}{G}$, that gives the total mass (DM + Baryonic) enclosed by the sphere with radius r.

Clearing up the parameter a² it is got $a^2 = \frac{G \cdot M_{TOTAL}}{\sqrt{r}}$ (1)

According that formula knowing a radius and the total mass enclosed by such sphere, then it is posible to know the parameter a².

It is common to consider R₂₀₀ and M₂₀₀ as the virial data and the researchers publish this data using the NFW method.

In the chapter 4 of paper [3] Abarca (2025) it is explained how to do such calculus knowing the parameters associated to the NFW method.

3.1 Calculus using the [4] Huang (2016) data

In the table 3 are the NFW parameters provided by [4] Huang et al. (2016) and the baryonic mass of the MW.

Table 3. The NFW parameters for M.W. according Huang (2016)	
Characteristic density D_0	Scale radius R_0
$8.196 \cdot 10^{-22} \text{ kg} \cdot \text{m}^{-3}$	14.39 kpc
Baryonic mass of MW $M_{BA} = 8.1 \cdot 10^{10} M_{SUN}$	

The formula for the concentration factor is $\frac{c^3}{f(c)} = \frac{4\pi G \rho_0}{100 \cdot H^2} = 1335.50$ whose solution is $c = 13.2046$ and $f(c) = 1.7240$

Therefore $R_{200} = 13.2046 \cdot 14.39 = 190.014 \text{ kpc}$ and $M_{200} = K_{NFW} \cdot f(c) = 7.81 \cdot 10^{11} M_{sun}$ where K_{NFW} is the characteristic mass of NFW profile, $K_{NFW} = 4.531 \cdot 10^{11} M_{sun}$.

As M_{200} is the DM enclosed by the sphere with radius R_{200} , to calculate the total mass it is added the baryonic mass, so $M_{200-TOTAL} = (7.81 + 0.81) \cdot 10^{11} M_{sun} = 8.62 \cdot 10^{11} M_{sun}$

Finally it is possible to calculate the parameter a^2 using $M_{200-TOTAL}$ and R_{200} getting

$$a^2 = 1.4949 \cdot 10^{21} \text{ m}^{2.5}/\text{s}^2$$

that it is incredibly close to the one got in the table 1, $a^2 = 1.4942 \cdot 10^{21} \text{ m}^{2.5}/\text{s}^2$.

In the paper [4] the authors give a different virial data because they have considered an overdensity = 95 instead 200. The procedure to calculate the parameter a^2 is the same.

Table 4 Calculus of the parameter a^2 using the virial data provided by [4] Huang et al.(2016)			
R_{VIR}	M_{VIR}	$M_{VIR-TOTAL}$	Parameter a^2
255.69 kpc	$9 \cdot 10^{11} M_{sun}$	$9.81 \cdot 10^{11} M_{sun}$	$1.467 \cdot 10^{21} \text{ m}^{2.5}/\text{s}^2$.

This value of parameter a^2 is only 1.8 % lower than the value one got by the Huang rotation curve data, see the table 1.

With a relative difference so low it is possible to state that both methods are equivalent i.e. the NFW DM profile is compatible with a velocity decaying law into the halo region equal to $v = a \cdot r^{-0.25}$

3.2 Calculus using the [5] Sofue (2020) data

In the table 5 there are the NFW parameters and the baryonic mass of MW. It is remarkable that the value for the baryonic mass of MW is more than twice the value calculated by Huang. However the total mass is similar and the parameter a^2 as well.

Table 5. The NFW parameters for M.W. according Sofue (2020)	
Characteristic density D_0	Scale radius R_0
$1.40 \cdot 10^{-21} \text{ kg} \cdot \text{m}^{-3}$	10.94 kpc
Baryonic mass of MW $M_{\text{BA}} = 1.8 \cdot 10^{11} M_{\text{SUN}}$	

The formula for the concentration factor is $\frac{c^3}{f(c)} = \frac{4\pi G \rho_0}{100 \cdot H^2} = 2281.24$ whose solution is $c = 16.334$ and $f(c) = 1.91$

Therefore $R_{200} = 16.334 \cdot 10.94 = 178.7 \text{ kpc}$ and $M_{200} = K_{\text{NFW}} \cdot f(c) = 6.495 \cdot 10^{11} M_{\text{sun}}$ where K_{NFW} is the characteristic mass of NFW profile, $K_{\text{NFW}} = 3.4 \cdot 10^{11} M_{\text{sun}}$.

