Title: Origin of the Lorentz Factor
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Abstract:

The purpose of writing this paper was to understand the origin of the Lorentz factor equation. Calculations were performed for various clock configurations and special relativity theory was used to compare the calculated dilated clock rate factors with the Lorentz factor.

The following clock configurations were examined:

Example 1	Clock mechanism used to measure time: Clock mechanism motion:	2-way light clock light motion *perpendicular* to spaceship motion
Example 2	Clock mechanism used to measure time: Clock mechanism motion:	2-way light clock light motion *parallel* to spaceship motion
Example 3	Clock mechanism used to measure time: Clock mechanism motion:	2-way ball clock ball motion *parallel* to spaceship motion
Example 4	Clock mechanism used to measure time: Clock mechanism motion:	1-way light clock light motion *parallel* to spaceship motion

Calculations were also performed in order to understand the Michelson & Morley 1887 interferometer experiment and how it relates to the Lorentz factor.

In examples 1 through 3, the calculated dilated clock rate factors are in agreement with the Lorentz factor equation. The dilated clock rate factor computed in example 4 (1-way light clock) does not agree with the Lorentz factor equation. Upon further examination it was found that for light clocks with the light motion parallel to the travel motion, the Lorentz factor is the average of the dilated clock rate factor of each leg.

In leg 1, light motion in same direction as travel motion, the dilated clock rate factor = $\sqrt{\frac{c+v}{c-v}}$.

In leg 2, light motion in the opposite direction as travel motion, the dilated clock rate factor = $\sqrt{\frac{c-v}{c+v}}$.

The overall Lorentz factor is the average of legs 1 & 2, $\gamma = \frac{\sqrt{\frac{c+v}{c-v}}+\sqrt{\frac{c-v}{c+v}}}{2} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$.

In leg 1 there is more than expected time dilation and in leg 2 there is time contraction. Time dilation should be uniform throughout the tick of the clock. It should not rely upon taking the average of the dilation factors of each leg. This example mathematically demonstrates that special relativity theory is invalid. An analysis of any range of the clock's tick should be able to be performed and the correct dilation rate should be able to be calculated. An expectation of uniform time dilation should not depend on the clock configuration, that is, whether the mechanism of the clock (light) is traveling perpendicular (in which case time dilation is uniform as expected) or parallel relative to the motion of the clock. According to the mathematics of special relativity theory, if someone is traveling fast with a parallel motion light clock, when the light is in the first leg of its tick (when light is moving in the same direction as the spaceship), their time would appear to over-dilate (compared to the Lorentz factor). When light is returning in the second leg, their time would appear to contract.

The Lorentz factor equation is an equation which was determined to solve the dilemma of the Michelson & Morley 1887 interferometer experiment. These calculations show that it is an artificially imposed reduction in the light path length for the light traveling in the direction of earth's assumed orbital velocity. After analyzing the light path lengths in Michelson's interferometer and ignoring the rules of special relativity theory, it is apparent how this factor was arrived at.

$d_1 := 100\ ft$ one-way clock length

$v := 0.9 \cdot c$ velocity of spaceship

(note, these are arbitrary values for d_1 and v, for the purposes of this quick example a high velocity is used so the differences are more apparent, the values corresponding to Michelson's experiment of 11m and 30 km/s are used in various examples throughout this paper)

$D_{perp} := 2 \cdot d_1 \cdot \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 458.831\ ft$ perpendicular light path distance

$t_{perp} := 2 \cdot d_1 \cdot \frac{1}{\sqrt{c^2-v^2}} = 0.000000466495496\ s$ perpendicular light path travel time

$D_{par} := 2 \cdot d_1 \cdot \frac{c^2}{c^2-v^2} = 1052.632\ ft$ parallel light path distance

$t_{par} := 2 \cdot d_1 \cdot \frac{c}{c^2-v^2} = 0.000001070214065\ s$ parallel light path travel time

(note: the derivations for these equations are provided within this paper)

To be in agreement with the accepted 0 km/s relative motion between the earth and the ether interpretation of the Michelson experiment, the two light paths must be equal and the rays of light must get back to the beam splitter at the exact same instant, thereby producing 0 fringe shift. As shown above (not considering special relativity theory), they are not.

If you multiply the parallel path distance equation by the reciprocal of how the perpendicular path was affected by the earth's motion (by $\sqrt{1-\frac{v^2}{c^2}}$), then the two path lengths become equal, $D_{par} := 2 \cdot d_1 \cdot \frac{c^2}{c^2-v^2} \cdot \sqrt{1-\frac{v^2}{c^2}} = 458.831\ ft$, hence length contraction. The ratio of the two travel times of the rays of light in the Michelson interferometer is $\frac{t_{par}}{t_{perp}} = 2.294$, which is the time dilation factor, $\gamma := \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 2.294$.

I conclude that the driving force behind Einstein's special relativity theory (in addition to what Lorentz and others were putting forth) was to salvage the Copernican (heliocentric) model of the solar system by developing theories which apparently forgot about our practical model of time (which requires clock's tick rates to be in accordance with celestial motions) and redefined time to be in terms of light rays performing round trip travel bouncing back and forth between moving walls because that is exactly what was occurring in Michelson's interferometer. This redefinition allowed him to establish the notion of time distortion and length contraction occurring in a moving reference frame as perceived from a stationary reference frame, unless you are Michelson who was somehow able to perceive the effects of these relativistic distortions (by measuring 0 fringe shift) in his own reference frame. Michelson was able to observe the same effects in his moving frame (earth at 30 km/s), that should only be observable from the relative stationary frame (the medium in which light waves propagate - the ether, or the nothingness in which light waves self-propagate - empty space, whichever you choose). All of this was theorized so that the unproven earth orbital velocity of 30 km/s could remain uninvestigated.

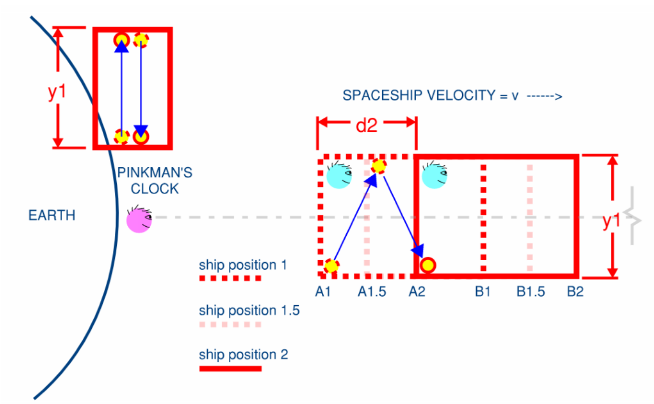
Example 1 Clock mechanism used to measure time: 2-way light clock
Clock mechanism motion: light motion *perpendicular* to spaceship motion

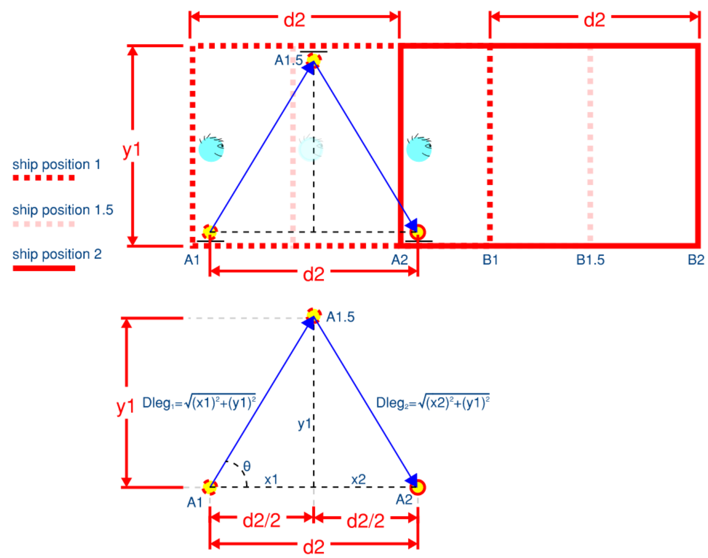
A common thought experiment, the light clock thought experiment, is used to illustrate how time can dilate.

In this experiment, a person is on earth and a person is on a spaceship traveling very fast. According to special relativity theory, a clock measures time (more specifically, a clock defined by light traveling between locations defines time). So to measure time, a person on earth has a clock and a person in the spaceship has a an identical clock.

Pinkman is on Earth. Blueman is in a spaceship. Blueman's spaceship is traveling directly overhead and away from Pinkman (along the grey axis).

They both have the same light clock. Light is emitted from the bottom of the ship, bounces off a mirror at the top, and returns to the bottom. That is how the clock ticks once.





Derivation of light path distance and light travel time for moving clock:

Leg 1

Calculate the distance light travels from A1 to A1.5.

The light travels a total distance of Dleg1 = $\sqrt{(x_1)^2 + (y_1)^2}$

y1 = vertical component of leg 1 distance
= height of clock (height of spaceship)
x1 = horizontal component of leg 1 distance
= half the distance the spaceship moves in one tick
= d2/2
d2/2 = distance the spaceship moves in leg 1
= velocity of spaceship * the time it takes for the light to travel from A1 to A1.5
= v * t1

So, Dleg1 = $\sqrt{(v \cdot t_1)^2 + (y_1)^2}$

The total distance the light travels can also be expressed in terms of the light's speed, c, and the time it takes for the light to travel from A1 to A1.5, t1.

Dleg1 = c*t1

Now we have two expressions for Dleg1 and can set them equal.

$$c \cdot t_1 = \sqrt{(v \cdot t_1)^2 + (y_1)^2}$$

$$t_1 = \frac{\sqrt{(v \cdot t_1)^2 + (y_1)^2}}{c}$$

Leg 2

Calculate the distance light travels from A1.5 to A2.

The light travels a total distance of Dleg2 = $\sqrt{(x_2)^2 + (y_1)^2}$

y1 = vertical component of leg 2 distance
= height of clock (height of spaceship)
x2 = horizontal component of leg 2 distance
= half the distance the spaceship moves in one tick
= d2/2
d2/2 = distance the spaceship moves in leg 2
= velocity of spaceship * the time it takes for the light to travel from A1.5 to A2
= v * t2

So, Dleg2 = $\sqrt{(v \cdot t_2)^2 + (y_1)^2}$

The total distance the light travels can also be expressed in terms of the light's speed, c, and the time it takes for the light to travel from A1.5 to A2, t2.

Dleg2 = c*t2

Now we have two expressions for Dleg2 and can set them equal.

$$c \cdot t_2 = \sqrt{(v \cdot t_2)^2 + (y_1)^2}$$

$$t_2 = \frac{\sqrt{(v \cdot t_2)^2 + (y_1)^2}}{c}$$

The times of each leg were derived as:

Leg 1 $t_1 = \frac{\sqrt{(v \cdot t_1)^2 + (y_1)^2}}{c}$ Leg 2 $t_2 = \frac{\sqrt{(v \cdot t_2)^2 + (y_1)^2}}{c}$

These equations will be algebraically rearranged to solve for t.

Using Mathcad, the program used for these calculations, symbolic evaluation works as shown in the example below:

Setting the number 3 symbolically equal to the number 3:

$3 = 3 = 1$ result is 1 (true)

$3 = 2 = 0$ result is 0 (false)

This ensures that the equation is rearranged correctly from step to step.

$t_1 = \frac{\sqrt{(v \cdot t_1)^2 + (y_1)^2}}{c} = 1$ --> $t_1 \cdot c = \sqrt{(v \cdot t_1)^2 + (y_1)^2} = 1$ --> $(t_1 \cdot c)^2 = (v \cdot t_1)^2 + y_1^2 = 1$ --> $(t_1 \cdot c)^2 - (v \cdot t_1)^2 = y_1^2 = 1$

$t_1^2 \cdot c^2 - v^2 \cdot t_1^2 = y_1^2 = 1$ --> $t_1^2 \cdot (c^2 - v^2) = y_1^2 = 1$ --> $t_1^2 = \frac{y_1^2}{(c^2 - v^2)} = 1$ --> $t_1 = \frac{y_1}{\sqrt{(c^2 - v^2)}} = 1$ --> $t_1 = \frac{y_1}{\sqrt{(c+v) \cdot (c-v)}} = 1$

$t_1 = y_1 \cdot \frac{1}{\sqrt{(c+v) \cdot (c-v)}} = 1$

The Lorentz factor = $\gamma := \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ or, in equivalent form: $\gamma = \frac{c \cdot \sqrt{(c+v) \cdot (c-v)}}{(c+v) \cdot (c-v)} = 1$

so, $\sqrt{(c+v) \cdot (c-v)} = \frac{\gamma \cdot (c+v) \cdot (c-v)}{c} = 1$

$t_1 = y_1 \cdot \frac{1}{\frac{\gamma \cdot (c+v) \cdot (c-v)}{c}} = 1$ --> $t_1 = y_1 \cdot \frac{c}{\gamma \cdot (c+v) \cdot (c-v)} = 1$ --> $t_1 = y_1 \cdot \frac{c \cdot \sqrt{1 - \frac{v^2}{c^2}}}{(c+v) \cdot (c-v)} = 1$ --> $t_1 = \frac{y_1}{c} \frac{c \cdot \sqrt{(c+v) \cdot (c-v)}}{(c+v) \cdot (c-v)} = 1$

$t_1 = \frac{y_1}{c} \cdot \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 1$ --> $t_1 = \frac{y_1}{c} \cdot \gamma = 1$ Due to the symmetry of the problem, the same equation will be true for t2.

$t_2 = \frac{y_1}{c} \cdot \gamma$ Note the equation highlighted in green will be used for the following calculation.

<u>Example 1 Calculations</u>
2-way Light Clock Experiment - light path perpendicular to spaceship motion

$u := c = 983571056 \frac{ft}{s}$ speed of "object" (light) used in clock mechanism

$y_1 := 100\ ft$ 1-way clock height (spaceship height)

$v := \begin{bmatrix} 0.20 \\ 0.50 \\ 0.9 \end{bmatrix} \cdot c = \begin{bmatrix} 196714211 \\ 491785528 \\ 885213951 \end{bmatrix} \frac{ft}{s}$ velocity of spaceship (a few examples provided)

Note: primed (') variables correspond to the observation of the moving reference frame (Blueman) from the perspective of the stationary reference frame (Pinkman). Unprimed (proper) variables correspond to the observations of each of their reference frames from their own reference frame.

$t'_1 := \frac{y_1}{\sqrt{c^2 - v^2}} = \begin{bmatrix} 0.000000103766852 \\ 0.000000117398792 \\ 0.000000233247748 \end{bmatrix} s$ time of leg1

$t'_2 := \frac{y_1}{\sqrt{c^2 - v^2}} = \begin{bmatrix} 0.000000103766852 \\ 0.000000117398792 \\ 0.000000233247748 \end{bmatrix} s$ time of leg2

$\Delta t_{moving} := t'_1 + t'_2 = \begin{bmatrix} 0.000000207533705 \\ 0.000000234797584 \\ 0.000000466495496 \end{bmatrix} s$ Dilated clock rate, what Pinkman sees Blueman's clock ticking at

$\Delta t_{stationary} := \frac{2 \cdot y_1}{c} = 0.000000203340672\ s$ Proper clock rate, what Pinkman and Blueman see their clock ticking at from their own perspectives

$$\frac{\Delta t_{moving}}{\Delta t_{stationary}} = \begin{bmatrix} 1.021 \\ 1.155 \\ 2.294 \end{bmatrix}$$

Ratio of dilated clock rate to proper clock rate, should match the Lorentz factor, γ

$$\gamma := \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \begin{bmatrix} 1.021 \\ 1.155 \\ 2.294 \end{bmatrix}$$

Lorentz factor

```
check := if γ = Δt_moving / Δt_stationary = "ok"
             ‖ "ok"
         else
             ‖ "no good"
```

Derivation of dilated clock rate factors for each leg

$$\frac{\Delta t_{moving}}{\Delta t_{stationary}} = \begin{bmatrix} 1.021 \\ 1.155 \\ 2.294 \end{bmatrix}$$

$$\frac{t'_1 + t'_2}{\frac{2 \cdot y_1}{c}} = \begin{bmatrix} 1.021 \\ 1.155 \\ 2.294 \end{bmatrix}$$

for leg 1

$$\gamma_{leg1} := \frac{t'_1}{\frac{y_1}{c}} = \begin{bmatrix} 1.021 \\ 1.155 \\ 2.294 \end{bmatrix}$$

for leg 2

$$\gamma_{leg2} := \frac{t'_2}{\frac{y_1}{c}} = \begin{bmatrix} 1.021 \\ 1.155 \\ 2.294 \end{bmatrix}$$

Side note: In this problem, the dilated clock rate factor is the same in leg 1 as it is for leg 2, which equals the overall dilated clock rate factor.

$$\gamma := \frac{\frac{t'_1}{\frac{y_1}{c}} + \frac{t'_2}{\frac{y_1}{c}}}{2} = \begin{bmatrix} 1.021 \\ 1.155 \\ 2.294 \end{bmatrix}$$

This is how one would expect time to dilate - uniformly. But we see the issue down in example 4, where time over-dilates in one leg and contracts in the other, and the Lorentz factor is only arrived at by taking the average of an over-dilated factor and contracted factor.

