

A simple proof of Riemann hypothesis

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Abstract

In the process of searching for counterexamples to the Riemann hypothesis, I unexpectedly proved it. Contrary to what modern mathematicians believe, although I did not create a new tool, I achieved this by constructing a sophisticated structure.

Keywords: Riemann hypothesis, Riemann Zeta function

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1. A strange question

Thanks to the current development of computers, we can clearly see the graph of the Riemann Zeta function. The first step is to list the Riemann Zeta functions. According to the definition of the Riemann Zeta function, within the critical band

$$\zeta(s) = \frac{1}{1 - 2^{1-s}} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^s} \quad (1)$$

The first figure depicts the variation of the real and imaginary parts of Zeta (s) along the critical line starting from the real axis

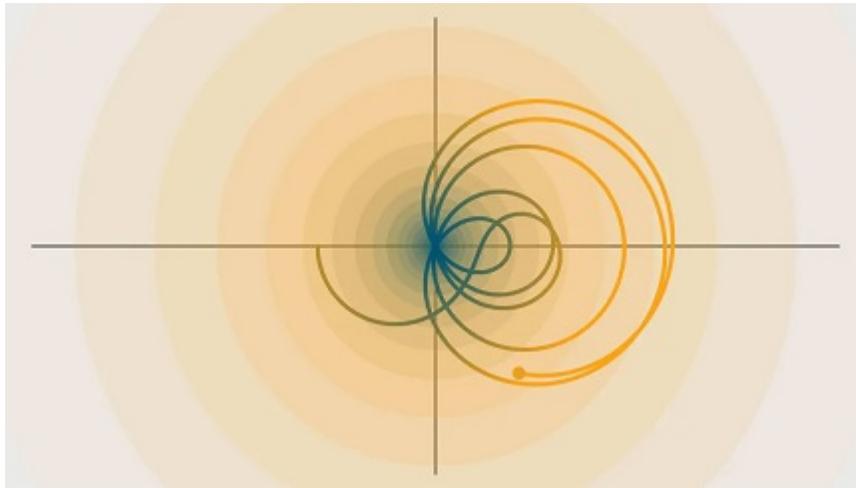


Figure 1:

We can see that this curve keeps circling in a clockwise direction. A strange question has come up, can it turn counterclockwise after multiple turns?

2. The rate of change of the real and imaginary parts of the Riemann Zeta function

In Figure 1, we can see that as t increases, the changes in the real and imaginary parts of Zeta (s) follow a pattern of constantly drawing circles. When the rate of change of the real part $\text{Re}(s)$ of Zeta (s) with respect to t approaches 0, the approximate value of the non trivial zero point can be calculated.

$$\zeta(s) = \frac{1}{1 - 2^{1-s}} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^s} \quad (2)$$

$$= \frac{1}{1 - 2^{1-r-it}} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{r+it}} \quad (3)$$

$$= \frac{1}{1 - (2^{1-r})(2^{-it})} \sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^{-it}}{n^r} \quad (4)$$

$$= \frac{1}{1 - (2^{1-r})(e^{\ln 2^{-it}})} \sum_{n=1}^{\infty} \frac{(-1)^{n+1} e^{\ln n^{-it}}}{n^r} \quad (5)$$

$$= \frac{1}{1 - (2^{1-r})(e^{-it \cdot \ln 2})} \sum_{n=1}^{\infty} \frac{(-1)^{n+1} e^{-it \cdot \ln n}}{n^r} \quad (6)$$

$$= \frac{1}{1 - (2^{1-r}) [\cos(-t \cdot \ln 2) + i \sin(-t \cdot \ln 2)]} \cdot \sum_{n=1}^{\infty} \frac{(-1)^{n+1} [\cos(-t \cdot \ln n) + i \sin(-t \cdot \ln n)]}{n^r} \quad (7)$$

$$= \frac{1}{[1 - (2^{1-r}) \cos(-t \cdot \ln 2)] - i (2^{1-r}) \sin(-t \cdot \ln 2)}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} [\cos(-t \cdot \ln n) + i \sin(-t \cdot \ln n)]}{n^r} \quad (8)$$