As M_{200} is the DM enclosed by the sphere with radius R_{200} , to calculate the total mass it is added the baryonic mass, so $M_{200\text{-TOTAL}} = (6.5 + 1.8) \cdot 10^{11} M_{\text{sun}} = 8.3 \cdot 10^{11} M_{\text{sun}}$

Finally it is possible to calculate the parameter a^2 using $M_{200\text{-TOTAL}}$ and R_{200} getting

$a^2 = 1.4842 \cdot 10^{21} \text{ m}^{2.5}/\text{s}^2$ that it is 3.9 % lower than the value one got by the Sofue rotation curve data, see the table 2.

With a relative difference so low it is possible to state that both methods are equivalent i.e. the NFW DM profile is compatible with a velocity decaying law into the halo region equal to $v = a \cdot r^{-0.25}$.

3.3 Calculus using the [6] karukes (2020) data

These authors do not provide rotation curve and they have used a new version more sophisticated of NFW called gNFW. However they have published some data pairs (radius and total mass), which are perfect to calculate the parameter a^2 .

In the page 25 of paper [6] Karukes, it is shown no only the virial data of table 6 but 10 data pairs of radius and total mass. In the table 7 are shown only 5 pairs.

Table 6 Virial data and total mass according [6] Karukes 2020			
$M_{200\text{-DM}}$	$M_{\text{TOTAL-R200}}$	R_{200}	Parameter a^2
M_{\odot}	M_{\odot}	kpc	$\times 10^{21} \text{ m}^{2.5}/\text{s}^2$
$8.26 \cdot 10^{11} M_{\text{sun}}$	$8.95 \cdot 10^{11} M_{\text{sun}}$	193.24	1.5391

By subtraction of both types of masses it is possible to calculate the baryonic mass of MW, $M_{\text{BA}} = 7 \cdot 10^{10} M_{\text{sun}}$

The result of parameter a^2 is quite close to the ones got by Sofue or Huang.

In addition in the table 7 it is calculated the parameter a^2 using another different data pairs. The only one to reject is the one associated to 28 kpc as it is too different to the others. In fact, if it is calculated the parameter using lower radius its parameter a^2 goes down.

This fact it is explained because the halo begins at 30 kpc. The direct mass is a formula so simple because its dominion is the halo region where the density of baryonic matter is negligible regarding the DM density.

As it is shown in the table 7, into the halo region the parameter a^2 is virtually constant.

Radius (kpc)	M_{TOTAL} ($\times 10^{11}$) Msun	Parameter a^2 $\times 10^{21} \text{ m}^{2.5}/\text{s}^2$
28.33	$3.04^{+0.10 (0.19)}$ $-0.08 (0.17)$	1.3653
45.79	$4.27^{+0.22 (0.43)}$ $-0.19 (0.37)$	1.5085
74.0	$5.68^{+0.40 (0.83)}$ $-0.37 (0.65)$	1.5784
119.57	$7.26^{+0.66 (1.40)}$ $-0.58 (1.03)$	1.5872
193.24	$8.95^{+0.98 (2.07)}$ $-0.84 (1.48)$	1.5391

4. Velocity decaying law for the M31 halo

To illustrate that the NFW DM profile is equivalent with the velocity decaying law $v = a \cdot r^{-0.25}$ for the M31 galaxy as well, it will be used the papers [7] Sofue (2015) and [8] Zhan et al. (2024) where are published two rotation curves of M31.

4.1 M31 Sofue data (2016)

4.1.1 Calculus of parameter a^2 using the rotation curve data

In the chapter 2 of paper [1] it is introduced the rotation curve data published by Sofue and in the chapter 8 it is calculated the parameter a using such rotation curve. Namely the parameter $a^2 = 2.235 \cdot 10^{21} \text{ m}^{5/2}/\text{s}^2$ is the one got by the rotation curve data of M31.