Back to derivation (leg 1 will be used, leg 2 is identical due to symmetry).

$$\frac{t'_1}{\frac{y_1}{c}} = \begin{bmatrix} 1.021 \\ 1.155 \\ 2.294 \end{bmatrix} \text{ --> } \frac{\sqrt{\frac{y_1^2}{\left(c^2 - v^2\right)}}}{\frac{y_1}{c}} = \begin{bmatrix} 1.021 \\ 1.155 \\ 2.294 \end{bmatrix} \text{ --> } \frac{\sqrt{\frac{y_1^2}{\left(c^2 - v^2\right)}}}{\frac{y_1}{c}} = \gamma_{leg1} = 1 \text{ --> } \sqrt{\frac{y_1^2}{\left(c^2 - v^2\right)}} \cdot \frac{c}{y_1} = \gamma_{leg1} = 1$$

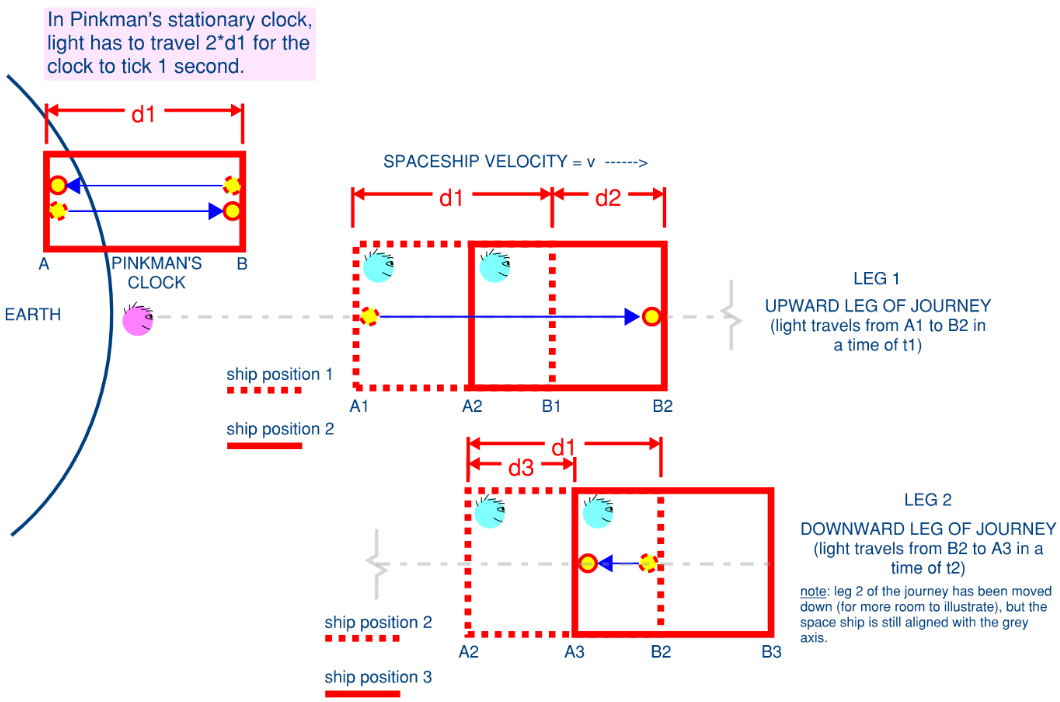
$$\frac{\sqrt{y_1^2}}{\sqrt{\left(c^2 - v^2\right)}} \cdot \frac{c}{y_1} = \gamma_{leg1} = 1 \text{ --> } \frac{y_1^2}{\left(c^2 - v^2\right)} \cdot \frac{c^2}{y_1^2} = \gamma_{leg1}^2 = 1 \text{ --> } \frac{c^2}{c^2 - v^2} = \gamma_{leg1}^2 = 1 \text{ --> } \sqrt{\frac{c^2}{c^2 - v^2}} = \gamma_{leg1} = 1$$

$$\sqrt{\frac{1}{\frac{c^2 - v^2}{c^2}}} = \gamma_{leg1} = 1 \text{ --> } \sqrt{\frac{1}{\frac{c^2}{c^2} - \frac{v^2}{c^2}}} = \gamma_{leg1} = 1 \text{ --> } \sqrt{\frac{1}{1 - \frac{v^2}{c^2}}} = \gamma_{leg1} = 1 \text{ --> } \frac{\sqrt{1}}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma_{leg1} = 1 \text{ --> } \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma_{leg1} = 1$$

Example 2 Clock mechanism used to measure time: 2-way light clock
Clock mechanism motion: light motion *parallel* to spaceship motion

Pinkman is on Earth. Blueman is in a spaceship. Blueman's spaceship is traveling directly overhead and away from Pinkman (along the grey axis).

They both have the same light clock. Light is emitted from A, bounces off a mirror at B, and returns to A. That is how the clock ticks once.



Derivation of light path distance and light travel time for moving clock:

Leg 1

Calculate the distance light travels from A1 to B2.

The light travels a total distance of Dleg1 = d1 + d2.

d1 = length of clock (similar to "d" in typical experiment above)

d2 = the distance the spaceship moves (front of spaceship moves from B1 to B2)
= velocity of the spaceship * the time it takes for the light to travel from A1 to B2
= v*t1

So, Dleg1 = d1 + v*t1. We need to calculate t1.

The total distance the light travels can also be expressed in terms of the light's speed, c, and the time it takes for the light to travel from A1 to B2, t1.

Dleg1 = velocity*time = c*t1

Now we have two expressions for Dleg1 and can set them equal.

c*t1 = d1 + v*t1 ---> c*t1 - v*t1 = d1 ---> t1(c-v) = d1
---> **t1 = d1 / (c-v)**

Leg 2

Calculate the distance light travels from B2 to A3.

The light travels a total distance of Dleg2 = d1 - d3.

d1 = length of clock

d3 = the distance the spaceship moves (back of spaceship moves from A2 to A3)
= velocity of the spaceship * the time it takes for the light to travel from B2 to A3
= v*t2

So, Dleg2 = d1 - v*t2. We need to calculate t2.

The total distance the light travels can also be expressed in terms of the light's speed, c, and the time it takes for the light to travel from B2 to A3, t2.

Dleg2 = velocity*time = c*t2

Now we have two expressions for Dleg2 and can set them equal.

c*t2 = d1 - v*t2 ---> c*t2 + v*t2 = d1 ---> t2(c+v) = d1
---> **t2 = d1 / (c+v)**

Example 2 Calculations
Adjusted 2-way Light Clock Experiment - light path parallel to spaceship motion

$u := c = 983571056 \frac{ft}{s}$ speed of "object" used in clock mechanism

$d_1 := 100\ ft$ clock length (1 way)

$v := \begin{bmatrix} 0.20 \\ 0.50 \\ 0.9 \end{bmatrix} \cdot c = \begin{bmatrix} 196714211 \\ 491785528 \\ 885213951 \end{bmatrix} \frac{ft}{s}$ velocity of spaceship (a few examples provided)

Note: primed (') variables correspond to the observation of the moving reference frame (Blueman) from the perspective of the stationary reference frame (Pinkman). Unprimed (proper) variables correspond to the observations of each of their reference frames from their own reference frame.

$d'_1 := d_1 \cdot \sqrt{1 - \frac{v^2}{c^2}} = \begin{bmatrix} 97.98 \\ 86.603 \\ 43.589 \end{bmatrix} ft$ clock length (1 way) length contracted

$u'_1 := \frac{v + u}{1 + \frac{v\ u}{c^2}} = \begin{bmatrix} 983571056 \\ 983571056 \\ 983571056 \end{bmatrix} \frac{ft}{s}$ speed of "object" used in clock mechanism in leg 1, relativistic velocity considered

$u'_2 := \frac{v - u}{1 - \frac{v \cdot u}{c^2}} = \begin{bmatrix} -983571056 \\ -983571056 \\ -983571056 \end{bmatrix} \frac{ft}{s}$ speed of "object" used in clock mechanism in leg 2, relativistic velocity considered

$u'_2 := \overrightarrow{|u'_2|} = \begin{bmatrix} 983571056 \\ 983571056 \\ 983571056 \end{bmatrix} \frac{ft}{s}$ using absolute value, sign convention is accounted for properly in equations which use this value

$t'_1 := \frac{d'_1}{u'_1 - v} = \begin{bmatrix} 0.000000124520223 \\ 0.000000176098188 \\ 0.000000443170721 \end{bmatrix} s$ time of leg 1

$d'_2 := \overrightarrow{v \cdot t'_1} = \begin{bmatrix} 24.495 \\ 86.603 \\ 392.301 \end{bmatrix} ft$

$D'_{leg1} := d'_1 + d'_2 = \begin{bmatrix} 122.474 \\ 173.205 \\ 435.89 \end{bmatrix} ft$ distance of leg 1

$t'_2 := \frac{d'_1}{u'_2 + v} = \begin{bmatrix} 0.000000083013482 \\ 0.000000058699396 \\ 0.000000023324775 \end{bmatrix} s$ time of leg 2

$d'_3 := \overrightarrow{v \cdot t'_2} = \begin{bmatrix} 16.33 \\ 28.868 \\ 20.647 \end{bmatrix} ft$

$D'_{leg2} := d'_1 - d'_3 = \begin{bmatrix} 81.65 \\ 57.735 \\ 22.942 \end{bmatrix} ft$ distance of leg 2

$D'_{leg1} = \begin{bmatrix} 122.474 \\ 173.205 \\ 435.89 \end{bmatrix} ft$ $D'_{leg2} = \begin{bmatrix} 81.65 \\ 57.735 \\ 22.942 \end{bmatrix} ft$ distance of leg 1 & 2 summary

$D'total := D'_{leg1} + D'_{leg2} = \begin{bmatrix} 204.124 \\ 230.94 \\ 458.831 \end{bmatrix} ft$ total distance light traveled to tick

$2 \cdot d_1 = 200\ ft$ $D'total = \begin{bmatrix} 204.124 \\ 230.94 \\ 458.831 \end{bmatrix} ft$ total distance light traveled to tick summary (proper on left)

$\Delta t_{stationary} := \frac{2 \cdot d_1}{u} = 0.000000203340672\ s$ — Proper clock rate, what Pinkman and Blueman see their clock ticking at from their own perspectives

$\Delta t_{moving} := \frac{D'_{leg1}}{u'_1} + \frac{D'_{leg2}}{u'_2} = \begin{bmatrix} 0.000000207533705 \\ 0.000000234797584 \\ 0.000000466495496 \end{bmatrix} s$ — Dilated clock rate, what Pinkman sees Blueman's clock ticking at

$\frac{\Delta t_{moving}}{\Delta t_{stationary}} = \begin{bmatrix} 1.021 \\ 1.155 \\ 2.294 \end{bmatrix}$ for, $v = \begin{bmatrix} 0.20 \\ 0.50 \\ 0.9 \end{bmatrix} \cdot c$ — Ratio of dilated clock rate to proper clock rate, should match Lorentz factor, γ

$\gamma := \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \begin{bmatrix} 1.021 \\ 1.155 \\ 2.294 \end{bmatrix}$ Lorentz factor

```
check := if γ = Δt_moving / Δt_stationary   = "ok"
             "ok"
         else
             "no good"
```

Check the general Lorentz transformations

General Lorentz transformations

$$x' = \gamma(x - vt), \quad x = \gamma(x' + vt'),$$
$$y' = y, \quad y = y',$$
$$z' = z, \quad z = z',$$
$$t' = \gamma(t - vx/c^2), \quad t = \gamma(t' + vx'/c^2).$$

	Leg 1		Leg 2
(1)	$t' = \gamma \cdot \left(t_1 + \frac{v \cdot d_1}{c^2}\right)$	+	$t' = \gamma \cdot \left(t_2 - \frac{v \cdot d_2}{c^2}\right)$
(2)	$d' = \gamma \cdot (d_1 + v \cdot t_1)$	+	$d' = \gamma \cdot (d_2 - v \cdot t_2)$

$t_1 := \frac{d_1}{u} = 0.000000101670336\ s$ $\quad t_2 := \frac{d_1}{u} = 0.000000101670336\ s$

$d_1 := t_1 \cdot c = 100\ ft$ $\quad d_2 := t_2 \cdot c = 100\ ft$

(1) $t' := \overrightarrow{\gamma \cdot \left(t_1 + \frac{v \cdot d_1}{c^2}\right) + \gamma \cdot \left(t_2 - \frac{v \cdot d_2}{c^2}\right)} = \begin{bmatrix} 0.000000207533705 \\ 0.000000234797584 \\ 0.000000466495496 \end{bmatrix} s$

$\Delta t_{moving} = \begin{bmatrix} 0.000000207533705 \\ 0.000000234797584 \\ 0.000000466495496 \end{bmatrix} s$

```
check := if t' = Δt_moving   = "ok"
             "ok"
         else
             "no good"
```

(2) $d' := \overrightarrow{\gamma \cdot (d_1 + v \cdot t_1) + \gamma \cdot (d_2 - v \cdot t_2)} = \begin{bmatrix} 204.124 \\ 230.94 \\ 458.831 \end{bmatrix} ft$

$D'total = \begin{bmatrix} 204.124 \\ 230.94 \\ 458.831 \end{bmatrix} ft$

```
check := if d' = D'total   = "ok"
             "ok"
         else
             "no good"
```

Derivation of Lorentz factor from calculations

First, the full equation (using the starting variables) is sought. Variables are plugged back in to the calculated Lorentz factor and worked backwards to get an equation which is composed of the starting variables.

$$\frac{\Delta t_{moving}}{\Delta t_{stationary}} = \begin{bmatrix}1.021\\1.155\\2.294\end{bmatrix} \qquad \gamma := \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \begin{bmatrix}1.021\\1.155\\2.294\end{bmatrix}$$

gamma shown for reference

$$\frac{\frac{D'_{leg1}}{u'_1}+\frac{D'_{leg2}}{u'_2}}{\frac{2\cdot d_1}{u}} = \begin{bmatrix}1.021\\1.155\\2.294\end{bmatrix}$$

showing the calculated values ensures that when plugging variables back in, the Lorentz factor is being computed properly

$$\frac{\frac{d'_1+d'_2}{u'_1}+\frac{d'_1-d'_3}{u'_2}}{\frac{2\cdot d_1}{u}} = \begin{bmatrix}1.021\\1.155\\2.294\end{bmatrix} \text{ --> } \frac{\frac{d'_1+\overrightarrow{v\cdot t'_1}}{u'_1}+\frac{d'_1-\overrightarrow{v\cdot t'_2}}{u'_2}}{\frac{2\cdot d_1}{u}} = \begin{bmatrix}1.021\\1.155\\2.294\end{bmatrix} \text{ --> } \frac{\frac{d'_1+v\cdot\overrightarrow{\frac{d'_1}{u'_1-v}}}{u'_1}+\frac{d'_1-v\cdot\overrightarrow{\frac{d'_1}{u'_2+v}}}{u'_2}}{\frac{2\cdot d_1}{u}} = \begin{bmatrix}1.021\\1.155\\2.294\end{bmatrix}$$

$$\frac{\frac{d'_1+v\cdot\overrightarrow{\frac{d'_1}{u'_1-v}}}{\frac{v+u}{1+\frac{v\,u}{c^2}}}+\frac{d'_1-v\cdot\overrightarrow{\frac{d'_1}{u'_2+v}}}{\left|\overrightarrow{\frac{v-u}{1-\frac{v\cdot u}{c^2}}}\right|}}{\frac{2\cdot d_1}{u}} = \begin{bmatrix}1.021\\1.155\\2.294\end{bmatrix} \text{ --> } \frac{\frac{d'_1+v\cdot\overrightarrow{\frac{d'_1}{\frac{v+u}{1+\frac{v\cdot u}{c^2}}-v}}}{\frac{v+u}{1+\frac{v\,u}{c^2}}}+\frac{d'_1-v\cdot\overrightarrow{\frac{d'_1}{\left|\overrightarrow{\frac{v-u}{1-\frac{v\cdot u}{c^2}}}\right|+v}}}{\left|\overrightarrow{\frac{v-u}{1-\frac{v\cdot u}{c^2}}}\right|}}{\frac{2\cdot d_1}{u}} = \begin{bmatrix}1.021\\1.155\\2.294\end{bmatrix}$$

$$\frac{\frac{d_1\cdot\sqrt{1-\frac{v^2}{c^2}}+v\cdot\overrightarrow{\frac{d_1\cdot\sqrt{1-\frac{v^2}{c^2}}}{\frac{v+u}{1+\frac{v\cdot u}{c^2}}-v}}}{\frac{v+u}{1+\frac{v\,u}{c^2}}}+\frac{d_1\cdot\sqrt{1-\frac{v^2}{c^2}}-v\cdot\overrightarrow{\frac{d_1\cdot\sqrt{1-\frac{v^2}{c^2}}}{\left|\overrightarrow{\frac{v-u}{1-\frac{v\cdot u}{c^2}}}\right|+v}}}{\left|\overrightarrow{\frac{v-u}{1-\frac{v\cdot u}{c^2}}}\right|}}{\frac{2\cdot d_1}{u}} = \begin{bmatrix}1.021\\1.155\\2.294\end{bmatrix}$$

for,

$$v = \begin{bmatrix}0.20\\0.50\\0.9\end{bmatrix}\cdot c$$

The full equation has now been expressed.

Now, using this full equation, it will be algebraically simplified to arrive at the Lorentz factor equation: $\gamma := \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \begin{bmatrix}1.021\\1.155\\2.294\end{bmatrix}$

$$\frac{\frac{d_1\cdot\sqrt{1-\frac{v^2}{c^2}}+v\cdot\overrightarrow{\frac{d_1\cdot\sqrt{1-\frac{v^2}{c^2}}}{\frac{v+u}{1+\frac{v\cdot u}{c^2}}-v}}}{\frac{v+u}{1+\frac{v\,u}{c^2}}}+\frac{d_1\cdot\sqrt{1-\frac{v^2}{c^2}}-v\cdot\overrightarrow{\frac{d_1\cdot\sqrt{1-\frac{v^2}{c^2}}}{\left|\overrightarrow{\frac{v-u}{1-\frac{v\cdot u}{c^2}}}\right|+v}}}{\left|\overrightarrow{\frac{v-u}{1-\frac{v\cdot u}{c^2}}}\right|}}{\frac{2\cdot d_1}{u}} = \gamma = 1$$

Equations are set equal to 1 (true) to ensure simplification is being performed correctly

$$\frac{d_1\cdot\sqrt{1-\frac{v^2}{c^2}}+v\cdot\overrightarrow{\frac{d_1\cdot\sqrt{1-\frac{v^2}{c^2}}}{\frac{v+u}{1+\frac{v\cdot u}{c^2}}-v}}}{\frac{v+u}{1+\frac{v\,u}{c^2}}}+\frac{d_1\cdot\sqrt{1-\frac{v^2}{c^2}}-v\cdot\overrightarrow{\frac{d_1\cdot\sqrt{1-\frac{v^2}{c^2}}}{\left|\overrightarrow{\frac{v-u}{1-\frac{v\cdot u}{c^2}}}\right|+v}}}{\left|\overrightarrow{\frac{v-u}{1-\frac{v\cdot u}{c^2}}}\right|}=\gamma\cdot\frac{2\cdot d_1}{u}=1$$

$$\frac{d_1\cdot\frac{1}{\gamma}+v\cdot\overrightarrow{\frac{d_1\cdot\frac{1}{\gamma}}{\frac{v+u}{1+\frac{v\,u}{c^2}}-v}}}{\frac{v+u}{1+\frac{v\,u}{c^2}}}+\frac{d_1\cdot\frac{1}{\gamma}-v\cdot\overrightarrow{\frac{d_1\cdot\frac{1}{\gamma}}{\left|\overrightarrow{\frac{v-u}{1-\frac{v\cdot u}{c^2}}}\right|+v}}}{\left|\overrightarrow{\frac{v-u}{1-\frac{v\cdot u}{c^2}}}\right|}=\gamma\cdot\frac{2\cdot d_1}{u}=1$$

$$\frac{d_1\cdot\frac{1}{\gamma}+v\cdot\overrightarrow{\frac{d_1\cdot\frac{1}{\gamma}}{\frac{c^2\,(v+u)}{c^2+v\cdot u}-v}}}{\frac{c^2\,(v+u)}{c^2+v\cdot u}}+\frac{d_1\cdot\frac{1}{\gamma}-v\cdot\overrightarrow{\frac{d_1\cdot\frac{1}{\gamma}}{\left|\overrightarrow{\frac{c^2\,(v-u)}{c^2-v\cdot u}}\right|+v}}}{\left|\overrightarrow{\frac{c^2\,(v-u)}{c^2-v\cdot u}}\right|}=\gamma\cdot\frac{2\cdot d_1}{u}=1$$

$$\frac{\overrightarrow{d_1\cdot\frac{1}{\gamma}\cdot\left(1+\frac{v}{\frac{c^2\,(v+u)}{c^2+v\cdot u}-v}\right)}}{\frac{c^2\,(v+u)}{c^2+v\cdot u}}+\frac{\overrightarrow{d_1\cdot\frac{1}{\gamma}\cdot\left(1-\frac{v}{\left|\frac{c^2\,(v-u)}{c^2-v\cdot u}\right|+v}\right)}}{\left|\overrightarrow{\frac{c^2\,(v-u)}{c^2-v\cdot u}}\right|}=\gamma\cdot\frac{2\cdot d_1}{u}=1$$