$$= \frac{[1 - (2^{1-r}) \cos(-t \cdot \ln 2)] + i (2^{1-r}) \sin(-t \cdot \ln 2)}{[1 - (2^{1-r}) \cos(-t \cdot \ln 2)]^2 + [(2^{1-r}) \sin(-t \cdot \ln 2)]^2}.$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} [\cos(-t \cdot \ln n) + i \sin(-t \cdot \ln n)]}{n^r} \quad (9)$$

$$= \frac{[1 - (2^{1-r}) \cos(-t \cdot \ln 2)] \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cos(-t \cdot \ln n)}{n^r}}{[1 - (2^{1-r}) \cos(-t \cdot \ln 2)]^2 + [(2^{1-r}) \sin(-t \cdot \ln 2)]^2} -$$

$$\frac{(2^{1-r}) \sin(-t \cdot \ln 2) \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \sin(-t \cdot \ln n)}{n^r}}{[1 - (2^{1-r}) \cos(-t \cdot \ln 2)]^2 + [(2^{1-r}) \sin(-t \cdot \ln 2)]^2} +$$

$$i \frac{[1 - (2^{1-r}) \cos(-t \cdot \ln 2)] \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \sin(-t \cdot \ln n)}{n^r}}{[1 - (2^{1-r}) \cos(-t \cdot \ln 2)]^2 + [(2^{1-r}) \sin(-t \cdot \ln 2)]^2} +$$

$$i \frac{(2^{1-r}) \sin(-t \cdot \ln 2) \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cos(-t \cdot \ln n)}{n^r}}{[1 - (2^{1-r}) \cos(-t \cdot \ln 2)]^2 + [(2^{1-r}) \sin(-t \cdot \ln 2)]^2} \quad (10)$$

Define

$$f(r, t) = \frac{[1 - (2^{1-r}) \cos(-t \cdot \ln 2)] \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cos(-t \cdot \ln n)}{n^r}}{[1 - (2^{1-r}) \cos(-t \cdot \ln 2)]^2 + [(2^{1-r}) \sin(-t \cdot \ln 2)]^2} -$$

$$\frac{(2^{1-r}) \sin(-t \cdot \ln 2) \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \sin(-t \cdot \ln n)}{n^r}}{[1 - (2^{1-r}) \cos(-t \cdot \ln 2)]^2 + [(2^{1-r}) \sin(-t \cdot \ln 2)]^2} \quad (11)$$

$$g(r, t) = \frac{[1 - (2^{1-r}) \cos(-t \cdot \ln 2)] \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \sin(-t \cdot \ln n)}{n^r}}{[1 - (2^{1-r}) \cos(-t \cdot \ln 2)]^2 + [(2^{1-r}) \sin(-t \cdot \ln 2)]^2} +$$

$$\frac{(2^{1-r}) \sin(-t \cdot \ln 2) \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cos(-t \cdot \ln n)}{n^r}}{[1 - (2^{1-r}) \cos(-t \cdot \ln 2)]^2 + [(2^{1-r}) \sin(-t \cdot \ln 2)]^2} \quad (12)$$

Then

$$\zeta(s) = f(r, t) + i \cdot g(r, t) \quad (13)$$

Define

$$\alpha(r, t) = (2^{1-r}) \cos(-t \cdot \ln 2) \quad (14)$$

$$\beta(r, t) = (2^{1-r}) \sin(-t \cdot \ln 2) \quad (15)$$

$$\chi(r, t) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cos(-t \cdot \ln n)}{n^r} \quad (16)$$

$$\delta(r, t) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \sin(-t \cdot \ln n)}{n^r} \quad (17)$$

So

$$f(r, t) = \frac{(1 - \alpha(r, t)) \cdot \chi(r, t) - \beta(r, t) \cdot \delta(r, t)}{(1 - \alpha(r, t))^2 + \beta^2(r, t)} \quad (18)$$

$$g(r, t) = \frac{(1 - \alpha(r, t)) \cdot \delta(r, t) + \beta(r, t) \cdot \chi(r, t)}{(1 - \alpha(r, t))^2 + \beta^2(r, t)} \quad (19)$$