4.1.2 Virial data and calculus of parameter a^2 by the direct mass formula

In the table 2 of paper [7] it is introduced the NFW parameters

Characteristic density D_0	Scale radius R_0
$1.51 \cdot 10^{-22} \text{ kg} \cdot \text{m}^{-3}$	34.6 kpc
Baryonic mass of M31 $M_{BA} = 1.6 \cdot 10^{11} M_{SUN}$	

The formula for the concentration factor is $\frac{c^3}{f(c)} = \frac{4\pi G \rho_0}{100 \cdot H^2} = 246.048$ whose solution is $c = 6.579$ and $f(c) = 1.1573$

Therefore $R_{200} = 6.579 \cdot 34.6 = 227.63 \text{ kpc}$ and $M_{200} = K_{NFW} \cdot f(c) = 1.342 \cdot 10^{12} \text{ Msun}$ where K_{NFW} is the characteristic mass of NFW profile, $K_{NFW} = 1.16 \cdot 10^{12} \text{ Msun}$.

As M_{200} is the DM enclosed by the sphere with radius R_{200} , to calculate the total mass it is added the baryonic mass, so $M_{200-TOTAL} = (1.34+0.16) \cdot 10^{12} \text{ Msun} = 1.50 \cdot 10^{12} \text{ Msun}$

Finally it is possible to calculate the parameter a^2 using $M_{200-TOTAL}$ and R_{200} getting $a^2 = 2.377 \cdot 10^{21} \text{ m}^{2.5}/\text{s}^2$ that it is 6 % bigger than the value one got by the Sofue rotation curve data, whose value is $a^2 = 2.235 \cdot 10^{21} \text{ m}^{5/2}/\text{s}^2$.

With a relative difference so low it is possible to state that both methods are equivalent i.e. the NFW DM profile is compatible with a velocity decaying law into the halo region equal to $v = a \cdot r^{-0.25}$.

4.2 Zhan et al. data (2024)

4.2.1 Calculus of parameter a^2 using the rotation curve data

In the page 9 of paper [8] Zhan, it is tabulated the R.C. data. In the chapter 5 of paper [1] Abarca it is estimated that for radius bigger than 40 kpc the baryonic density is negligible regarding the DM density, so it is right to calculate the parameter a since 40 kpc. However the data for 46 and 52 kpc have a velocity too low, so both values have been rejected.

In the table 9 are tabulated the other values. The average a^2 is a 19% lower than the one calculated by the Sofue rotation curve data, but both are compatibles because the error data are even bigger than 20%.

Radius	Velocity	radius	velocity	Param. a^2
kpc	km/s	m	m/s	$m^{2.5}/s^2$
54,85	196,28	1,6925E+21	196280,00	1,5850E+21
67,26	202,02	2,0754E+21	202020,00	1,8593E+21
98,74	192,59	3,0468E+21	192590,00	2,0473E+21
123,56	168,53	3,8127E+21	168530,00	1,7538E+21
			Total sum	7,2453E+21
			Average a^2	1,8113E+21

4.2.2 Virial data and calculus of parameter a^2 by the direct mass formula

In the graphic of page 8 of paper [8], it is plotted the RC associated to the disc and the bulge mass, so by a simple calculus it is got the baryonic mass of M31, $M_{BA} = 10.5 \cdot 10^{10}$ Msun. In addition, in the page 11 are given the virial data associated to DM purely.

The radius error is about 11% but the upper error of mass is 45%, very high.

M_{200}	R_{200}	Baryonic mass
$M_{vir} = 1.14^{+0.51}_{-0.35} \times 10^{12} M_{\odot}$	$r_{vir} = 220 \pm 25 \text{ kpc.}$	$M_{BA} = 10.5 \cdot 10^{10} \text{ Msun.}$

Using such data it is got the 200-Total mass $M_{200-TOTAL} = 1.245 \cdot 10^{12}$ Msun and as in previous epigraphs, the parameter a^2 it is calculated rightly, $a^2 = 2.006 \cdot 10^{21}$. This value is 10% bigger regarding the value got by the RC method (Epigraph 4.2.1) and it is a 10% lower regarding the value got using the R.C. data provided by Sofue, $a^2 = 2.235 \cdot 10^{21}$.