$$\overrightarrow{d_1\cdot\frac{1}{\gamma}\cdot\frac{\left(1+\frac{v}{\frac{c^2\,(v+u)}{c^2+v\cdot u}-v}\right)}{\frac{c^2\,(v+u)}{c^2+v\cdot u}}+d_1\cdot\frac{1}{\gamma}\cdot\left(\frac{1-\frac{v}{\left|\frac{c^2\,(v-u)}{c^2-v\cdot u}\right|+v}}{\left|\frac{c^2\,(v-u)}{c^2-v\cdot u}\right|}\right)}=d_1\cdot 2\cdot\frac{\gamma}{u}=1$$

$$\overrightarrow{\frac{1}{\gamma}\cdot\frac{\left(1+\frac{v}{\frac{c^2\,(v+u)}{c^2+v\cdot u}-v}\right)}{\frac{c^2\,(v+u)}{c^2+v\cdot u}}+\frac{1}{\gamma}\cdot\left(\frac{1-\frac{v}{\left|\frac{c^2\,(v-u)}{c^2-v\cdot u}\right|+v}}{\left|\frac{c^2\,(v-u)}{c^2-v\cdot u}\right|}\right)}=2\cdot\frac{\gamma}{u}=1$$

$$\overrightarrow{\frac{1}{\gamma}\cdot\left(\frac{1+\frac{v}{\frac{c^2\,(v+u)}{c^2+v\cdot u}-v}}{\frac{c^2\,(v+u)}{c^2+v\cdot u}}+\frac{1-\frac{v}{\left|\frac{c^2\,(v-u)}{c^2-v\cdot u}\right|+v}}{\left|\frac{c^2\,(v-u)}{c^2-v\cdot u}\right|}\right)}=2\cdot\frac{\gamma}{u}=1$$

$$\overrightarrow{\frac{1+\frac{v}{\frac{c^2\,(v+u)}{c^2+v\cdot u}-v}}{\frac{c^2\,(v+u)}{c^2+v\cdot u}}+\frac{1-\frac{v}{\left|\frac{c^2\,(v-u)}{c^2-v\cdot u}\right|+v}}{\left|\frac{c^2\,(v-u)}{c^2-v\cdot u}\right|}}=2\cdot\frac{\gamma^2}{u}=1$$

$$\overrightarrow{\left(1+\frac{v}{\frac{c^2\,(v+u)}{c^2+v\cdot u}-v}\right)\cdot\left(\frac{c^2+v\cdot u}{c^2\,(v+u)}\right)+\left(1-\frac{v}{\left|\frac{c^2\,(v-u)}{c^2-v\cdot u}\right|+v}\right)\cdot\left(\left|\frac{c^2-v\cdot u}{c^2\,(v-u)}\right|\right)}=2\cdot\frac{\gamma^2}{u}=1$$

$$\overrightarrow{\left(1+\frac{v}{\frac{u\cdot(c+v)\,(c-v)}{c^2+v\cdot u}}\right)\cdot\left(\frac{c^2+v\cdot u}{c^2\,(v+u)}\right)+\left(1-\frac{v}{\frac{u\cdot(c+v)\,(c-v)}{c^2-v\cdot u}}\right)\cdot\left(\left|\frac{c^2-v\cdot u}{c^2\,(v-u)}\right|\right)}=2\cdot\frac{\gamma^2}{u}=1$$

$$\overrightarrow{\left(1+\frac{c^2\cdot v+v^2\cdot u}{u\cdot(c+v)\,(c-v)}\right)\cdot\left(\frac{c^2+v\cdot u}{c^2\,(v+u)}\right)+\left(1-\frac{c^2\cdot v-v^2\cdot u}{u\cdot(c+v)\,(c-v)}\right)\cdot\left(\left|\frac{c^2-v\cdot u}{c^2\,(v-u)}\right|\right)}=2\cdot\frac{\gamma^2}{u}=1$$

$$\overrightarrow{\frac{c^2+v\cdot u}{u\cdot(c+v)\cdot(c-v)}+\frac{c^2-v\cdot u}{u\cdot(c+v)\cdot(c-v)}}=2\cdot\frac{\gamma^2}{u}=1$$

$\overrightarrow{\frac{2\cdot c^2}{u\cdot(c+v)\cdot(c-v)}}=2\cdot\frac{\gamma^2}{u}=1$ --> $\overrightarrow{\frac{c^2}{(c+v)\cdot(c-v)}}=\gamma^2=1$ --> $\overrightarrow{\frac{c\cdot\sqrt{(c+v)\cdot(c-v)}}{(c+v)\cdot(c-v)}}=\gamma=1$ --> $\overrightarrow{\frac{c}{\sqrt{(c+v)\cdot(c-v)}}}=\gamma=1$

$\frac{c}{\sqrt{c^2-v^2}}=\gamma=1$ --> $\frac{1}{\frac{\sqrt{c^2-v^2}}{c}}=\gamma=1$ --> $\frac{1}{\sqrt{\frac{c^2}{c^2}-\frac{v^2}{c^2}}}=\gamma=1$ --> $\frac{1}{\sqrt{\frac{c}{c}-\frac{v^2}{c^2}}}=\gamma=1$ --> $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}=\gamma=1$

Derive the time transformation

General Lorentz transformations

$$x'=\gamma(x-vt),\qquad x=\gamma(x'+vt'),$$
$$y'=y,\qquad y=y',$$
$$z'=z,\qquad z=z',$$
$$t'=\gamma(t-vx/c^2),\qquad t=\gamma(t'+vx'/c^2).\qquad(1)$$

Leg 1: $t'=\gamma\cdot\left(t_1+\frac{v\cdot d_1}{c^2}\right)$ + Leg 2: $t'=\gamma\cdot\left(t_2-\frac{v\cdot d_2}{c^2}\right)$

$t'=t'(ofLeg1)+t'(ofLeg2)$

Transformed clock rate (what Pinkman sees Blueman's clock ticking at)

$$t':=\overrightarrow{\gamma\cdot\left(t_1+\frac{v\cdot d_1}{c^2}\right)+\gamma\cdot\left(t_2-\frac{v\cdot d_2}{c^2}\right)}=\begin{bmatrix}0.000000207533705\\0.000000234797584\\0.000000466495496\end{bmatrix}s$$

From calculations in example above, this value was computed to be

$$\Delta t_{moving}=\begin{bmatrix}0.000000207533705\\0.000000234797584\\0.000000466495496\end{bmatrix}s$$

Setting $t'=\Delta t_{moving}$, derive the general transformation:

$t'=\Delta t_{moving}=1$ --> $t'=\frac{D'_{leg1}}{u'_1}+\frac{D'_{leg2}}{u'_2}=1$ --> $t'=\frac{d'_1+d'_2}{u'_1}+\frac{d'_1-d'_3}{u'_2}=1$ --> $t'=\frac{d_1\cdot\sqrt{1-\frac{v^2}{c^2}}+d'_2}{u'_1}+\frac{d_1\cdot\sqrt{1-\frac{v^2}{c^2}}-d'_3}{u'_2}=1$

$t'=\frac{d_1\cdot\sqrt{1-\frac{v^2}{c^2}}+\overrightarrow{v\cdot t'_1}}{u'_1}+\frac{d_1\cdot\sqrt{1-\frac{v^2}{c^2}}-\overrightarrow{v\cdot t'_2}}{u'_2}=1$ --> $t'=\frac{\frac{d_1}{\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}}+\overrightarrow{v\cdot t'_1}}{u'_1}+\frac{\frac{d_1}{\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}}-\overrightarrow{v\cdot t'_2}}{u'_2}=1$

$$t'=\frac{\frac{d_1}{\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}}+v\cdot\overrightarrow{\frac{d_1\cdot\sqrt{1-\frac{v^2}{c^2}}}{u'_1-v}}}{u'_1}+\frac{\frac{d_1}{\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}}-v\cdot\overrightarrow{\frac{d_1\cdot\sqrt{1-\frac{v^2}{c^2}}}{u'_1+v}}}{u'_2}=1$$

$$t' = \frac{\dfrac{d_1}{\dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}} + \dfrac{v}{\dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}} \cdot \dfrac{d_1}{u'_1 - v}}{u'_1} + \frac{\dfrac{d_1}{\dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}} - \dfrac{v}{\dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}} \cdot \dfrac{d_1}{u'_1 + v}}{u'_2} = 1$$

$$t' = \frac{1}{u'_1} \cdot \left(\frac{d_1}{\dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}} + \frac{v}{\dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}} \cdot \frac{d_1}{u'_1 - v} \right) + \frac{1}{u'_2} \cdot \left(\frac{d_1}{\dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}} - \frac{v}{\dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}} \cdot \frac{d_1}{u'_1 + v} \right) = 1$$

$$t' = \left(\frac{1}{u'_1} + \frac{1}{u'_2} \right) \cdot \left(\frac{d_1}{\dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}} \right) + \frac{1}{u'_1} \cdot \left(\frac{v}{\dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}} \cdot \frac{d_1}{u'_1 - v} \right) - \frac{1}{u'_2} \cdot \left(\frac{v}{\dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}} \cdot \frac{d_1}{u'_1 + v} \right) = 1$$

$$t' = \left(\frac{1}{u'_1} + \frac{1}{u'_2} \right) \cdot \left(\frac{d_1}{\dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}} \right) + \frac{v \cdot \sqrt{(c+v) \cdot (c-v)}}{u'_1 \cdot c \cdot (u'_1 - v)} \cdot d_1 - \frac{v \cdot \sqrt{(c+v) \cdot (c-v)}}{u'_2 \cdot c \cdot (u'_2 + v)} \cdot d_1 = 1$$

$$t' = \frac{\sqrt{(c+v) \cdot (c-v)}}{u'_1 \cdot u'_2 \cdot c} \cdot (u'_1 + u'_2) \cdot d_1 + \frac{v \cdot \sqrt{(c+v) \cdot (c-v)}}{u'_1 \cdot c \cdot (u'_1 - v)} \cdot d_1 - \frac{v \cdot \sqrt{(c+v) \cdot (c-v)}}{u'_2 \cdot c \cdot (u'_2 + v)} \cdot d_1 = 1$$

$$t' = \sqrt{(c+v) \cdot (c-v)} \cdot d_1 \cdot \left(\frac{u'_1 + u'_2}{u'_1 \cdot u'_2 \cdot c} + \frac{v}{u'_1 \cdot c \cdot (u'_1 - v)} - \frac{v}{u'_2 \cdot c \cdot (u'_2 + v)} \right) = 1$$

$$t' = \sqrt{(c+v) \cdot (c-v)} \cdot d_1 \cdot \left(\frac{u'_1 + u'_2}{c \cdot (u'_1 - v) \cdot (u'_2 + v)} \right) = 1$$ --> $$t' = \frac{d_1 \cdot (u'_1 + u'_2)}{(u'_1 - v) \cdot (u'_2 + v)} \cdot \frac{\sqrt{(c+v) \cdot (c-v)}}{c} = 1$$

$$t' = \frac{2 \cdot d_1 \cdot c}{(u'_1 - v) \cdot (u'_2 + v)} \cdot \frac{\sqrt{(c+v) \cdot (c-v)}}{c} = 1$$ --> $$t' = \frac{2 \cdot d_1}{c} \cdot \frac{c \cdot \sqrt{(c+v) \cdot (c-v)}}{(c+v) \cdot (c-v)} = 1$$

γ has another form, $\gamma = \frac{c \cdot \sqrt{(c+v) \cdot (c-v)}}{(c+v) \cdot (c-v)} = 1$, substitute γ for this $t' = \frac{2 \cdot d_1}{c} \cdot \gamma = 1$

The proper time of of legs 1 and 2 are $t_1 := \frac{d_1}{c}$ $t_2 := \frac{d_2}{c}$ and, $d_1 = d_2$
so, $t_1 = t_2$

$t' = 2 \cdot t_1 \cdot \gamma = 1$

$t' = \gamma \cdot (t_1 + t_2) = 1$

Simplifying the general Lorentz transformation for this example,

$$t' = \gamma \cdot \left(t_1 + \frac{v \cdot d_1}{c^2} \right) + \gamma \cdot \left(t_2 - \frac{v \cdot d_1}{c^2} \right) = 1$$ --> $$t' = \gamma \cdot t_1 + \gamma \cdot \frac{v \cdot d_1}{c^2} + \gamma \cdot t_2 - \gamma \cdot \frac{v \cdot d_1}{c^2} = 1$$ --> $$t' = \gamma \cdot t_1 + \gamma \cdot \frac{v \cdot d_1}{c^2} + \gamma \cdot t_2 - \gamma \cdot \frac{v \cdot d_1}{c^2} = 1$$

$t' = \gamma \cdot t_1 + \gamma \cdot t_2 = 1$ --> $t' = \gamma \cdot (t_1 + t_2) = 1$

Derive the position transformation

General Lorentz transformations

$$x' = \gamma(x - vt), \qquad x = \gamma(x' + vt'),$$
$$y' = y, \qquad y = y',$$
$$z' = z, \qquad z = z',$$
$$t' = \gamma(t - vx/c^2), \qquad t = \gamma(t' + vx'/c^2).$$

	Leg 1		Leg 2
(2)	$d' = \gamma \cdot (d_1 + v \cdot t_1)$	+	$d' = \gamma \cdot (d_2 - v \cdot t_2)$

Transformed distance light has to travel to "tick" once (how far Pinkman observes Blueman's light in his clock to travel to tick)

$$d' = d'(ofLeg1) + d'(ofLeg2)$$

$$d' := \overrightarrow{\gamma \cdot (d_1 + v \cdot t_1) + \gamma \cdot (d_2 - v \cdot t_2)} = \begin{bmatrix} 204.124 \\ 230.94 \\ 458.831 \end{bmatrix} ft$$

From calculations in example above, this value was computed to be

$$D'total = \begin{bmatrix} 204.124 \\ 230.94 \\ 458.831 \end{bmatrix} ft$$

Setting $d' = D'total$, derive the general transformation:

$$d' = D'total = 1 \quad \text{-->} \quad d' = D'_{leg1} + D'_{leg2} = 1 \quad \text{-->} \quad d' = d'_1 + d'_2 + d'_1 - d'_3 = 1 \quad \text{-->} \quad d' = d_1 \cdot \sqrt{1 - \frac{v^2}{c^2}} + d'_2 + d_1 \cdot \sqrt{1 - \frac{v^2}{c^2}} - d'_3 = 1$$

$$d' = d_1 \cdot \sqrt{1 - \frac{v^2}{c^2}} + \overrightarrow{v \cdot t'_1} + d_1 \cdot \sqrt{1 - \frac{v^2}{c^2}} - \overrightarrow{v \cdot t'_2} = 1 \quad \text{-->} \quad d' = \frac{d_1}{\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}} + \overrightarrow{v \cdot t'_1} + \frac{d_1}{\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}} - \overrightarrow{v \cdot t'_2} = 1$$

$$d' = \frac{d_1}{\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}} + v \cdot \overrightarrow{\frac{d_1 \cdot \sqrt{1 - \frac{v^2}{c^2}}}{u'_1 - v}} + \frac{d_1}{\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}} - v \cdot \overrightarrow{\frac{d_1 \cdot \sqrt{1 - \frac{v^2}{c^2}}}{u'_1 + v}} = 1$$

$$d' = \frac{d_1}{\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}} + \overrightarrow{\frac{v}{\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}} \cdot \frac{d_1}{u'_1 - v}} + \frac{d_1}{\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}} - \overrightarrow{\frac{v}{\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}} \cdot \frac{d_1}{u'_1 + v}} = 1 \quad \text{-->} \quad d' = 2 \cdot \left(\frac{d_1}{\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}} \right) + \overrightarrow{\frac{v}{\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}} \cdot \frac{d_1}{u'_1 - v}} - \overrightarrow{\frac{v}{\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}} \cdot \frac{d_1}{u'_1 + v}} = 1$$

$$d' = 2 \cdot \left(\frac{d_1}{\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}} \right) + \overrightarrow{\frac{v}{\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}} \cdot \left(\frac{d_1}{u'_1 - v} - \frac{d_1}{u'_1 + v} \right)} = 1 \quad \text{-->} \quad d' = 2 \cdot \left(\frac{d_1}{\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}} \right) + \overrightarrow{\frac{v \cdot \sqrt{(c+v) \cdot (c-v)}}{c} \cdot \left(\frac{d_1}{u'_1 - v} - \frac{d_1}{u'_1 + v} \right)} = 1$$

$$d' = \overrightarrow{2 \cdot \left(\frac{d_1}{\frac{c \cdot \sqrt{(c+v) \cdot (c-v)}}{(c+v) \cdot (c-v)}} \right) + \frac{v \cdot \sqrt{(c+v) \cdot (c-v)}}{c} \cdot \left(\frac{d_1}{u'_1 - v} - \frac{d_1}{u'_1 + v} \right)} = 1$$

$$d' = \overrightarrow{2 \cdot \left(d_1 \cdot \frac{(c+v) \cdot (c-v)}{c \cdot \sqrt{(c+v) \cdot (c-v)}} \right) + \frac{v \cdot \sqrt{(c+v) \cdot (c-v)}}{c} \cdot \left(\frac{d_1}{u'_1 - v} - \frac{d_1}{u'_1 + v} \right)} = 1$$

$$d' = \overrightarrow{2 \cdot d_1 \cdot \frac{(c+v) \cdot (c-v)}{c \cdot \sqrt{(c+v) \cdot (c-v)}} + 2 \cdot \frac{\sqrt{(c+v) \cdot (c-v)}}{c \cdot (c+v) \cdot (c-v)} \cdot d_1 \cdot v^2} = 1 \quad \text{-->} \quad d' = \overrightarrow{2 \cdot d_1 \cdot \frac{(c+v) \cdot (c-v)}{c \cdot \sqrt{(c+v) \cdot (c-v)}} + 2 \cdot \frac{\sqrt{(c+v) \cdot (c-v)}}{c \cdot (c+v) \cdot (c-v)} \cdot d_1 \cdot v^2} = 1$$

$$d' = \overrightarrow{2 \cdot d_1 \cdot \left(\frac{(c+v) \cdot (c-v)}{c \cdot \sqrt{(c+v) \cdot (c-v)}} + \frac{\sqrt{(c+v) \cdot (c-v)}}{c \cdot (c+v) \cdot (c-v)} \cdot v^2 \right)} = 1 \quad \text{-->} \quad d' = \overrightarrow{2 \cdot d_1 \cdot \frac{c \cdot \sqrt{(c+v) \cdot (c-v)}}{(c+v) \cdot (c-v)}} = 1$$

γ has another form, $\gamma = \overrightarrow{\dfrac{c \cdot \sqrt{(c+v) \cdot (c-v)}}{(c+v) \cdot (c-v)}} = 1$, substitute γ for this $d' = 2 \cdot d_1 \cdot \gamma = 1$

The proper distance of legs 1 and 2 is $d_1 := t_1 \cdot c$ $d_2 := t_2 \cdot c$ and, $t_1 = t_2 = 1$

so, $d_1 = d_2 = 1$

$d' = 2 \cdot d_1 \cdot \gamma = 1$

$d' = \gamma \cdot (d_1 + d_2) = 1$

Simplifying the general Lorentz transformation for this example,

$d' = \overrightarrow{\gamma \cdot (d_1 + v \cdot t_1) + \gamma \cdot (d_2 - v \cdot t_2)} = 1$

$d' = \gamma \cdot d_1 + \gamma \cdot v \cdot t_1 + \gamma \cdot d_2 - \gamma \cdot v \cdot t_2 = 1$

$d' = \gamma \cdot d_1 + \gamma \cdot d_2 = 1$

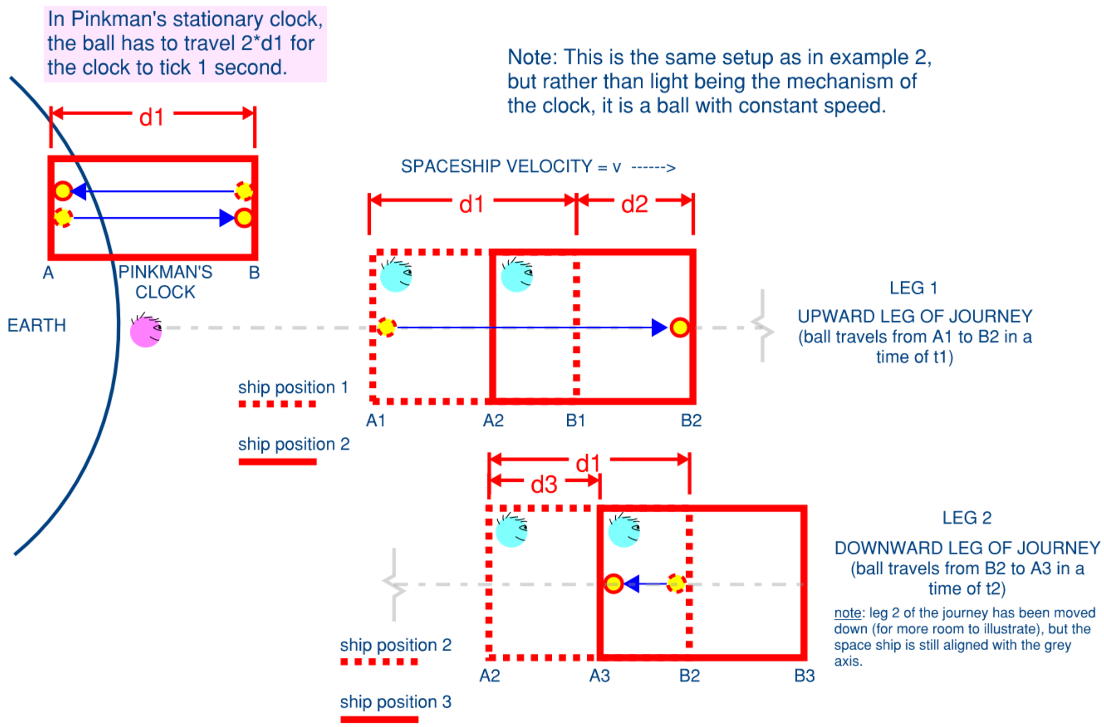
$d' = \gamma \cdot (d_1 + d_2) = 1$

The Lorentz factor and Lorentz general transformations were able to be derived for this example of a light clock with light motion parallel to the spaceship's motion.