Derive that

$$\frac{\partial \alpha(r, t)}{\partial t} = \frac{\partial [(2^{1-r}) \cos(-t \cdot \ln 2)]}{\partial t} \quad (20)$$

$$= - (2^{1-r}) \sin(-t \cdot \ln 2) \frac{\partial (-t \cdot \ln 2)}{\partial t} \quad (21)$$

$$= \ln 2 \cdot (2^{1-r}) \sin(-t \cdot \ln 2) \quad (22)$$

$$= \ln 2 \cdot \beta(r, t) \quad (23)$$

$$\frac{\partial \beta(r, t)}{\partial t} = \frac{\partial [(2^{1-r}) \sin(-t \cdot \ln 2)]}{\partial t} \quad (24)$$

$$= - \ln 2 \cdot (2^{1-r}) \cos(-t \cdot \ln 2) \quad (25)$$

$$= - \ln 2 \cdot \alpha(r, t) \quad (26)$$

$$\frac{\partial \chi(r, t)}{\partial t} = \frac{\partial \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cos(-t \cdot \ln n)}{n^r}}{\partial t} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \ln n \sin(-t \cdot \ln n)}{n^r} \quad (27)$$

$$\frac{\partial^2 \chi(r, t)}{\partial t^2} = \frac{\partial \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \ln n \sin(-t \cdot \ln n)}{n^r}}{\partial t} = - \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \ln^2 n \cos(-t \cdot \ln n)}{n^r} \quad (28)$$

$$\frac{\partial \delta(r, t)}{\partial t} = \frac{\partial \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \sin(-t \cdot \ln n)}{n^r}}{\partial t} = - \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \ln n \cos(-t \cdot \ln n)}{n^r} \quad (29)$$

$$\frac{\partial^2 \delta(r, t)}{\partial t^2} = \frac{-\partial \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \ln n \cos(-t \cdot \ln n)}{n^r}}{\partial t} = - \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \ln^2 n \sin(-t \cdot \ln n)}{n^r} \quad (30)$$

Then

$$\frac{\partial f(r, t)}{\partial t} = \frac{\partial \frac{(1-\alpha(r, t)) \cdot \chi(r, t) - \beta(r, t) \cdot \delta(r, t)}{(1-\alpha(r, t))^2 + \beta^2(r, t)}}{\partial t} \quad (31)$$

$$\begin{aligned} &= \frac{\left[(1 - \alpha(r, t))^2 + \beta^2(r, t) \right] \frac{\partial [(1-\alpha(r, t)) \cdot \chi(r, t) - \beta(r, t) \cdot \delta(r, t)]}{\partial t}}{\left[(1 - \alpha(r, t))^2 + \beta^2(r, t) \right]^2} - \\ &= \frac{[(1 - \alpha(r, t)) \cdot \chi(r, t) - \beta(r, t) \cdot \delta(r, t)] \frac{\partial [(1-\alpha(r, t))^2 + \beta^2(r, t)]}{\partial t}}{\left[(1 - \alpha(r, t))^2 + \beta^2(r, t) \right]^2} \quad (32) \\ &= \frac{\left[(1 - \alpha(r, t))^2 + \beta^2(r, t) \right] \left[(1 - \alpha(r, t)) \frac{\partial \chi(r, t)}{\partial t} + \chi(r, t) \frac{\partial (1-\alpha(r, t))}{\partial t} \right]}{\left[(1 - \alpha(r, t))^2 + \beta^2(r, t) \right]^2} + \\ &\quad \frac{\left[(1 - \alpha(r, t))^2 + \beta^2(r, t) \right] \left[-\beta(r, t) \frac{\partial \delta(r, t)}{\partial t} - \delta(r, t) \frac{\partial \beta(r, t)}{\partial t} \right]}{\left[(1 - \alpha(r, t))^2 + \beta^2(r, t) \right]^2} - \\ &= \frac{[(1 - \alpha(r, t)) \cdot \chi(r, t) - \beta(r, t) \cdot \delta(r, t)] \left[2 \cdot (1 - \alpha(r, t)) \cdot \frac{\partial (1-\alpha(r, t))}{\partial t} \right]}{\left[(1 - \alpha(r, t))^2 + \beta^2(r, t) \right]^2} - \end{aligned}$$

$$\frac{[(1 - \alpha(r, t)) \cdot \chi(r, t) - \beta(r, t) \cdot \delta(r, t)] \left[2 \cdot \beta(r, t) \cdot \frac{\partial(\beta(r, t))}{\partial t} \right]}{\left[(1 - \alpha(r, t))^2 + \beta^2(r, t) \right]^2} \quad (33)$$