5. Calculus of the virial data associated to the total mass, using the parameter a^2

In the framework of DMbQG it is possible to know the virial data for the total mass with these formulas:

See in the paper [3] formulas (3.4) and (3.5)

$$M_{200-TOTAL} = \frac{a^{12/5}}{G \cdot (10 \cdot H)^{2/5}} \quad \text{and} \quad R_{200-TOTAL} = \left[\frac{a^2}{100 \cdot H^2} \right]^{2/5} \quad \text{being } H = 70 \text{ km/s/Mpc}$$

The first one gives the total mass enclosed by a sphere whose average density is 200 times the critic density of the universe, and the second one gives its radius.

Both values are bigger than the DM virial data because the average density remain equal to $200 \rho_C$ but the total mass is the DM plus the baryonic mass.

In the following tables are calculated the virial data for the total mass associated to its parameter a^2 . For each author are written the two results of parameter a^2 calculated using the rotation curve data (R.C.) and using the virial data associated to DM plus the baryonic mass (V. Data). In the last row of each table is written the relative difference between both results.

5.1 Milky Way galaxy

In the tables 11, 12, 13 are summarized the total virial data using the parameter a^2 calculated by the data provided by the three different authors.

Table 11 Calculus of $M_{200-TOTAL}$ and $R_{200-TOTAL}$ using the parameter a^2 for the MW galaxy. Huang data tables 1&4		
parameter a^2	$R_{200-TOTAL}$	$M_{200-TOTAL}$
$a^2 = 1.4942 \cdot 10^{21} \text{ m}^{5/2}/\text{s}^2$ (by R.C.)	197.61 kpc	$8.786 \cdot 10^{11} \text{ Msun}$
$a^2 = 1.4949 \cdot 10^{21} \text{ m}^{5/2}/\text{s}^2$ (by V. Data)	197.65	$8.791 \cdot 10^{11} \text{ Msun}$
Relative difference %	Negligible	Negligible

Table 12 Calculus of $M_{200-TOTAL}$ and $R_{200-TOTAL}$ using the parameter a^2 for the MW galaxy. Sofue data - table 2		
parameter a^2	$R_{200-TOTAL}$	$M_{200-TOTAL}$
$a^2 = 1.5432 \cdot 10^{21} \text{ m}^{5/2}/\text{s}^2$ (by R.C.)	200.18 kpc	$9.13 \cdot 10^{11} \text{ Msun}$.
$a^2 = 1.4842 \cdot 10^{21} \text{ m}^{5/2}/\text{s}^2$ (by V. Data)	197.08 kpc	$8.72 \cdot 10^{11} \text{ Msun}$
Relative difference %	1.5 %	4.5 %

Table 13 Calculus of $M_{200-TOTAL}$ and $R_{200-TOTAL}$ using the parameter a^2 for the MW galaxy. Karukes data - table 6		
parameter a^2	$R_{200-TOTAL}$	$M_{200-TOTAL}$
$a^2 = 1.5391 \cdot 10^{21} \text{ m}^{5/2}/\text{s}^2$ (by V. Data)	199.97 kpc	$9.10 \cdot 10^{11} \text{ Msun}$.
The authors did not publish R.C.		

The relative difference for the radius is lower than 1.6 % and for the mass is lower than 4.5 %. This little values are a strong back to the tesis of this work, because in general the error of the experimental data are about 15% or even bigger.

It is remarkable the Huang data, table 11, where the results of virial data calculated by the two different methods are virtually the same.

5.2 M31 galaxy

In the tables 14 and 15 are summarized the total virial data using the parameter a^2 calculated by the data provided by the two authors.