Example 3 Clock mechanism used to measure time: 2-way ball clock
Clock mechanism motion: ball motion *parallel* to spaceship motion

In this scenario we will use a different type of clock. This clock will be very similar to the parallel motion light clock, but with the following changes:

1. Light will not be used, but rather a ball.
2. The ball's motion will follow the same motion as the parallel motion light clock.
3. Like the light, the ball is emitted from A, bounces off B, and returns to A. This is how the clock ticks.
4. The velocity of the ball will be set equal to 0.5*the speed of light, c



Derivation of light path distance and light travel time for moving clock:

Leg 1

Calculate the distance the ball travels from A1 to B2.

The ball travels a total distance of Dleg1 = d1 + d2.

d1 = length of clock

d2 = the distance the spaceship moves (front of spaceship moves from B1 to B2)
= velocity of the spaceship * the time it takes for the ball to travel from A1 to B2
= v*t1

So, Dleg1 = d1 + v*t1. We need to calculate t1.

The total distance the ball travels can also be expressed in terms of the ball's speed, u', and the time it takes for the ball to travel from A1 to B2, t1.

Dleg1 = velocity*time = u'*t1

Now we have two expressions for Dleg1 and can set them equal.

u'*t1 = d1 + v*t1 ---> u'*t1 - v*t1 = d1 ---> (u'-v)*t1 = d1
---> **t1 = d1 / (u'-v)**

Leg 2

Calculate the distance ball travels from B2 to A3.

The ball travels a total distance of Dleg2 = d1 - d3.

d1 = length of clock

d3 = the distance the spaceship moves (back of spaceship moves from A2 to A3)
= velocity of the spaceship * the time it takes for the ball to travel from B2 to A3
= v*t2

So, Dleg2 = d1 - v*t2. We need to calculate t2.

The total distance the ball travels can also be expressed in terms of the ball's speed, u', and the time it takes for the ball to travel from B2 to A3, t2.

Dleg2 = velocity*time = u'*t2

Now we have two expressions for Dleg2 and can set them equal.

u'*t2 = d1 - v*t2 ---> u'*t2 + v*t2 = d1 ---> (u'+v)*t2 = d1
---> **t2 = d1 / (u'+v)**

Example 3 Calculations
Adjusted 2-way Ball Clock Experiment - ball path parallel to spaceship motion

$u := 0.5 \cdot c = 491785528.215 \ \frac{ft}{s}$ speed of object (ball) used in clock mechanism

$d_1 := 100 \ ft$ clock length (1 way)

$v := \begin{bmatrix} 0.10 \\ 0.15 \\ 0.20 \end{bmatrix} \cdot c = \begin{bmatrix} 98357106 \\ 147535658 \\ 196714211 \end{bmatrix} \frac{ft}{s}$ velocity of spaceship

Note: primed (') variables correspond to the observation of the moving reference frame (Blueman) from the perspective of the stationary reference frame (Pinkman). Unprimed (proper) variables correspond to the observations of each of their reference frames from their own reference frame.

$d'_1 := d_1 \cdot \sqrt{1 - \frac{v^2}{c^2}} = \begin{bmatrix} 99.499 \\ 98.869 \\ 97.98 \end{bmatrix} ft$ clock length (1 way) length contracted

$u'_1 := \frac{v + u}{1 + \frac{v \cdot u}{c^2}} = \begin{bmatrix} 562040604 \\ 594717383 \\ 625908854 \end{bmatrix} \frac{ft}{s}$ speed of object used in clock mechanism in leg 1, relativistic velocity considered

$u'_2 := \frac{v - u}{1 - \frac{v \cdot u}{c^2}} = \begin{bmatrix} -414135182 \\ -372162021 \\ -327857019 \end{bmatrix} \frac{ft}{s}$ speed of object used in clock mechanism in leg 2, relativistic velocity considered

$u'_2 := \overrightarrow{|u'_2|} = \begin{bmatrix} 414135182 \\ 372162021 \\ 327857019 \end{bmatrix} \frac{ft}{s}$ using absolute value, sign convention is accounted for properly in equations which use this value

$t'_1 := \frac{d'_1}{u'_1 - v} = \begin{bmatrix} 0.000000214583318 \\ 0.000000221092666 \\ 0.000000228287075 \end{bmatrix} s$ time of leg 1

$d'_2 := \overrightarrow{v \cdot t'_1} = \begin{bmatrix} 21.106 \\ 32.619 \\ 44.907 \end{bmatrix} ft$

$D'_{leg1} := d'_1 + d'_2 = \begin{bmatrix} 120.605 \\ 131.488 \\ 142.887 \end{bmatrix} ft$ distance of leg 1

$t'_2 := \frac{d'_1}{u'_2 + v} = \begin{bmatrix} 0.000000194146812 \\ 0.000000190242527 \\ 0.000000186780334 \end{bmatrix} s$ time of leg 2

$d'_3 := \overrightarrow{v \cdot t'_2} = \begin{bmatrix} 19.096 \\ 28.068 \\ 36.742 \end{bmatrix} ft$

$D'_{leg2} := d'_1 - d'_3 = \begin{bmatrix} 80.403 \\ 70.801 \\ 61.237 \end{bmatrix} ft$ distance of leg 2

$D'_{leg1} = \begin{bmatrix} 120.605 \\ 131.488 \\ 142.887 \end{bmatrix} ft \qquad D'_{leg2} = \begin{bmatrix} 80.403 \\ 70.801 \\ 61.237 \end{bmatrix} ft$ distance of leg 1 & 2 summary

$D'total := D'_{leg1} + D'_{leg2} = \begin{bmatrix} 201.008 \\ 202.289 \\ 204.124 \end{bmatrix} ft$ total distance ball traveled to tick

$2 \cdot d_1 = 200 \ ft \qquad D'total = \begin{bmatrix} 201.008 \\ 202.289 \\ 204.124 \end{bmatrix} ft$ total distance ball traveled to tick summary (proper on left)

$\Delta t_{stationary} := \frac{2 \cdot d_1}{u} = 0.000000406681345 \ s$ Proper clock rate, what Pinkman and Blueman see their clock ticking at from their own perspectives

$$\Delta t_{moving} := \frac{D'_{leg1}}{u'_1} + \frac{D'_{leg2}}{u'_2} = \begin{bmatrix} 0.00000040873013 \\ 0.000000411335193 \\ 0.00000041506741 \end{bmatrix} s$$

Dilated clock rate, what Pinkman sees Blueman's clock ticking at

$$\frac{\Delta t_{moving}}{\Delta t_{stationary}} = \begin{bmatrix} 1.005 \\ 1.011 \\ 1.021 \end{bmatrix} \quad \text{for,} \quad v = \begin{bmatrix} 0.10 \\ 0.15 \\ 0.20 \end{bmatrix} \cdot c$$

Ratio of dilated clock rate to proper clock rate, should match Lorentz factor, γ

$$\gamma := \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \begin{bmatrix} 1.005 \\ 1.011 \\ 1.021 \end{bmatrix}$$

$check :=$ if $\frac{\Delta t_{moving}}{\Delta t_{stationary}} = \gamma$ = "ok"
  "ok"
else
  "no good"

Check the general Lorentz transformations

General Lorentz transformations

$$x' = \gamma(x - vt), \qquad x = \gamma(x' + vt'),$$
$$y' = y, \qquad y = y',$$
$$z' = z, \qquad z = z',$$
$$t' = \gamma(t - vx/c^2), \quad t = \gamma(t' + vx'/c^2).$$

	Leg 1		Leg 2
(1)	$t' = \gamma \cdot \left(t_1 + \frac{v \cdot d_1}{c^2}\right)$	+	$t' = \gamma \cdot \left(t_2 - \frac{v \cdot d_2}{c^2}\right)$
(2)	$d' = \gamma \cdot (d_1 + v \cdot t_1)$	+	$d' = \gamma \cdot (d_2 - v \cdot t_2)$

$t_1 := \frac{d_1}{u} = 0.000000203340672\ s$ $\qquad t_2 := \frac{d_2}{u} = 0.000000203340672\ s$

$d_1 := t_1 \cdot u = 100\ ft$ $\qquad d_2 := t_2 \cdot u = 100\ ft$

(1) $$t' := \overrightarrow{\gamma \cdot \left(t_1 + \frac{v \cdot d_1}{c^2}\right) + \gamma \cdot \left(t_2 - \frac{v \cdot d_2}{c^2}\right)} = \begin{bmatrix} 0.00000040873013 \\ 0.000000411335193 \\ 0.00000041506741 \end{bmatrix} s$$

$$\Delta t_{moving} = \begin{bmatrix} 0.00000040873013 \\ 0.000000411335193 \\ 0.00000041506741 \end{bmatrix} s$$

$check :=$ if $t' = \Delta t_{moving}$ = "ok"
  "ok"
else
  "no good"

(2) $$d' := \overrightarrow{\gamma \cdot (d_1 + v \cdot t_1) + \gamma \cdot (d_2 - v \cdot t_2)} = \begin{bmatrix} 201.008 \\ 202.289 \\ 204.124 \end{bmatrix} ft$$

$$D'total = \begin{bmatrix} 201.008 \\ 202.289 \\ 204.124 \end{bmatrix} ft$$

$check :=$ if $d' = D'total$ = "ok"
  "ok"
else
  "no good"

Derivation of Lorentz factor from calculations (note, this is the same derivation as ex. 2)

First, the full equation (using the starting variables) is sought. Variables are plugged back in to the calculated Lorentz factor and worked backwards to get an equation which is composed of the starting variables.

$$\frac{\Delta t_{moving}}{\Delta t_{stationary}} = \begin{bmatrix}1.005\\1.011\\1.021\end{bmatrix} \quad \text{-->} \quad \frac{\frac{D'_{leg1}}{u'_1}+\frac{D'_{leg2}}{u'_2}}{\frac{2\cdot d_1}{u}} = \begin{bmatrix}1.005\\1.011\\1.021\end{bmatrix} \quad \text{-->} \quad \frac{\frac{d'_1+d'_2}{u'_1}+\frac{d'_1-d'_3}{u'_2}}{\frac{2\cdot d_1}{u}} = \begin{bmatrix}1.005\\1.011\\1.021\end{bmatrix} \quad \text{-->} \quad \frac{\frac{d'_1+\overrightarrow{v\cdot t'_1}}{u'_1}+\frac{d'_1-\overrightarrow{v\cdot t'_2}}{u'_2}}{\frac{2\cdot d_1}{u}} = \begin{bmatrix}1.005\\1.011\\1.021\end{bmatrix}$$

$$\frac{\frac{d'_1+\overrightarrow{v\cdot\frac{d'_1}{u'_1-v}}}{u'_1}+\frac{d'_1-\overrightarrow{v\cdot\frac{d'_1}{u'_2+v}}}{u'_2}}{\frac{2\cdot d_1}{u}} = \begin{bmatrix}1.005\\1.011\\1.021\end{bmatrix} \quad \text{-->} \quad \frac{\frac{d'_1+\overrightarrow{v\cdot\frac{d'_1}{u'_1-v}}}{\frac{v+u}{1+\frac{v\cdot u}{c^2}}}+\frac{d'_1-\overrightarrow{v\cdot\frac{d'_1}{u'_2+v}}}{\overrightarrow{\left|\frac{v-u}{1-\frac{v\cdot u}{c^2}}\right|}}}{\frac{2\cdot d_1}{u}} = \begin{bmatrix}1.005\\1.011\\1.021\end{bmatrix}$$

$$\frac{\frac{d'_1+v\cdot\overrightarrow{\frac{d_1\cdot\sqrt{1-\frac{v^2}{c^2}}}{\frac{v+u}{1+\frac{v\cdot u}{c^2}}-v}}}{\frac{v+u}{1+\frac{v\cdot u}{c^2}}}+\frac{d'_1-v\cdot\overrightarrow{\frac{d_1\cdot\sqrt{1-\frac{v^2}{c^2}}}{\overrightarrow{\left|\frac{v-u}{1-\frac{v\cdot u}{c^2}}\right|}+v}}}{\overrightarrow{\left|\frac{v-u}{1-\frac{v\cdot u}{c^2}}\right|}}}{\frac{2\cdot d_1}{u}} = \begin{bmatrix}1.005\\1.011\\1.021\end{bmatrix}$$

$$\frac{\frac{d_1\cdot\sqrt{1-\frac{v^2}{c^2}}+v\cdot\overrightarrow{\frac{d_1\cdot\sqrt{1-\frac{v^2}{c^2}}}{\frac{v+u}{1+\frac{v\cdot u}{c^2}}-v}}}{\frac{v+u}{1+\frac{v\cdot u}{c^2}}}+\frac{d_1\cdot\sqrt{1-\frac{v^2}{c^2}}-v\cdot\overrightarrow{\frac{d_1\cdot\sqrt{1-\frac{v^2}{c^2}}}{\overrightarrow{\left|\frac{v-u}{1-\frac{v\cdot u}{c^2}}\right|}+v}}}{\overrightarrow{\left|\frac{v-u}{1-\frac{v\cdot u}{c^2}}\right|}}}{\frac{2\cdot d_1}{u}} = \begin{bmatrix}1.005\\1.011\\1.021\end{bmatrix} \quad \text{for,} \quad v = \begin{bmatrix}0.10\\0.15\\0.20\end{bmatrix}\cdot c$$

The full equation has now been expressed.

A derivation of the calculated factor will be attempted

While going through the simplification process, each step is symbolically set equal to the calculated dilated factor, γ_{ball}. The procedure is performed similar to as described in example 2.

$$\gamma_{ball} := \frac{\Delta t_{moving}}{\Delta t_{stationary}} = \begin{bmatrix}1.005\\1.011\\1.021\end{bmatrix} \quad \text{for,} \quad v = \begin{bmatrix}0.10\\0.15\\0.20\end{bmatrix}\cdot c$$

$$\frac{\frac{d_1\cdot\sqrt{1-\frac{v^2}{c^2}}+v\cdot\overrightarrow{\frac{d_1\cdot\sqrt{1-\frac{v^2}{c^2}}}{\frac{v+u}{1+\frac{v\cdot u}{c^2}}-v}}}{\frac{v+u}{1+\frac{v\cdot u}{c^2}}}+\frac{d_1\cdot\sqrt{1-\frac{v^2}{c^2}}-v\cdot\overrightarrow{\frac{d_1\cdot\sqrt{1-\frac{v^2}{c^2}}}{\overrightarrow{\left|\frac{v-u}{1-\frac{v\cdot u}{c^2}}\right|}+v}}}{\overrightarrow{\left|\frac{v-u}{1-\frac{v\cdot u}{c^2}}\right|}}}{\frac{2\cdot d_1}{u}} = \gamma_{ball} = 1$$

$$\frac{d_1\cdot\sqrt{1-\frac{v^2}{c^2}}+v\cdot\overrightarrow{\frac{d_1\cdot\sqrt{1-\frac{v^2}{c^2}}}{\frac{v+u}{1+\frac{v\cdot u}{c^2}}-v}}}{\frac{v+u}{1+\frac{v\,u}{c^2}}}+\frac{d_1\cdot\sqrt{1-\frac{v^2}{c^2}}-v\cdot\overrightarrow{\frac{d_1\cdot\sqrt{1-\frac{v^2}{c^2}}}{\left|\overrightarrow{\frac{v-u}{1-\frac{v\cdot u}{c^2}}}\right|+v}}}{\left|\overrightarrow{\frac{v-u}{1-\frac{v\cdot u}{c^2}}}\right|}=\gamma_{ball}\cdot\frac{2\cdot d_1}{u}=1$$

$$\frac{d_1\cdot\frac{1}{\gamma}+v\cdot\overrightarrow{\frac{d_1\cdot\frac{1}{\gamma}}{\frac{v+u}{1+\frac{v\,u}{c^2}}-v}}}{\frac{v+u}{1+\frac{v\,u}{c^2}}}+\frac{d_1\cdot\frac{1}{\gamma}-v\cdot\overrightarrow{\frac{d_1\cdot\frac{1}{\gamma}}{\left|\overrightarrow{\frac{v-u}{1-\frac{v\cdot u}{c^2}}}\right|+v}}}{\left|\overrightarrow{\frac{v-u}{1-\frac{v\cdot u}{c^2}}}\right|}=\gamma_{ball}\cdot\frac{2\cdot d_1}{u}=1 \quad \text{-->} \quad \frac{d_1\cdot\frac{1}{\gamma}+v\cdot\overrightarrow{\frac{d_1\cdot\frac{1}{\gamma}}{\frac{c^2\,(v+u)}{c^2+v\cdot u}-v}}}{\frac{c^2\,(v+u)}{c^2+v\cdot u}}+\frac{d_1\cdot\frac{1}{\gamma}-v\cdot\overrightarrow{\frac{d_1\cdot\frac{1}{\gamma}}{\left|\overrightarrow{\frac{c^2\,(v-u)}{c^2-v\cdot u}}\right|+v}}}{\left|\overrightarrow{\frac{c^2\,(v-u)}{c^2-v\cdot u}}\right|}=\gamma_{ball}\cdot\frac{2\cdot d_1}{u}=1$$