$$= \frac{(2 - 2 \cdot \alpha(r, t)) \left[(1 - \alpha(r, t)) \frac{\partial \chi(r, t)}{\partial t} - \ln 2 \cdot \chi(r, t) \beta(r, t) \right]}{\left[(1 - \alpha(r, t))^2 + \beta^2(r, t) \right]^2} +$$

$$\frac{(2 - 2 \cdot \alpha(r, t)) \left[-\beta(r, t) \frac{\partial \delta(r, t)}{\partial t} + \ln 2 \cdot \delta(r, t) \alpha(r, t) \right]}{\left[(1 - \alpha(r, t))^2 + \beta^2(r, t) \right]^2} +$$

$$\frac{[(1 - \alpha(r, t)) \cdot \chi(r, t) - \beta(r, t) \cdot \delta(r, t)] (2 \cdot \ln 2 \cdot \beta(r, t))}{\left[(1 - \alpha(r, t))^2 + \beta^2(r, t) \right]^2} \quad (34)$$

$$\frac{\partial g(r, t)}{\partial t} = \frac{\partial \frac{(1 - \alpha(r, t)) \cdot \delta(r, t) + \beta(r, t) \cdot \chi(r, t)}{(1 - \alpha(r, t))^2 + \beta^2(r, t)}}{\partial t} \quad (35)$$

$$= \frac{\left[(1 - \alpha(r, t))^2 + \beta^2(r, t) \right] \frac{\partial [(1 - \alpha(r, t)) \cdot \delta(r, t) + \beta(r, t) \cdot \chi(r, t)]}{\partial t}}{\left[(1 - \alpha(r, t))^2 + \beta^2(r, t) \right]^2} -$$

$$\frac{[(1 - \alpha(r, t)) \cdot \delta(r, t) + \beta(r, t) \cdot \chi(r, t)] \frac{\partial [(1 - \alpha(r, t))^2 + \beta^2(r, t)]}{\partial t}}{\left[(1 - \alpha(r, t))^2 + \beta^2(r, t) \right]^2} \quad (36)$$

$$= \frac{\left[(1 - \alpha(r, t))^2 + \beta^2(r, t) \right] \left[(1 - \alpha(r, t)) \frac{\partial \delta(r, t)}{\partial t} + \delta(r, t) \frac{\partial (1 - \alpha(r, t))}{\partial t} \right]}{\left[(1 - \alpha(r, t))^2 + \beta^2(r, t) \right]^2} +$$

$$\frac{\left[(1 - \alpha(r, t))^2 + \beta^2(r, t) \right] \left[+\beta(r, t) \frac{\partial \chi(r, t)}{\partial t} + \chi(r, t) \frac{\partial \beta(r, t)}{\partial t} \right]}{\left[(1 - \alpha(r, t))^2 + \beta^2(r, t) \right]^2} -$$

$$\frac{[(1 - \alpha(r, t)) \cdot \delta(r, t) + \beta(r, t) \cdot \chi(r, t)] \left[2 \cdot (1 - \alpha(r, t)) \cdot \frac{\partial(1 - \alpha(r, t))}{\partial t} \right]}{\left[(1 - \alpha(r, t))^2 + \beta^2(r, t) \right]^2} - \frac{[(1 - \alpha(r, t)) \cdot \delta(r, t) + \beta(r, t) \cdot \chi(r, t)] \left[2 \cdot \beta(r, t) \cdot \frac{\partial(\beta(r, t))}{\partial t} \right]}{\left[(1 - \alpha(r, t))^2 + \beta^2(r, t) \right]^2} \quad (37)$$

$$= \frac{(2 - 2 \cdot \alpha(r, t)) \left[(1 - \alpha(r, t)) \frac{\partial \delta(r, t)}{\partial t} - \ln 2 \cdot \delta(r, t) \beta(r, t) \right]}{\left[(1 - \alpha(r, t))^2 + \beta^2(r, t) \right]^2} +$$

$$\frac{(2 - 2 \cdot \alpha(r, t)) \left[\beta(r, t) \frac{\partial \chi(r, t)}{\partial t} - \ln 2 \cdot \chi(r, t) \alpha(r, t) \right]}{\left[(1 - \alpha(r, t))^2 + \beta^2(r, t) \right]^2} +$$

$$\frac{[(1 - \alpha(r, t)) \cdot \delta(r, t) + \beta(r, t) \cdot \chi(r, t)] (2 \cdot \ln 2 \cdot \beta(r, t))}{\left[(1 - \alpha(r, t))^2 + \beta^2(r, t) \right]^2} \quad (38)$$