Table 14 Calculus of $M_{200-TOTAL}$ and $R_{200-TOTAL}$ using the parameter a^2 for the M31 galaxy. Sofue data.		
parameter a^2	$R_{200-TOTAL}$	$M_{200-TOTAL}$
$a^2 = 2.235 \cdot 10^{21} \text{ m}^{5/2}/\text{s}^2$ (by R.C.)	232.15 kpc	$1.42 \cdot 10^{12} \text{ Msun}$.
$a^2 = 2.377 \cdot 10^{21} \text{ m}^{5/2}/\text{s}^2$ (by V. Data)	237.94 kpc	$1.53 \cdot 10^{12} \text{ Msun}$
Relative difference %	2.5 %	7.6 %

These relative differences are an excellent result because in general the data error of radius and velocities are about 10% or 15% and these errors generate bigger errors associated to the mass.

Table 15 Calculus of $M_{200-TOTAL}$ and $R_{200-TOTAL}$ using the parameter a^2 for the M31 galaxy. Zhang data. Table 9		
parameter a^2	$R_{200-TOTAL}$	$M_{200-TOTAL}$
$a^2 = 1.81 \cdot 10^{21} \text{ m}^{5/2}/\text{s}^2$ (by R.C.)	213.36 kpc	$1.11 \cdot 10^{12} \text{ Msun}$.
$a^2 = 2.006 \cdot 10^{21} \text{ m}^{5/2}/\text{s}^2$ (by V. Data)	222.32 kpc	$1.25 \cdot 10^{12} \text{ Msun}$
Relative difference %	4 %	11.2 %

The 11 % as relative difference is a good result by the same reason explained in the previous paragraph. For example in the table 10, according Zhang, the relative difference of the virial mass is 44 %.

If it is taken the four virial radius of M31 as a set, the relative difference of radius is about 8%, but when these data are differentiated by authors the relative differences drop to 2.5 % and 4 %.

Similarly, If it is taken the four virial masses of M31 as a set, its relative difference is about 27%, but when these data are differentiated by authors the relative differences drop to 7.6 % and 11.2 % .

Regarding the thesis of this work, it is more important the relative differences of the virial data linked to the same author, so these results are excellent because the error of radius data or velocity ones provided by the two authors are about 15% or 20%. However the error linked to virial masses are bigger. Namely the virial mass upper error published by Zhang is $+0.51 \cdot 10^{12} M_{\text{sun}}$ that it is equivalent to a relative error of 44%.

This high error is due to the accumulation of errors. Namely the formula for the relative error for the dynamical mass, $M_{\text{DYN}} = v^2 \cdot r / G$, is $dm/m = 2dv/v + dr/r$, where dm , dv and dr are the errors of its respective measures.

For example in the page 9 of paper [8] are tabulated the data and its errors. If it is considered only the four last data at 54, 67, 98 and 123 kpc and it is calculated the average of the mass relative error, then the result is an unexpected error equal to 56%.

In the table 16 are tabulated the measures, its errors and the calculus of the mass relative errors.

Radius	Rad. Error	Radius	Rad. Error	Velocity	Vel. Error	dm/m	dm/m
kpc	kpc	m	m	m/s	m/s	dimensionless	%
54,85	1,67	1,6925E+21	5,1531E+19	196280	38810	4,259E-01	4,259E+01
67,26	14,95	2,0754E+21	4,6131E+20	202020	40670	6,249E-01	6,249E+01
98,74	17,19	3,0468E+21	5,3043E+20	192590	42230	6,126E-01	6,126E+01
123,56	11,6	3,8127E+21	3,5794E+20	168530	41340	5,845E-01	5,845E+01
						Total sum	2,2479E+02
						Average error	56,2 %

As it was pointed in the chapter 2, the dynamical mass is equivalent to the direct mass, so its errors are equivalent as well.

Therefore, even the bigger relative difference of the virial mass linked to the Zhang data (11%) is a result quite good that back the thesis of this work about the compatibility of the decaying velocity law into the halo region with the virial data of DM provided by the NFW method.