$$\frac{\overrightarrow{d_1\cdot\frac{1}{\gamma}\cdot\left(1+\frac{v}{\frac{c^2\,(v+u)}{c^2+v\cdot u}-v}\right)}}{\frac{c^2\,(v+u)}{c^2+v\cdot u}}+\frac{\overrightarrow{d_1\cdot\frac{1}{\gamma}\cdot\left(1-\frac{v}{\left|\frac{c^2\,(v-u)}{c^2-v\cdot u}\right|+v}\right)}}{\left|\overrightarrow{\frac{c^2\,(v-u)}{c^2-v\cdot u}}\right|}=\gamma_{ball}\cdot\frac{2\cdot d_1}{u}=1$$

$$\overrightarrow{d_1\cdot\frac{1}{\gamma}\cdot\frac{\left(1+\frac{v}{\frac{c^2\,(v+u)}{c^2+v\cdot u}-v}\right)}{\frac{c^2\,(v+u)}{c^2+v\cdot u}}+d_1\cdot\frac{1}{\gamma}\cdot\left(\frac{1-\frac{v}{\left|\frac{c^2\,(v-u)}{c^2-v\cdot u}\right|+v}}{\left|\frac{c^2\,(v-u)}{c^2-v\cdot u}\right|}\right)}=d_1\cdot 2\cdot\frac{\gamma_{ball}}{u}=1 \quad \text{-->} \quad \overrightarrow{\frac{1}{\gamma}\cdot\frac{\left(1+\frac{v}{\frac{c^2\,(v+u)}{c^2+v\cdot u}-v}\right)}{\frac{c^2\,(v+u)}{c^2+v\cdot u}}+\frac{1}{\gamma}\cdot\left(\frac{1-\frac{v}{\left|\frac{c^2\,(v-u)}{c^2-v\cdot u}\right|+v}}{\left|\frac{c^2\,(v-u)}{c^2-v\cdot u}\right|}\right)}=2\cdot\frac{\gamma_{ball}}{u}=1$$

$$\overrightarrow{\frac{1}{\gamma}\cdot\left(\frac{1+\frac{v}{\frac{c^2\,(v+u)}{c^2+v\cdot u}-v}}{\frac{c^2\,(v+u)}{c^2+v\cdot u}}+\frac{1-\frac{v}{\left|\frac{c^2\,(v-u)}{c^2-v\cdot u}\right|+v}}{\left|\frac{c^2\,(v-u)}{c^2-v\cdot u}\right|}\right)}=2\cdot\frac{\gamma_{ball}}{u}=1 \quad \text{-->} \quad \overrightarrow{\frac{1+\frac{v}{\frac{c^2\,(v+u)}{c^2+v\cdot u}-v}}{\frac{c^2\,(v+u)}{c^2+v\cdot u}}+\frac{1-\frac{v}{\left|\frac{c^2\,(v-u)}{c^2-v\cdot u}\right|+v}}{\left|\frac{c^2\,(v-u)}{c^2-v\cdot u}\right|}}=2\cdot\frac{{\gamma_{ball}}^2}{u}=1$$

$$\overrightarrow{\left(1+\frac{v}{\frac{c^2\,(v+u)}{c^2+v\cdot u}-v}\right)\cdot\left(\frac{c^2+v\cdot u}{c^2\,(v+u)}\right)+\left(1-\frac{v}{\left|\frac{c^2\,(v-u)}{c^2-v\cdot u}\right|+v}\right)\cdot\left(\left|\frac{c^2-v\cdot u}{c^2\,(v-u)}\right|\right)}=2\cdot\frac{{\gamma_{ball}}^2}{u}=1$$

$$\overrightarrow{\left(1+\frac{v}{\frac{u\cdot(c+v)\,(c-v)}{c^2+v\cdot u}}\right)\cdot\left(\frac{c^2+v\cdot u}{c^2\,(v+u)}\right)+\left(1-\frac{v}{\frac{u\cdot(c+v)\,(c-v)}{c^2-v\cdot u}}\right)\cdot\left(\left|\frac{c^2-v\cdot u}{c^2\,(v-u)}\right|\right)}=2\cdot\frac{{\gamma_{ball}}^2}{u}=1$$

$$\overrightarrow{\left(1+\frac{c^2\cdot v+v^2\cdot u}{u\cdot(c+v)\,(c-v)}\right)\cdot\left(\frac{c^2+v\cdot u}{c^2\,(v+u)}\right)+\left(1-\frac{c^2\cdot v-v^2\cdot u}{u\cdot(c+v)\,(c-v)}\right)\cdot\left(\left|\frac{c^2-v\cdot u}{c^2\,(v-u)}\right|\right)}=2\cdot\frac{{\gamma_{ball}}^2}{u}=1$$

$$\overrightarrow{\frac{c^2+v\cdot u}{u\cdot(c+v)\cdot(c-v)}+\frac{c^2-v\cdot u}{u\cdot(c+v)\cdot(c-v)}}=2\cdot\frac{{\gamma_{ball}}^2}{u}=1 \quad \text{-->} \quad \overrightarrow{\frac{2\cdot c^2}{u\cdot(c+v)\cdot(c-v)}}=2\cdot\frac{{\gamma_{ball}}^2}{u}=1 \quad \text{-->} \quad \overrightarrow{\frac{c^2}{(c+v)\cdot(c-v)}}={\gamma_{ball}}^2=1$$

$$\overrightarrow{\frac{c\cdot\sqrt{(c+v)\cdot(c-v)}}{(c+v)\cdot(c-v)}}=\gamma_{ball}=1 \quad \text{-->} \quad \overrightarrow{\frac{c}{\sqrt{(c+v)\cdot(c-v)}}}=\gamma_{ball}=1 \quad \text{-->} \quad \frac{c}{\sqrt{c^2-v^2}}=\gamma_{ball}=1 \quad \text{-->} \quad \frac{1}{\frac{\sqrt{c^2-v^2}}{c}}=\gamma_{ball}=1$$

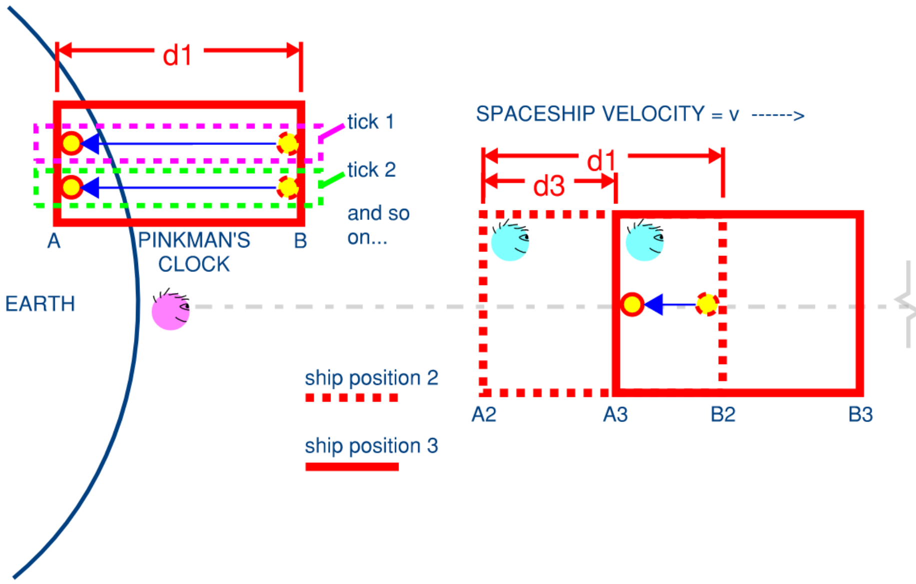
$$\frac{1}{\sqrt{\frac{c^2}{c^2}-\frac{v^2}{c^2}}}=\gamma_{ball}=1 \quad \text{-->} \quad \frac{1}{\sqrt{\frac{c}{c}-\frac{v^2}{c^2}}}=\gamma_{ball}=1 \quad \text{-->} \quad \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}=\gamma_{ball}=1$$

The general transformations do not need to be derived for this example. The derivations will be the same as example 2.

Example 4 Clock mechanism used to measure time: 1-way light clock
Clock mechanism motion: light motion *parallel* to spaceship motion

For this example, the clock repeatedly performs the same motion of leg 2 of example number 2.

Light is emitted from B2 and travels to A3, this is 1 tick. For the next tick, the same motion is performed, and so on.



The time it takes for each tick will be the same equation of leg 2 from example 2: **t = d1 / (u'+v)** .

Example 4 Calculations
Adjusted 1-way Light Clock Experiment - light path parallel to spaceship motion

$u := c = 983571056 \, \frac{ft}{s}$ speed of "object" used in clock mechanism

$d_1 := 100 \, ft$ clock length (1 way)

$v := \begin{bmatrix} 0.20 \\ 0.50 \\ 0.9 \end{bmatrix} \cdot c = \begin{bmatrix} 196714211 \\ 491785528 \\ 885213951 \end{bmatrix} \frac{ft}{s}$ velocity of spaceship (a few examples provided)

Note: primed (') variables correspond to the observation of the moving reference frame (Blueman) from the perspective of the stationary reference frame (Pinkman). Unprimed (proper) variables correspond to the observations of each of their reference frames from their own reference frame.

$d'_1 := d_1 \cdot \sqrt{1 - \frac{v^2}{c^2}} = \begin{bmatrix} 97.98 \\ 86.603 \\ 43.589 \end{bmatrix} ft$ clock length (1 way) length contracted

$u' := \frac{v - u}{1 - \frac{v \cdot u}{c^2}} = \begin{bmatrix} -983571056 \\ -983571056 \\ -983571056 \end{bmatrix} \frac{ft}{s}$ speed of "object" used in clock mechanism

$u' := \overrightarrow{|u'|} = \begin{bmatrix} 983571056 \\ 983571056 \\ 983571056 \end{bmatrix} \frac{ft}{s}$ using absolute value, sign convention is accounted for properly in equations which use this value

$t' := \frac{d'_1}{u' + v} = \begin{bmatrix} 0.000000083013482 \\ 0.000000058699396 \\ 0.000000023324775 \end{bmatrix} s$ time light takes to tick

$d'_2 := \overrightarrow{v \cdot t'} = \begin{bmatrix} 16.33 \\ 28.868 \\ 20.647 \end{bmatrix} ft$

$D'_{leg} := d'_1 - d'_2 = \begin{bmatrix} 81.65 \\ 57.735 \\ 22.942 \end{bmatrix} ft$ distance light travels to tick

$D'total := D'_{leg} = \begin{bmatrix} 81.65 \\ 57.735 \\ 22.942 \end{bmatrix} ft$ total distance light traveled to tick

$d_1 = 100\ ft \qquad D'total = \begin{bmatrix} 81.65 \\ 57.735 \\ 22.942 \end{bmatrix} ft$ total distance light traveled to tick summary (proper on left)

$\Delta t_{stationary} := \frac{d_1}{u} = 0.000000101670336\ s$ proper clock rate, what Pinkman and Blueman see their clock ticking at from their own perspectives

$\Delta t_{moving} := \frac{D'_{leg}}{u'} = \begin{bmatrix} 0.000000083013482 \\ 0.000000058699396 \\ 0.000000023324775 \end{bmatrix} s$ dilated clock rate, what Pinkman sees Blueman's clock ticking at

$\frac{\Delta t_{moving}}{\Delta t_{stationary}} = \begin{bmatrix} 0.816 \\ 0.577 \\ 0.229 \end{bmatrix}$ for, $v = \begin{bmatrix} 0.20 \\ 0.50 \\ 0.9 \end{bmatrix} \cdot c$ ratio of dilated clock rate to proper clock rate, should match Lorentz factor, γ

$\gamma := \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \begin{bmatrix} 1.021 \\ 1.155 \\ 2.294 \end{bmatrix}$ Lorentz factor

$check :=$ if $\gamma = \frac{\Delta t_{moving}}{\Delta t_{stationary}}$ = "no good"
  "ok"
else
  "no good"

From Pinkman's perspective of Blueman's clock, time is contracting.

Check the general Lorentz transformations

General Lorentz transformations

$$x' = \gamma(x - vt), \qquad x = \gamma(x' + vt'),$$
$$y' = y, \qquad y = y',$$
$$z' = z, \qquad z = z',$$
$$t' = \gamma(t - vx/c^2), \qquad t = \gamma(t' + vx'/c^2).$$

(1) $t' = \gamma \cdot \left(t - \frac{v \cdot d_1}{c^2}\right)$

(2) $d' = \gamma \cdot (d_1 - v \cdot t)$

$t := \frac{d_1}{u} = 0.000000101670336\ s$

$d := t \cdot c = 100\ ft$

(1) $t' := \overrightarrow{\gamma \cdot \left(t - \frac{v \cdot d}{c^2}\right)} = \begin{bmatrix} 0.000000083013482 \\ 0.000000058699396 \\ 0.000000023324775 \end{bmatrix} s$

$\Delta t_{moving} = \begin{bmatrix} 0.000000083013482 \\ 0.000000058699396 \\ 0.000000023324775 \end{bmatrix} s$

$check :=$ if $t' = \Delta t_{moving}$ = "ok"
  "ok"
else
  "no good"

(2) $d' := \overrightarrow{\gamma \cdot (d - v \cdot t)} = \begin{bmatrix} 81.65 \\ 57.735 \\ 22.942 \end{bmatrix} ft$

$D'total = \begin{bmatrix} 81.65 \\ 57.735 \\ 22.942 \end{bmatrix} ft$

$check :=$ if $d' = D'total$ = "ok"
  "ok"
else
  "no good"

Derivation of Lorentz factor from calculations

First, the full equation (using the starting variables) is sought. Variables are plugged back in to the calculated Lorentz factor and worked backwards to get an equation which is composed of the starting variables.

$$\frac{\Delta t_{moving}}{\Delta t_{stationary}} = \begin{bmatrix} 0.816 \\ 0.577 \\ 0.229 \end{bmatrix} \quad \text{--->} \quad \frac{\frac{D'_{leg}}{u'}}{\frac{d_1}{u}} = \begin{bmatrix} 0.816 \\ 0.577 \\ 0.229 \end{bmatrix} \quad \text{--->} \quad \frac{\frac{d'_1 - d'_2}{u'}}{\frac{d_1}{u}} = \begin{bmatrix} 0.816 \\ 0.577 \\ 0.229 \end{bmatrix} \quad \text{--->} \quad \frac{\dfrac{d_1 \cdot \sqrt{1 - \frac{v^2}{c^2}} - \overrightarrow{v \cdot t'}}{\overrightarrow{\left| \frac{v-u}{1 - \frac{v \cdot u}{c^2}} \right|}}}{\frac{d_1}{u}} = \begin{bmatrix} 0.816 \\ 0.577 \\ 0.229 \end{bmatrix}$$

$$\frac{\dfrac{d_1 \cdot \sqrt{1 - \frac{v^2}{c^2}} - \overrightarrow{v \cdot \frac{d'_1}{u' + v}}}{\overrightarrow{\left| \frac{v-u}{1 - \frac{v \cdot u}{c^2}} \right|}}}{\frac{d_1}{u}} = \begin{bmatrix} 0.816 \\ 0.577 \\ 0.229 \end{bmatrix} \quad \text{--->} \quad \frac{\dfrac{d_1 \cdot \sqrt{1 - \frac{v^2}{c^2}} - v \cdot \overrightarrow{\frac{d_1 \cdot \sqrt{1 - \frac{v^2}{c^2}}}{u' + v}}}{\overrightarrow{\left| \frac{v-u}{1 - \frac{v \cdot u}{c^2}} \right|}}}{\frac{d_1}{u}} = \begin{bmatrix} 0.816 \\ 0.577 \\ 0.229 \end{bmatrix}$$

$$\frac{\dfrac{d_1 \cdot \sqrt{1 - \frac{v^2}{c^2}} - v \cdot \overrightarrow{\dfrac{d_1 \cdot \sqrt{1 - \frac{v^2}{c^2}}}{\overrightarrow{\left| \frac{v-u}{1 - \frac{v \cdot u}{c^2}} \right|} + v}}}{\overrightarrow{\left| \frac{v-u}{1 - \frac{v \cdot u}{c^2}} \right|}}}{\frac{d_1}{u}} = \begin{bmatrix} 0.816 \\ 0.577 \\ 0.229 \end{bmatrix} \qquad \text{for,} \quad v = \begin{bmatrix} 0.20 \\ 0.50 \\ 0.9 \end{bmatrix} \cdot c$$

The full equation has now been expressed.

While going through the simplification process, each step is symbolically set equal to the calculated dilated factor, γ_{1way}.