We can calculate the rate of change of the real and imaginary parts of the Riemann Zeta function for variable t using equations 34 and 38

3. The condition for the curve of Figure 1 to reverse

We refer to points that satisfy the following equations as Zhiyang Zhang Points, and according to the concept of analytical extension, these points do not exist in isolation. If these points are connected, a curve is formed, which is the curve I defined Zhiyang Zhang Curve.

$$\frac{\partial f(r, t)}{\partial t} = \frac{\partial g(r, t)}{\partial t} = 0 \quad (39)$$

If there exists t, which makes equation 39 hold, then the curve in Figure 1 will reverse

4. Solutions to some basic equations

To solve the problem with equation 39, we must start studying some simple equations. For

$$\sin(2x) = \sin(3x) = 0 \quad (40)$$

We know that $x=k\pi$ ($k=0, 1, 2, 3\dots$) is the solution to the above system of equations

For

$$\sin(x \ln 2) = \sin(x \ln 3) = 0 \quad (41)$$

We know that $x=0$ is the solution to the above system of equations. Besides that, we can't find a second solution. It can be proven through the method of contradiction.

$$\sin(x \ln 2) = 0 \quad (42)$$

$$\sin(x \ln 3) = 0 \quad (43)$$

The solution of equation 42 is

$$x_m = \frac{m\pi}{\ln 2} (m = 0, 1, 2, 3\dots) \quad (44)$$

The solution of equation 43 is

$$x_n = \frac{n\pi}{\ln 3} (n = 0, 1, 2, 3\dots) \quad (45)$$

Therefore, we conclude that

$$\frac{m}{n} = \frac{\ln 2}{\ln 3} \quad (46)$$

The equation does not hold as the left-hand side is rational and the right-hand side is irrational. Therefore, equation system 41 only has one solution $x=0$

5. The movement of function images

For the following system of equations

$$\sin(x \ln 2) = \cos(x \ln 3) = 0 \quad (47)$$

The above equation system has no solution, but in order to solve more general cases, we need to study it

$$\sin(x \ln 2) = \sin(x \ln 3 + t) = 0 \quad (48)$$

Essentially, one of the functions has been moved, and the points that were originally coincident at the origin no longer coincide. Can a new solution still be found?

For equations 44 and 45, when

$$t = x_m - x_n \quad (49)$$

Equation system 48 has a solution. Meanwhile, we can prove through the method of contradiction that this solution is unique. Assuming existence

$$t = x_{m1} - x_{n1} = x_{m2} - x_{n2} \quad (50)$$

We can obtain

$$\frac{m_1\pi}{\ln 2} - \frac{n_1\pi}{\ln 3} = \frac{m_2\pi}{\ln 2} - \frac{n_2\pi}{\ln 3} \quad (51)$$

$$\frac{m_1 - m_2}{n_1 - n_2} = \frac{\ln 2}{\ln 3} \quad (52)$$

The equation does not hold as the left-hand side is rational and the right-hand side is irrational. Therefore, equation system 48 and 49 only has one solution

6. The number of solutions to a composite function

For equation system 39, we already know that there exists a solution between 0 and the first non trivial zero point. Because there was a reversal in Figure 1. From the content of Chapter 5, we can know that this solution is unique, so the image will not undergo a new reversal. A system of equations similar to the following can only have at most one solution

$$\sin(x \ln 2) + \sin(x \ln 3) = \sin(x \ln 4 + t) + \sin(x \ln 5 + t) = 0 \quad (53)$$

$$\sin(x \ln 2) \cdot \sin(x \ln 3) = \sin(x \ln 4 + t) \cdot \sin(x \ln 5 + t) = 0 \quad (54)$$

7. Topology proves the validity of Riemann hypothesis

Due to the fact that the curve in Figure 1 will no longer reverse, we may not be able to construct a counterexample to the Riemann hypothesis. The Riemann hypothesis has been proven over 160 years after its proposal

References

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