6. Concluding Remarks

This work have demonstrated mathematically that the NFW DM profile is compatible with the velocity decaying law power radius with -0.25 as exponent into the halo region up to the virial radius for MW and M31. It is stated a finite dominion because the NFW DM profile establish a finite dominion although for the DMbQG theory the halo region is unbounded.

This law was deduced into the framework of DMbQG theory, therefore it is posible to rewrite the previous sentence in this way: The NFW DM profile is compatible with the Direct mass into the halo region up to the virial radius.

According the hypothesis of DMbQG theory, the DM is generated by the gravitational field, therefore the halo is unbounded. In the final chapter of [1] Abarca, M (2024) it is demonstrated that it is the D. energy the phenomenon able to counterbalance the increase without limit of DM calculated by the direct mass formula.

In the paper [2] the theory is extended to cluster of galaxies using to the fullest the coupling DM-DE, and the theoretical findings obtained are tested successfully with results published for the Local Group and the Virgo cluster.

If it is considered that our Galaxy and M31 have the rotation curves better studied with the maximum of accuracy data it is right to generalize the equivalence between the two theories for all the galaxies in the framework of DMbQG th.

It is clear that the NFW is a trustable method tested in thousand of galaxies, used widely for the scientific community during several decades, so the mathematical proof got in the paper [3] about the equivalence between the two methods means a strong back for the DMbQG theory.

The direct mass depend on only of one parameter whereas the NFW profile depend on the two ones. The uniqueness of the parameter may be explained by the fact that according the DMbQG the DM is generated by the own field, and supposing spherical symmetry a gravitational field is defined by the parameter a^2 , according the DMbQG theory.

If the DM phenomenon may be explained with a more simple formula over an unbounded halo is why the DMbQG theory is based on a trustable hypothesis about the DM physic nature.

The findings got in the paper [3] and the other two previous papers, allows to state that the DMbQG theory is a step forward to understand the DM phenomenon.

7. BIBLIOGRAPHY

- [1]Abarca, M. (2024) A Dark Matter Theory by Quantum Gravitation for Galaxies and Clusters. *Journal of High Energy Physics, Gravitation and Cosmology*, **10**, 1749-1784. <https://doi.org/10.4236/jhepgc.2024.104100>
- [2]Abarca, M. (2024) Solving the Conundrum of Dark Matter and Dark Energy in Galaxy Clusters. *Journal of High Energy Physics, Gravitation and Cosmology*, **10**, 1785-1805. <https://doi.org/10.4236/jhepgc.2024.104101>
- [3] Abarca,M.(2025) Equivalence between Direct Mass and NFW-Total Mass Formula in MW and M31 Galaxies. *Journal of High Energy Physics, Gravitation and Cosmology*. Vol.11 No.3, July 2025. <https://doi.org/10.4236/jhepgc.2025.113073>
- [4] Huang, Y. et al. (2016) The Milky Way's rotation curve out to 100 kpc and its constraint on the Galactic mass distribution. *MNRAS* **463**, 2623–2639 (2016). <https://doi:10.1093/mnras/stw2096>
- [5]Sofue, Y. (2020) Rotation Curve of the Milky Way and the Dark Matter Density. *Galaxies*, **8**, Article 37. <https://doi.org/10.3390/galaxies8020037>
- [6] Karukes, E.V., Benito, M., Iocco, F., Trotta, R. and Geringer-Sameth, A. (2020) A Robust Estimate of the Milky Way Mass from Rotation Curve Data. *Journal of Cosmology and Astroparticle Physics* , 2020, Article 33. <https://doi.org/10.1088/1475-7516/2020/05/033>
- [7] Sofue, Y. (2015) Dark Halos of M31 and the Milky Way. *Publications of the Astronomical Society of Japan*, **67**, 75. <https://doi.org/10.1093/pasj/psv042>
- [8] Zhang, X., Chen, B., Chen, P., Sun, J. and Tian, Z. (2024) The Rotation Curve and Mass Distribution of M31. *Monthly Notices of the Royal Astronomical Society* , **528**, 2653-2666. <https://doi.org/10.1093/mnras/stae025>