The procedure is performed similar to the prior examples.

$$\gamma_{1way} := \frac{\Delta t_{moving}}{\Delta t_{stationary}} \quad \text{--->} \quad \frac{\dfrac{d_1 \cdot \sqrt{1 - \frac{v^2}{c^2}} - v \cdot \overrightarrow{\dfrac{d_1 \cdot \sqrt{1 - \frac{v^2}{c^2}}}{\overrightarrow{\left| \frac{v-u}{1 - \frac{v \cdot u}{c^2}} \right|} + v}}}{\overrightarrow{\left| \frac{v-u}{1 - \frac{v \cdot u}{c^2}} \right|}}}{\frac{d_1}{u}} = \gamma_{1way} = 1 \quad \text{--->} \quad \frac{d_1 \cdot \frac{1}{\gamma} - v \cdot \overrightarrow{\dfrac{d_1 \cdot \frac{1}{\gamma}}{\overrightarrow{\left| \frac{v-u}{1 - \frac{v \cdot u}{c^2}} \right|} + v}}}{\overrightarrow{\left| \frac{v-u}{1 - \frac{v \cdot u}{c^2}} \right|}} = \gamma_{1way} \cdot \frac{d_1}{u} = 1$$

$$\frac{d_1 \cdot \frac{1}{\gamma} - v \cdot \overrightarrow{\dfrac{d_1 \cdot \frac{1}{\gamma}}{\overrightarrow{\left| \frac{c^2 (v-u)}{c^2 - v \cdot u} \right|} + v}}}{\overrightarrow{\left| \frac{c^2 (v-u)}{c^2 - v \cdot u} \right|}} = \gamma_{1way} \cdot \frac{d_1}{u} = 1 \quad \text{--->} \quad \frac{\overrightarrow{d_1 \cdot \frac{1}{\gamma} \cdot \left(1 - \dfrac{v}{\left| \frac{c^2 (v-u)}{c^2 - v \cdot u} \right| + v} \right)}}{\overrightarrow{\left| \frac{c^2 (v-u)}{c^2 - v \cdot u} \right|}} = \gamma_{1way} \cdot \frac{d_1}{u} = 1$$

$$\overrightarrow{d_1 \cdot \frac{1}{\gamma} \cdot \left(\frac{1 - \frac{v}{\left|\frac{c^2\,(v-u)}{c^2 - v \cdot u}\right| + v}}{\left|\frac{c^2\,(v-u)}{c^2 - v \cdot u}\right|} \right)} = \gamma_{1way} \cdot \frac{d_1}{u} = 1$$

--->

$$\overrightarrow{\left(\frac{1 - \frac{v}{\left|\frac{c^2\,(v-u)}{c^2 - v \cdot u}\right| + v}}{\left|\frac{c^2\,(v-u)}{c^2 - v \cdot u}\right|} \right)} = \overrightarrow{\gamma_{1way} \cdot \gamma} \cdot \frac{1}{u} = 1$$

$$\overrightarrow{\left(1 - \frac{v}{\left|\frac{c^2\,(v-u)}{c^2 - v \cdot u}\right| + v}\right) \cdot \left(\left|\frac{c^2 - v \cdot u}{c^2\,(v-u)}\right|\right)} = \overrightarrow{\gamma_{1way} \cdot \gamma} \cdot \frac{1}{u} = 1$$

--->

$$\overrightarrow{\left(1 - \frac{v}{\frac{u \cdot (c+v)\,(c-v)}{c^2 - v \cdot u}}\right) \cdot \left(\left|\frac{c^2 - v \cdot u}{c^2\,(v-u)}\right|\right)} = \overrightarrow{\gamma_{1way} \cdot \gamma} \cdot \frac{1}{u} = 1$$

$$\overrightarrow{\left(1 - \frac{c^2 \cdot v - v^2 \cdot u}{u \cdot (c+v)\,(c-v)}\right) \cdot \left(\left|\frac{c^2 - v \cdot u}{c^2\,(v-u)}\right|\right)} = \overrightarrow{\gamma_{1way} \cdot \gamma} \cdot \frac{1}{u} = 1$$

--->

$$\overrightarrow{\frac{c^2 - v \cdot u}{u \cdot (c+v) \cdot (c-v)}} = \overrightarrow{\gamma_{1way} \cdot \gamma} \cdot \frac{1}{u} = 1$$

--->

$$\overrightarrow{\frac{c^2 - v \cdot u}{(c+v) \cdot (c-v)}} = \overrightarrow{\gamma_{1way} \cdot \gamma} = 1$$

$$\overrightarrow{\frac{c^2 - v \cdot u}{(c+v) \cdot (c-v)} \cdot \frac{1}{\gamma}} = \overrightarrow{\gamma_{1way}} = 1$$

--->

$$\overrightarrow{\frac{c^2 - v \cdot u}{(c+v) \cdot (c-v)} \cdot \sqrt{1 - \frac{v^2}{c^2}}} = \overrightarrow{\gamma_{1way}} = 1$$

--->

$$\overrightarrow{\frac{\sqrt{(c+v) \cdot (c-v)}}{c \cdot (c+v) \cdot (c-v)} \cdot \left(c^2 - v \cdot u\right)} = \gamma_{1way} = 1$$

$$\overrightarrow{\frac{\gamma}{c^2} \cdot \left(c^2 - v \cdot u\right)} = \gamma_{1way} = 1$$

--->

$$\overrightarrow{\frac{\left(c^2 - v \cdot u\right)}{c^2}} = \frac{\gamma_{1way}}{\gamma} = 1$$

--->

$$\overrightarrow{\frac{\left(c^2 - v \cdot u\right)}{c^2}} = \frac{\gamma_{1way}}{\gamma} = 1$$

--->

$$\overrightarrow{1 - \frac{v \cdot u}{c^2}} = \frac{\gamma_{1way}}{\gamma} = 1$$

$$\overrightarrow{\left(1 - \frac{v \cdot u}{c^2}\right) \cdot \gamma} = \gamma_{1way} = 1$$

--->

$$\overrightarrow{\left(1 - \frac{v \cdot c}{c^2}\right) \cdot \gamma} = \gamma_{1way} = 1$$

(u = c, for light clock) --->

$$\overrightarrow{\left(1 - \frac{v}{c}\right) \cdot \gamma} = \gamma_{1way} = 1$$

$$\overrightarrow{\frac{\left(1 - \frac{v}{c}\right)}{\sqrt{1 - \frac{v^2}{c^2}}}} = \gamma_{1way} = 1$$

--->

$$\overrightarrow{\frac{\sqrt{(c+v) \cdot (c-v)}}{(c+v)}} = \gamma_{1way} = 1$$

--->

$$\overrightarrow{\sqrt{\frac{(c-v)}{(c+v)}}} = \gamma_{1way} = 1$$

For this 1-way light clock experiment the calculated clock dilation factor is:

$$\gamma_{1way} = \overrightarrow{\sqrt{\frac{(c-v)}{(c+v)}}} \qquad \gamma_{1way.return} := \gamma_{1way} = \begin{bmatrix} 0.816 \\ 0.577 \\ 0.229 \end{bmatrix}$$

The clock setup in this 1-way experiment is equivalent to leg 2 of example 2 (the return trip of the light from B to A).

If you were to redo this example, but only use leg 1 from example 2 (the outward trip of light from A to B) as the 1-way clock, the resulting clock dilation factor would be:

$$\gamma_{1way.outward} := \overrightarrow{\sqrt{\frac{(c+v)}{(c-v)}}} = \begin{bmatrix} 1.225 \\ 1.732 \\ 4.359 \end{bmatrix}$$

for,

$$v = \begin{bmatrix} 0.20 \\ 0.50 \\ 0.9 \end{bmatrix} \cdot c$$

Taking the average of each of these legs, yields: the Typical Lorentz factor:

$$\gamma_{average} := \frac{\sqrt{\frac{(c+v)}{(c-v)}} + \sqrt{\frac{(c-v)}{(c+v)}}}{2} = \begin{bmatrix} 1.021 \\ 1.155 \\ 2.294 \end{bmatrix}$$

for,

$$v = \begin{bmatrix} 0.20 \\ 0.50 \\ 0.9 \end{bmatrix} \cdot c \qquad \gamma := \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \begin{bmatrix} 1.021 \\ 1.155 \\ 2.294 \end{bmatrix}$$

The analysis of a **parallel motion** light clock reveals that the dilated clock factors in each leg are:

leg 1 leg 2

$$\sqrt{\frac{(c+v)}{(c-v)}} \quad \& \quad \sqrt{\frac{(c-v)}{(c+v)}}$$

which results in more than expected time dilation in leg 1 and time contraction in leg 2.

round-trip: $\gamma = \dfrac{\sqrt{\frac{(c+v)}{(c-v)}} + \sqrt{\frac{(c-v)}{(c+v)}}}{2}$ which is equal to: $\gamma = \dfrac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

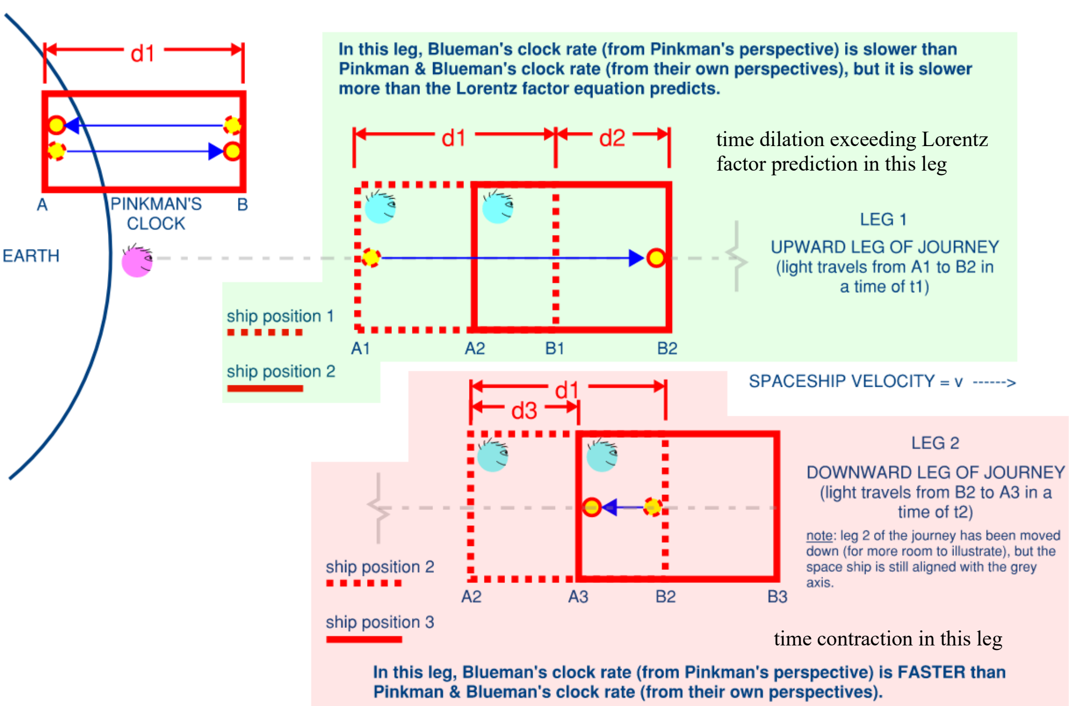
The analysis of a **perpendicular motion** light clock (as shown in example 1) reveals that the dilated clock factors in each leg are:

leg 1 leg 2

$$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \& \quad \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

which results in uniform time dilation in each leg.

round-trip: $\gamma = \dfrac{\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}}{2}$ which is equal to: $\gamma = \dfrac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$



Time should be dilated by the same rate for each leg of the clock (as it is in example 1 - each leg was shown to dilate according to the Lorentz factor). Time dilation should not depend on taking the average of the dilated clock rate factors of each leg to arrive at the expected time dilation factor, nor should it depend on the clock configuration (perpendicular vs. parallel motion or 2-way clock vs. 1-way clock).

The time dilation rate in leg 1 should = the time dilation rate in leg 2, which should = the round trip time dilation rate factor.

Discussion on the origin of the Lorentz factor

Background and Assumptions of the Day

Einstein said that he started developing special relativity theory due to the "strange" result of the Michelson 1887 interferometer experiment[1].

So although he does not mention the word Michelson or interferometer once in his 1905 paper on special relativity theory[2], the purpose of his theory is to explain the results of the 0 fringe shift. He is attempting to do exactly what Lorentz was doing, develop a mathematical theory for how the two rays of light in Michelson's interferometer (one going in the perpendicular and one going in the parallel direction, with respect to the direction of earth's orbital velocity) both arrived back at the beam splitter at the same time, thus showing 0 fringe shift.

Since these theories were attempting to explain Michelson's results, it was important to clearly understand Michelson's experiment, therefore some background and discussion is provided regarding this experiment.

The following assumptions are listed to help understand how many physicists were thinking in 1887.

1. There is an ether in which electromagnetic phenomena propagate as waves.
   - 1678 - Christiaan Huygens introduced the wave theory of light. Due to this, a medium was assumed for propagation.
   - 1801 - Thomas Young performed the double slit experiment in which light interference was observed, lending credence to Huygens's wave theory of light.
   - 1864 - James Clerk Maxwell formulated mathematical equations which describe the mechanics of electromagnetic phenomena and that the medium in which these mechanics take place is the ether.

2. The ether is at rest.

   This is stated in Michelson's 1887 paper: "On the undulatory (wave-like) theory, according to Fresnel, first, the ether is supposed to be at rest except in the interior of transparent media...(break)...The experimental trial of the first hypothesis forms the subject of the present paper"[3].

3. The velocity of light is finite and has been measured.

   Many folks, including Romer, Huygens, Bradley, Foucalt, Fizeau, Fresnel, Michelson, and others performed experiments to measure the velocity of light. All of these experiments measured velocities of light in the vicinity of each other, and were close to the currently accepted velocity of: $c = 299792458 \frac{m}{s}$. Maxwell provides a good summary of this information in his 1865 paper[4].

4. Earth's orbital velocity around the sun is 30 km/s.

   I cannot find record of an experiment measuring this. The top search results mention stellar aberration and stellar parallax, meaning there is evidence of relative motion and relative position changes between the earth and the stars, but these observations do not prove which is moving, just that there is relative motion. It appears to just be deduced from the assumed distance between the earth and the sun.

   $d_e := 149597870700\ m = 92955807\ mi$ assumed distance between earth and the sun

   $t := 1\ yr$ time it takes earth to make the assumed revolution around the sun

   $v_e := \frac{2 \cdot \pi \cdot d_e}{t} = 30 \frac{km}{s}$ earth orbital velocity

   The Michelson 1887 paper attempted to measure earth's velocity with respect to the ether and concluded that "the relative velocity of the earth and the ether is certainly less than one-fourth of the earth's orbital velocity" (that is 0.25*30km/s = 7.5 km/s). Based on their experiment, they detected an earth velocity of 7.5 km/s (upper limit) relative to whatever the medium is in which light propagates.

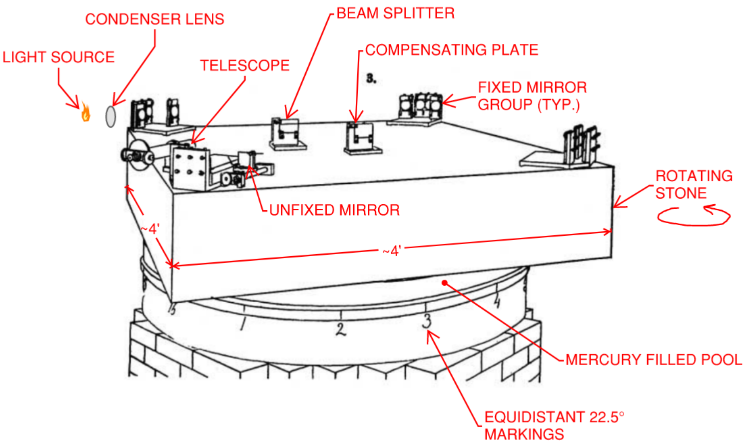
   There are experiments which allegedly measured no motion. Francois Arago in 1810 performed a prism experiment. This was the first attempt to measure the absolute motion of the earth. The results showed no motion. I cannot find the direct paper online, just papers discussing his experiment[5].

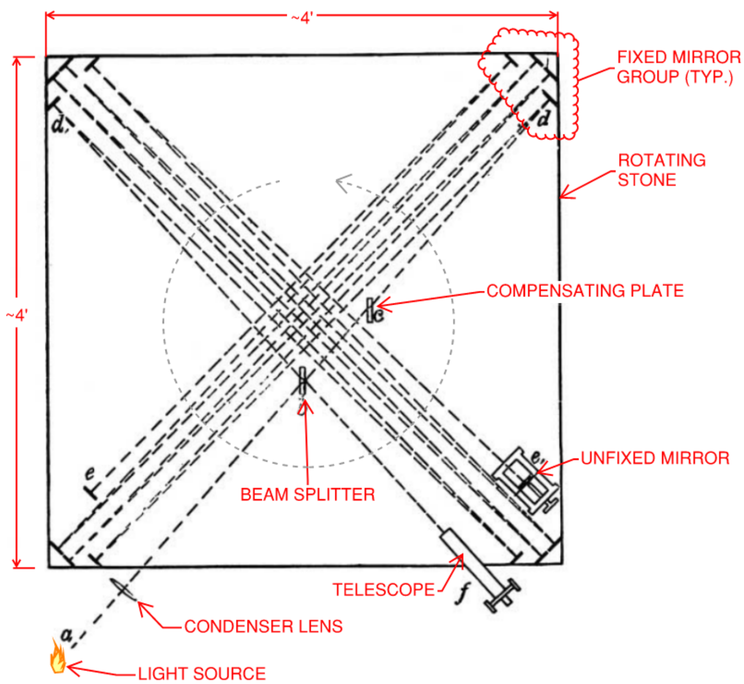
   George Biddell Airy performed an experiment[6] in 1871 that had to do with the observed aberration angles of star light. A star's position appears to move depending on which part of the year it is viewed through a telescope. This indicates that there is relative motion between the star and the earth. Airy's experiment was an attempt to figure if the star was moving or if earth was moving. It involved viewing the aberration (deviation from normal) of star light through a telescope which was configured such that the light had to travel through different mediums - water or air. If the earth was in motion, then the light would refract (bend) at different angles as it went through each configuration of the telescope (water or air) and the apparent position of the source of light would be different for each. However, his experiment demonstrated the same apparent position for each medium, which would indicate a motionless earth.

The Michelson & Morley Experiments

They performed an experiment which attempted to measure the relative motion between the earth and the ether. They constructed an apparatus (called an interferometer) shown below. The experiment was performed in 1887 in the basement of a dormitory at the Case School of Applied Science in Cleveland, Ohio.

Breakdown of the components of the interferometer.



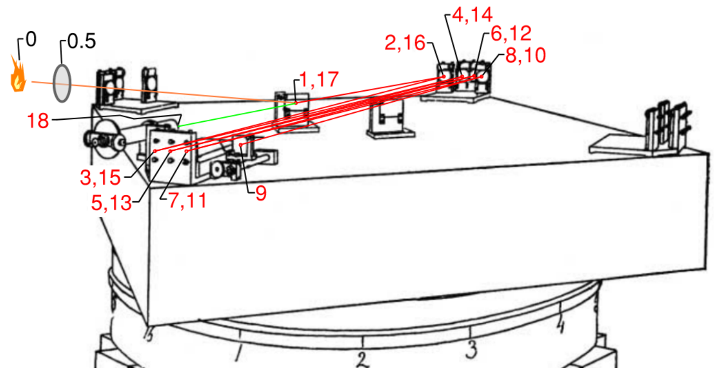


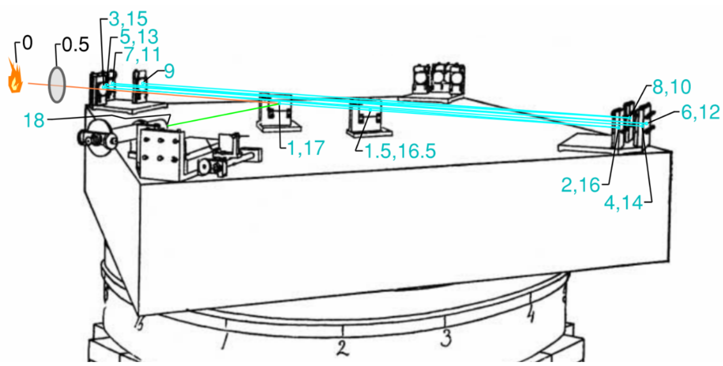
View from top

Components	Description
Light source	Flame from an argand burner (incoherent light source with varying wavelengths).
Condenser lens	Convex lens used to make the non-parallel flame light roughly parallel.
Beam splitter	Half silvered mirror. Allows light to "split" - some waves are reflected to one light path, other waves are transmitted through to the other light path. (1.25 cm thick).
Compensating plate	Glass for one light path to pass through. This was to ensure the light paths were equal. (1.25cm thick).
Fixed mirrors	Mirrors (5 cm diameter) were use to extend the light paths. According to M&M, this reduced experimental error due to vibration.
Unfixed mirror	This mirror had additional degrees of freedom than the fixed mirrors. It allowed fine adjustments to be made to the light path.
Telescope	The fringe patterns due to light interference were viewed through the telescope.
Rotating stone	Provides a sturdy base for the components. Sits atop a wooden float in the mercury pool.
Mercury pool	The stone rotated atop the mercury surface. According to M&M, supporting the apparatus on mercury reduced distortion.
Markings	16 equidistant markings for positioning the apparatus.

The sketches above of the apparatus (black linework) are from Michelson's 1887 paper.

Light paths





Red Path Point	Blue Path Point	Description
0	0	Light waves emitted from flame.
0.5	0.5	Waves pass through condenser plate, becoming more parallel with respect to each other.
1	1	Waves are split. The split waves travel down the red path (reflected by the splitter) or blue path (transmitted through the splitter).
-	1.5	Waves along the blue path travel through a compensation plate (see note 1).
2-9	2-9	Waves bounce off mirrors to point 9.
9-16	9-16	Waves then retrace their steps and bounce off the same mirrors.
-	16.5	Waves along the blue path travel through the compensation plate again (see note 1).
17	17	The blue waves return to the beam splitter, some wave are reflected towards the telescope. The red waves return to the beam splitter, some waves are transmitted through towards the telescope.
18	18	The interference from the blue path waves and red path waves which were reflected by and transmitted through the beam splitter are observed through the telescope.

Notes:

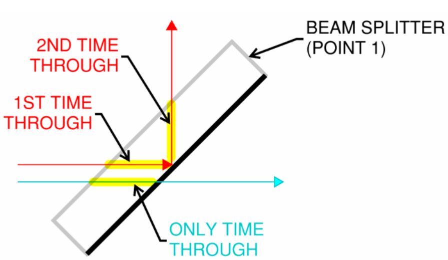
The compensating plate's purpose is to ensure the equality of the two different light paths (red and blue). As said in Michelson's 1887 paper, "The second of these (rectangular glass components) was placed in the path of one of the pencils to compensate for the passage of the other through the same thickness of glass".

The thickness of the glass of the beam splitter and the compensating plate were each 1.25 cm.

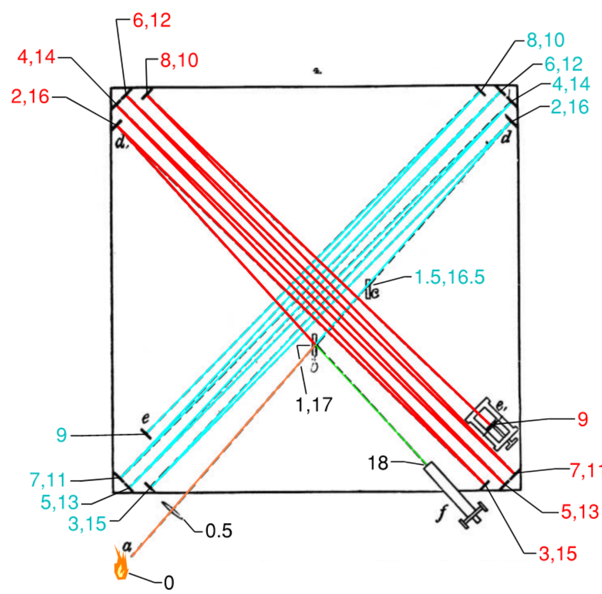
They wanted to make sure that along each light path, the light traveled through the same cumulative thickness of glass.

	Red path	Blue path
Number of times through the beam splitter glass	2* (point 1) 1 (point 17)	1 (point 1)
Number of times through the compensating plate	0	2 (points 1.5&16.5)
Total times going through 1.25 cm glass	3	3

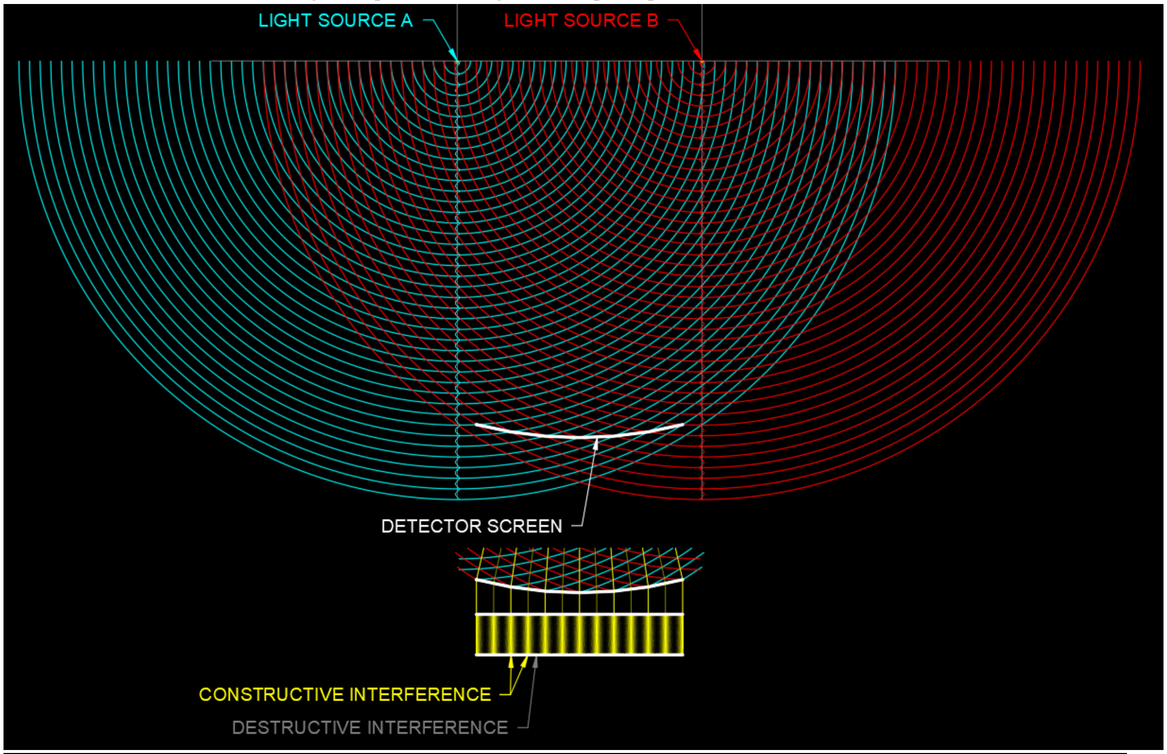
*

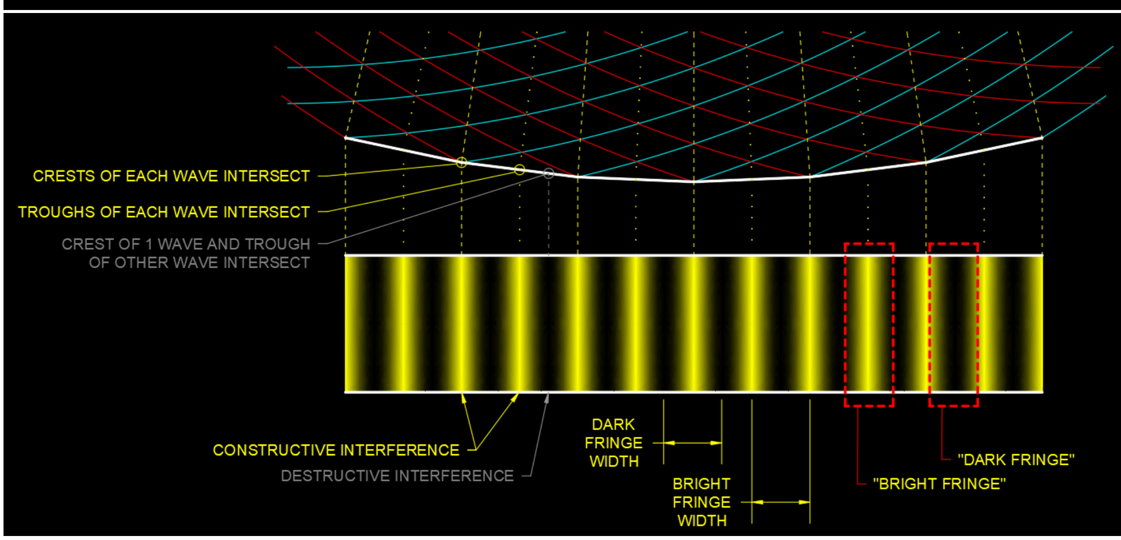


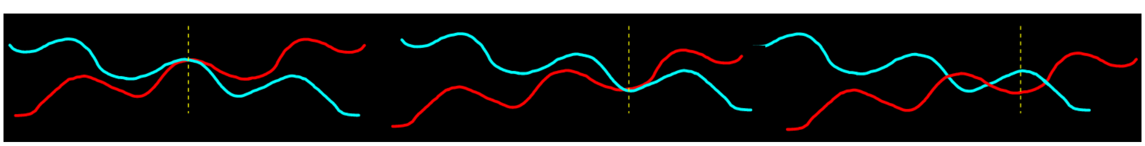
In the sketch above, the effects due to refraction are not illustrated (does not affect concept being displayed).



The thing being measured using the interferometer is the interference (fringe) pattern created when the light waves interfere. Shown below is an example of an interference pattern from light waves from two sources of light A & B. The interference pattern is viewed on a detector screen. Note this is not representative of the Michelson interferometer. This diagram represents Young's double split experiment. It is used because it demonstrates interference clearly.





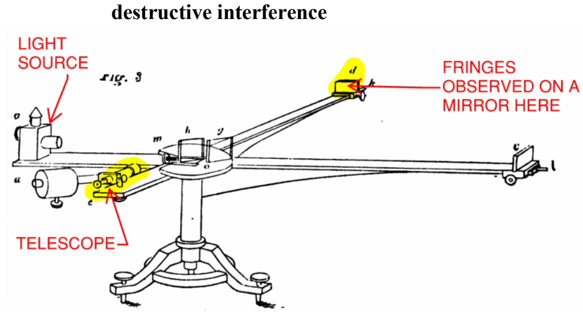


crests of each wave intersect
constructive interference

troughs of each wave intersect
constructive interference

crest of one wave and trough of other wave intersect
destructive interference

It is worth discussing where the interference fringes were viewed in Michelson's 1887 experiment. The 1887 paper is quite vague regarding this. Michelson's 1881 similar interferometer experiment and paper[7] specifically states the telescope was focused on the surface of mirror d (shown to the right), so it is assumed a similar observation also occurred in the 1887 experiment. The interference is occurring when the two light rays get back to the beam splitter and it is being viewed by focusing the telescope on the mirror beyond the beam spitter.



The sketch above of the apparatus (black linework) is from Michelson's 1881 paper.

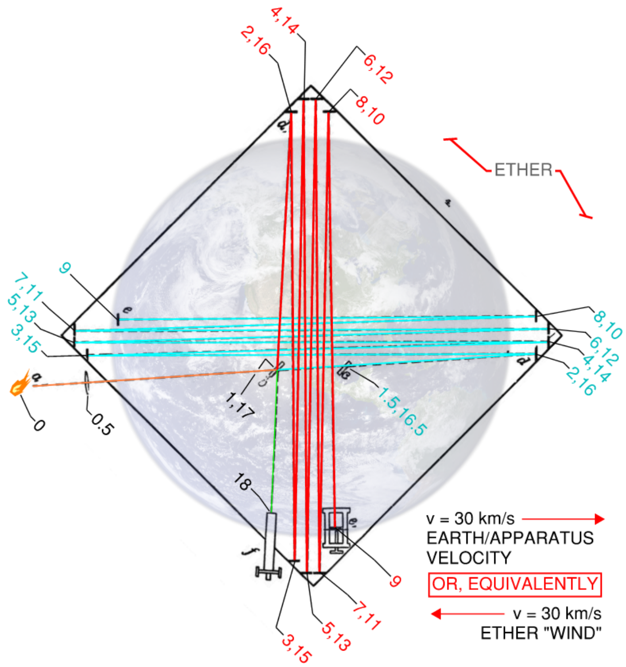
There is a direct relationship between the observed light interference pattern in their (Michelson & Morley, abbreviated as M&M occasionally) experiment and the velocity of the earth.

The diagram to the right shows the interferometer in a specific position. The position of the interferometer is oriented such that the light traveling along the blue path is parallel to the velocity of earth and the light traveling along the red path is perpendicular to the velocity of the earth.

In this position, each light path (blue and red) is going to be affected by the velocity of the earth differently. Each light path is going to increase in length (compared to the "at-rest" light path, if say v = 0 km/s), but the blue light path length will increase more than the red light path will increase.

This is shown mathematically further down in this document, and the difference in light path lengths is calculated and compared to M&M's calculation.

The resulting comparison of difference in light paths is shown below. Δ_D = the length of the blue light path - the length of the red light path.



These calcs $$\Delta_D = \frac{2 \cdot d_1 \cdot c \cdot \left(c - \sqrt{(c+v) \cdot (c-v)}\right)}{(c+v) \cdot (c-v)}$$

M&M 1887 calcs $$\Delta_{D.MM.1887} = d_1 \cdot \frac{v^2}{c^2}$$

These equations are different, due to M&M neglecting terms of the fourth order (as they intended) - which means their equation is a more simplified approximation, but it does not lose accuracy for the velocity involved, 30 km/s).

Values will be plugged in and the path difference will be computed to display this. Two velocities are provided to show that the above two equations are equivalent for lower velocities, but not equivalent for very high velocities.

$$v := \begin{bmatrix} 30 \dfrac{km}{s} \\ 0.5 \cdot c \end{bmatrix} = \begin{bmatrix} 30 \\ 149896 \end{bmatrix} \frac{km}{s} \quad \begin{bmatrix} \text{“earth orbital velocity”} \\ \text{“half the speed of light”} \end{bmatrix} \qquad c = 299792 \frac{km}{s} \text{ speed of light} \qquad d_1 := 11\, m \text{ (light path)}$$

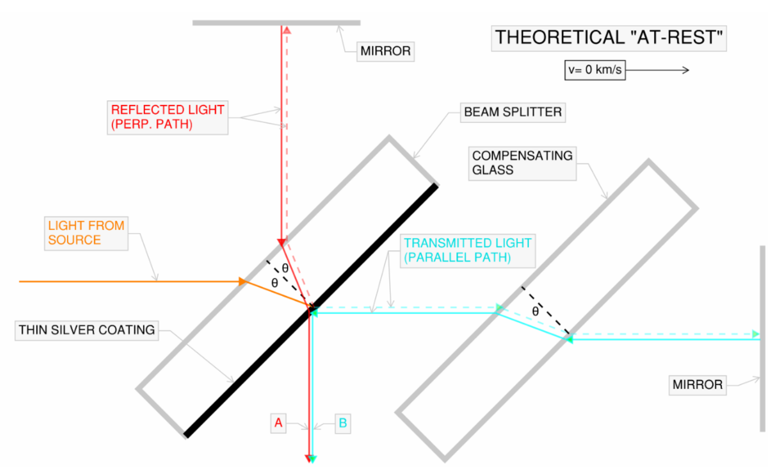
These calcs $$\Delta_D := \overrightarrow{\frac{2 \cdot d_1 \cdot c \cdot \left(c - \sqrt{(c+v) \cdot (c-v)}\right)}{(c+v) \cdot (c-v)}} = \begin{bmatrix} 0.000000361392247 \\ 12.8934432053471 \end{bmatrix} ft \quad \begin{bmatrix} \text{“earth orbital velocity”} \\ \text{“half the speed of light”} \end{bmatrix}$$

M&M 1887 $$\Delta_{D.MM.1887} := d_1 \cdot \frac{v^2}{c^2} = \begin{bmatrix} 0.000000361392243 \\ 9.02230971128609 \end{bmatrix} ft \quad \begin{bmatrix} \text{“earth orbital velocity”} \\ \text{“half the speed of light”} \end{bmatrix}$$

Difference $$diff := \Delta_D - \Delta_{D.MM.1887} = \begin{bmatrix} 0.000000000000005 \\ 3.87113349406103 \end{bmatrix} ft \quad \begin{bmatrix} \text{“earth orbital velocity”} \\ \text{“half the speed of light”} \end{bmatrix}$$

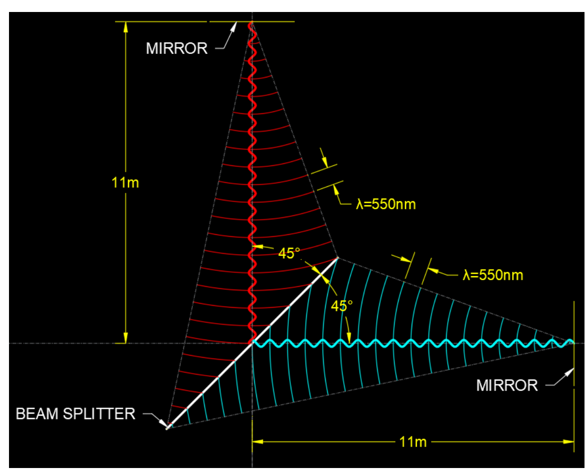
As can be seen, for an experiment using earth orbital velocity as v, their approximated equation is essentially the same as the un-simplified equation from these calculations. The difference between the two is $5 \cdot 10^{-15}$ ft, negligible.

In an at-rest scenario (earth velocity, v, = 0 km/s), the light waves, after meeting again at the beam splitter, will interfere in such a way where the crests, troughs, and all other points of the light wave traveling perpendicularly will align with the corresponding crests, troughs, and all other points of the light traveling in the parallel direction (crests align with crests, troughs with troughs, etc.). . In this scenario, nothing is moving except the light waves.



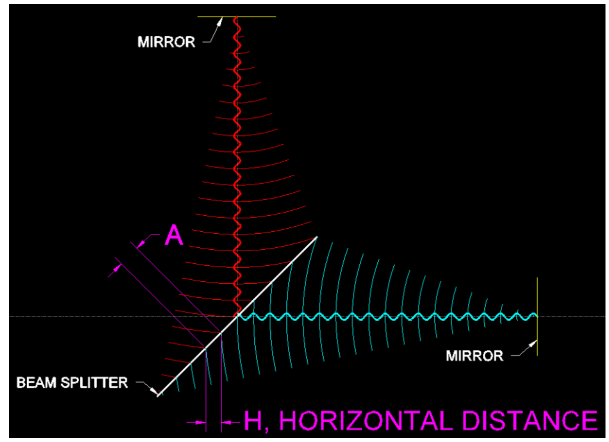
(telescope)

Note: these diagrams resemble the simpler 1881 interferometer where the light only bounced off 1 mirror for each path. In the 1887 interferometer, the light would return to the beam splitter at a slight angle due to the 1887 configuration (additional mirrors).



The diagram above represents this same at-rest scenario. The light shown (for simplicity) begins at the point it is reflected off the mirrors and travels back to the beam splitter. The lengths of each light path are equal and the light arrives at the same time at the splitter.

The beam splitter is at a 45 degree angle with respect to the telescope so the measurements of distances between fringes and fringe shifts are being made of the horizontal projection (H) of the actual interference patterns (A).



These horizontal distances between wavefronts can be calculated by setting up a system of equations: one equation for the beam splitter (equation of a line) and one equation for the light wavefront (equation of a circle).

This is shown for the at-rest scenario in order to establish a baseline of what the horizontal distance (viewed distance) is between the interference points of the crests of the blue and red waves.

For the parallel light path, the two equations are:

$y = x$ equation for beam splitter, equation of a line

$(x-h)^2 + (y-k)^2 = r^2$ equation for light wavefront, equation of a circle

$(x-h)^2 + (x-k)^2 = r^2$ substitute y for x, then solve for x

$$(x-h)\cdot(x-h) + (x-k)\cdot(x-k) = r^2$$

$$x^2 - 2\cdot h\cdot x + h^2 + x^2 - 2\cdot k\cdot x + k^2 = r^2$$

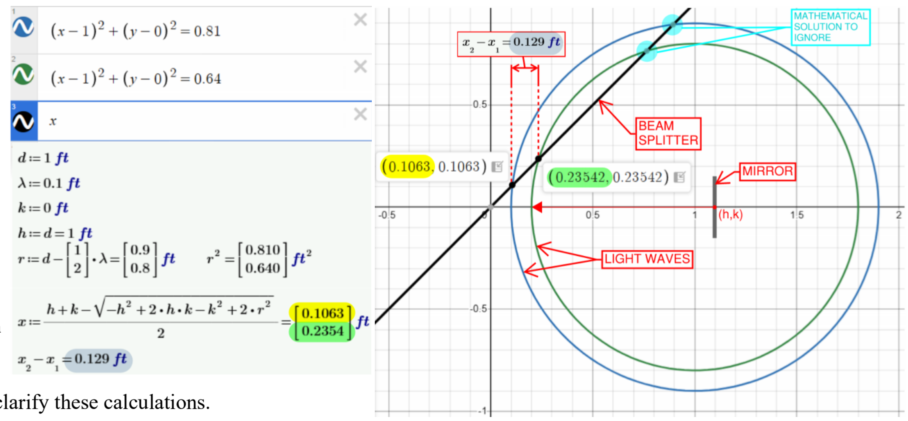
$$2\cdot x^2 - 2\cdot h\cdot x - 2\cdot k\cdot x + h^2 + k^2 = r^2$$

$$2\cdot x^2 - 2\cdot h\cdot x - 2\cdot k\cdot x + h^2 + k^2 - r^2 = 0$$

$$2\cdot x^2 - (2\cdot h + 2\cdot k)\cdot x + h^2 + k^2 - r^2 = 0$$

$$x = \frac{h + k + \sqrt{-h^2 + 2\cdot h\cdot k - k^2 + 2\cdot r^2}}{2}$$ <-- this solution is the other point of intersection and is ignored (it is a point far away from the splitter) (blue points in diagram to the right)

$$x = \frac{h + k - \sqrt{-h^2 + 2\cdot h\cdot k - k^2 + 2\cdot r^2}}{2}$$ <-- use this solution



An example shown in the images to the right is provided to clarify these calculations.

Plugging in the parameters associated with the M&M experiment:

$d := 11\ m = 11000000000\ nm$ 1-way dist. of light path

$\lambda := 550\ nm$ light wavelength

$k := 0\ nm$ origin of light wave is along the x axis, so origin offset in y direction is 0

$h := d = 11000000000\ nm$ origin of light wave is the point of reflection off the mirror, so h = d

$wf :=$ (Wavefront #), $r := d - wf\cdot\lambda =$ (Radii of wavefronts, nm), $x := \frac{h + k - \sqrt{-h^2 + 2\cdot h\cdot k - k^2 + 2\cdot r^2}}{2} =$ (Horizontal distance from origin to point of wavefront intersection with beam splitter, nm)

Wavefront #	Radii of wavefronts (nm)	Horizontal distance from origin to point of wavefront intersection with beam splitter (nm)
−9	11000004950	−4949.9988854507
−8	11000004400	−4399.9991206078
−7	11000003850	−3849.9993273433
−6	11000003300	−3299.9995047689
−5	11000002750	−2749.9996564373
−4	11000002200	−2199.9997805722
−3	11000001650	−1649.9998780617
−2	11000001100	−1099.9999453532
−1	11000000550	−549.9999859993
0	11000000000	0.0000000000
1	10999999450	550.0000126446
2	10999998900	1100.0000554873
3	10999998350	1650.0001240871
4	10999997800	2200.0002211087
5	10999997250	2750.0003438874
6	10999996700	3300.0004941997
7	10999996150	3850.0006747100
8	10999995600	4400.0008800893
9	10999995050	4950.0011130021

Note: wavefront #0 represents the wave front that intersects the beam splitter along the x-axis (wave front radius = d).

Radii of wavefronts — Wave front

Hor. dist. between adjacent points of wavefront intersection with beam splitter

$n := 2 .. 19$

$$r = \begin{bmatrix} 11000004950 \\ 11000004400 \\ 11000003850 \\ 11000003300 \\ 11000002750 \\ 11000002200 \\ 11000001650 \\ 11000001100 \\ 11000000550 \\ 11000000000 \\ 10999999450 \\ 10999998900 \\ 10999998350 \\ 10999997800 \\ 10999997250 \\ 10999996700 \\ 10999996150 \\ 10999995600 \\ 10999995050 \end{bmatrix} nm \qquad \begin{bmatrix} -9 \\ -8 \\ -7 \\ -6 \\ -5 \\ -4 \\ -3 \\ -2 \\ -1 \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{bmatrix}$$

Wave front minus Wave front

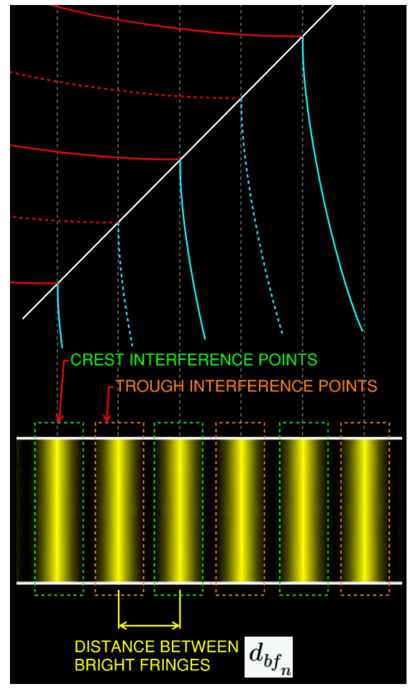
$$x_n - x_{n-1} = \begin{bmatrix} 549.99976484 \\ 549.99979326 \\ 549.99982257 \\ 549.99984833 \\ 549.99987587 \\ 549.99990251 \\ 549.99993271 \\ 549.99995935 \\ 549.99998600 \\ 550.00001264 \\ 550.00004284 \\ 550.00006860 \\ 550.00009702 \\ 550.00012278 \\ 550.00015031 \\ 550.00018051 \\ 550.00020538 \\ 550.00023291 \end{bmatrix} nm \qquad \begin{bmatrix} -8 \\ -7 \\ -6 \\ -5 \\ -4 \\ -3 \\ -2 \\ -1 \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{bmatrix} - \begin{bmatrix} -9 \\ -8 \\ -7 \\ -6 \\ -5 \\ -4 \\ -3 \\ -2 \\ -1 \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}$$

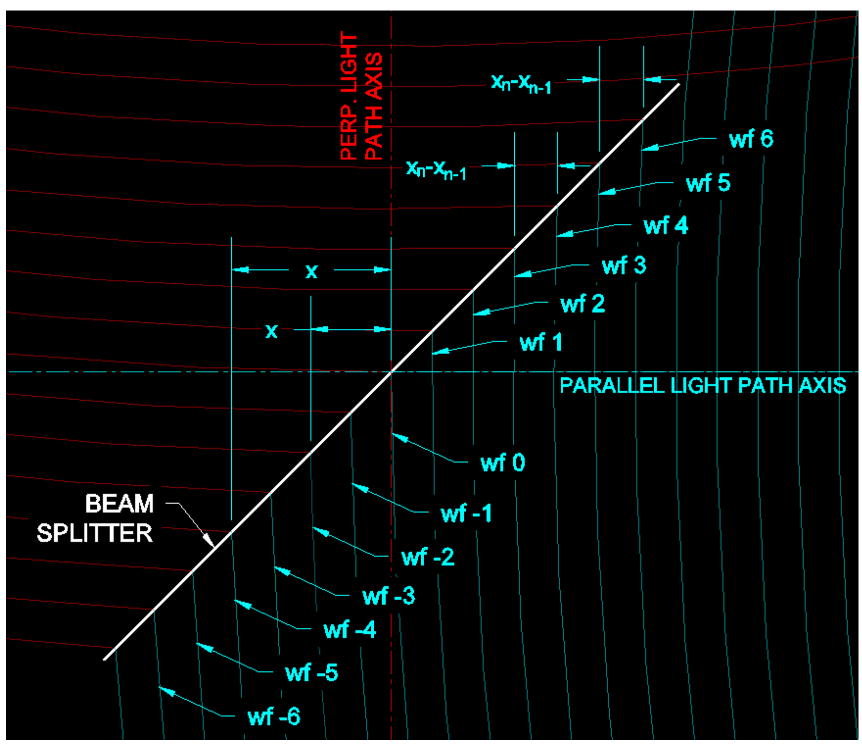
The distances between crests of the wavefronts is $x_n - x_{n-1}$.

Since constructive interference occurs at the intersection of the wavefront troughs also, the distances between bright fringes would be $\frac{x_n - x_{n-1}}{2}$.

Distance between bright fringes:

$$d_{bf_n} := \frac{x_n - x_{n-1}}{2} = \begin{bmatrix} 274.99988242 \\ 274.99989663 \\ 274.99991129 \\ 274.99992417 \\ 274.99993793 \\ 274.99995126 \\ 274.99996635 \\ 274.99997968 \\ 274.999993 \\ 275.00000632 \\ 275.00002142 \\ 275.0000343 \\ 275.00004851 \\ 275.00006139 \\ 275.00007516 \\ 275.00009026 \\ 275.00010269 \\ 275.00011646 \end{bmatrix} nm$$





For the perpendicular light path, the two equations are: (note, the equations are the same as the parallel path, except h and k are swapped)

$k := d = 11000000000\ nm$ origin of light wave is the point of reflection off the mirror, so k = d

$h := 0\ nm$ origin of light wave is along y axis, so origin offset in x direction is 0

$$x := \frac{h + k - \sqrt{-h^2 + 2 \cdot h \cdot k - k^2 + 2 \cdot r^2}}{2} = \begin{bmatrix} -4949.99888545 \\ -4399.99912061 \\ -3849.99932734 \\ -3299.99950477 \\ -2749.99965644 \\ -2199.99978057 \\ -1649.99987806 \\ -1099.99994535 \\ -549.999986 \\ 0 \\ 550.00001264 \\ 1100.00005549 \\ 1650.00012409 \\ 2200.00022111 \\ 2750.00034389 \\ 3300.0004942 \\ 3850.00067471 \\ 4400.00088009 \\ 4950.001113 \end{bmatrix} nm \qquad \frac{x_n - x_{n-1}}{2} = \begin{bmatrix} 274.99988242 \\ 274.99989663 \\ 274.99991129 \\ 274.99992417 \\ 274.99993793 \\ 274.99995126 \\ 274.99996635 \\ 274.99997968 \\ 274.999993 \\ 275.00000632 \\ 275.00002142 \\ 275.0000343 \\ 275.00004851 \\ 275.00006139 \\ 275.00007516 \\ 275.00009026 \\ 275.00010269 \\ 275.00011646 \end{bmatrix} nm$$

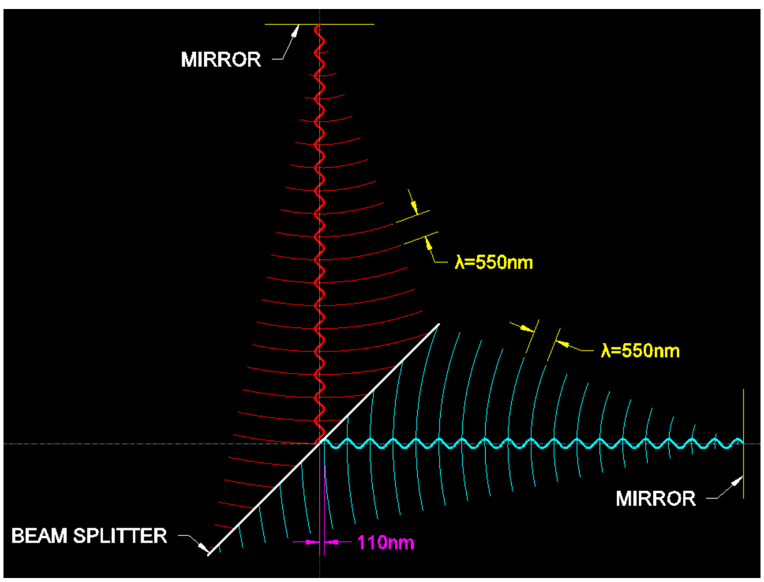
(same as parallel path)

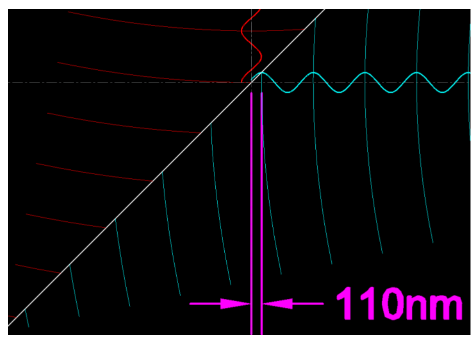
This scenario establishes a baseline of what the distances between fringes are if earth velocity = 0 km/s. We can do the same analysis to determine the distance between fringes using an earth velocity of 30 km/s, and see if there is a 1:1 correlation between fringe shift and light path difference.

$v := 30 \frac{km}{s}$ velocity of earth

note: equation is from M&M 1887 paper

$\Delta_{D.MM.1887} := d_1 \cdot \frac{v^2}{c^2} = 110\ nm$ difference in light path travel distance, blue path (parallel) - red path (perpendicular)





As shown, the blue (parallel) light has to travel an additional 110 nm (or in light wavelength terms, 0.2* 1 wavelength) to get back to the beam splitter than the red (perpendicular) light.

The crests of the wavefronts no longer hit the beam splitter at the same point as they did in the at-rest scenario.

$d := d_1 = 11000000000\ nm$ 1-way dist. of light path

$\lambda := 550\ nm$ light wavelength

(note: the light path distance in the parallel direction is altered due to earth's orbital velocity. This is accounted for by shifting the origin of the mirror, the value of h below. This is mathematically equivalent because it results in the light path difference expected, $0.2^* \lambda$)

$k := 0\ nm$ origin of light wave is along the x axis, so origin offset in y direction is 0

$\Delta_{D.MM.1887} = 110\ nm$ difference in light path travel distance, blue path (parallel) - red path (perpendicular)

$h := d + \Delta_{D.MM.1887} = 11000000110\ nm$ origin of light wave is the point of reflection off the mirror

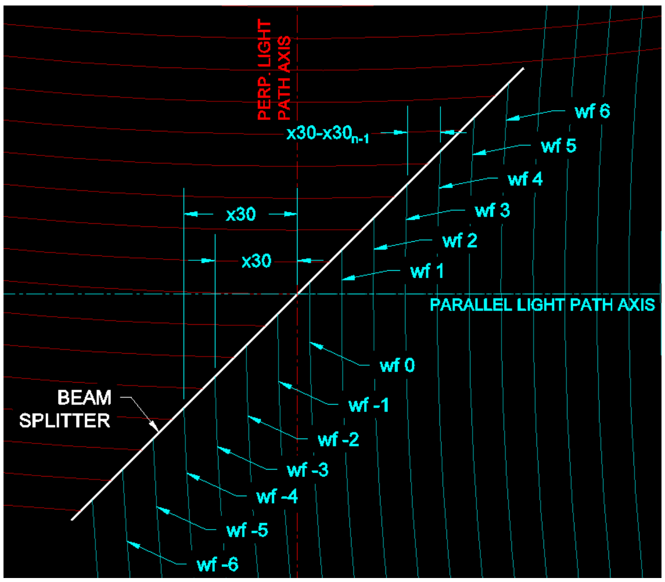
(this is the main difference in comparison to the at-rest scenario - the additional distance due to the velocity of the earth of 30 km/s)

Horizontal distance from origin to point of wavefront intersection with beam splitter

$$x_{30} := \frac{h + k - \sqrt{-h^2 + 2 \cdot h \cdot k - k^2 + 2 \cdot r^2}}{2} = \begin{bmatrix} -4839.8465795074 \\ -4289.8468093355 \\ -3739.8470098538 \\ -3189.8471819503 \\ -2639.8473282896 \\ -2089.8474470954 \\ -1539.8475392559 \\ -989.8476012182 \\ -439.8476356471 \\ 110.1523556812 \\ 660.1523736549 \\ 1210.1524218267 \\ 1760.1524957556 \\ 2310.1525981062 \\ 2860.1527271022 \\ 3410.1528827435 \\ 3960.1530685829 \\ 4510.1532792913 \\ 5060.1535184214 \end{bmatrix} nm$$

Distance between bright fringes:

$$d_{bf.30_n} := \frac{x_{30_n} - x_{30_{n-1}}}{2} = \begin{bmatrix} 274.99988509 \\ 274.99989974 \\ 274.99991395 \\ 274.99992683 \\ 274.9999406 \\ 274.99995392 \\ 274.99996902 \\ 274.99998279 \\ 274.99999566 \\ 275.00000899 \\ 275.00002409 \\ 275.00003696 \\ 275.00005118 \\ 275.0000645 \\ 275.00007782 \\ 275.00009292 \\ 275.00010535 \\ 275.00011957 \end{bmatrix} nm$$



To figure out what the fringe shift is between the 30 km/s and 0km/s scenarios, the difference in the horizontal distances from the origin to point of wavefront intersection with beam splitter is calculated.

Horizontal distance from origin to point of wavefront intersection with beam splitter:

earth velocity = 0 km/s

$$x = \begin{bmatrix} -4949.99888545 \\ -4399.99912061 \\ -3849.99932734 \\ -3299.99950477 \\ -2749.99965644 \\ -2199.99978057 \\ -1649.99987806 \\ -1099.99994535 \\ -549.999986 \\ 0 \\ 550.00001264 \\ 1100.00005549 \\ 1650.00012409 \\ 2200.00022111 \\ 2750.00034389 \\ 3300.0004942 \\ 3850.00067471 \\ 4400.00088009 \\ 4950.001113 \end{bmatrix} nm$$

earth velocity = 30 km/s

$$x_{30} = \begin{bmatrix} -4839.84657951 \\ -4289.84680934 \\ -3739.84700985 \\ -3189.84718195 \\ -2639.84732829 \\ -2089.8474471 \\ -1539.84753926 \\ -989.84760122 \\ -439.84763565 \\ 110.15235568 \\ 660.15237365 \\ 1210.15242183 \\ 1760.15249576 \\ 2310.15259811 \\ 2860.1527271 \\ 3410.15288274 \\ 3960.15306858 \\ 4510.15327929 \\ 5060.15351842 \end{bmatrix} nm$$

fringe shift between at-rest and 30 km/s scenarios:

$$b_{shift} := x_{30} - x = \begin{bmatrix} 110.15231 \\ 110.15231 \\ 110.15232 \\ 110.15232 \\ 110.15233 \\ 110.15233 \\ 110.15234 \\ 110.15234 \\ 110.15235 \\ 110.15236 \\ 110.15236 \\ 110.15237 \\ 110.15237 \\ 110.15238 \\ 110.15238 \\ 110.15239 \\ 110.15239 \\ 110.15240 \\ 110.15241 \end{bmatrix} nm$$

Difference in light path travel distance, blue path (parallel) - red path (perpendicular) $\Delta_{D.MM.1887} := d_1 \cdot \frac{v^2}{c^2} = 110.15236\ nm$

These calculations agree with Michelson & Morley's statement that the light path difference, $\Delta_{D.MM.1887}$, is equal to the expected fringe shift, b_{shift}.

In reality, M&M did not have an at-rest baseline to compare to, because the apparatus is always moving at 30 km/s. So they compared two interferometer positions, more specifically they compared two interferometer positions that effectively double the expected fringe shift noted above.

	Comparing interferometer position to at-rest scenario (cannot perform in reality)	Comparing two interferometer positions (what M&M did)
Expected fringe shift	$\Delta_{D\lambda.1} := \frac{d}{\lambda} \cdot \frac{v^2}{c^2} = 0.200$	$\Delta_{D\lambda.2} := 2 \cdot \frac{d}{\lambda} \cdot \frac{v^2}{c^2} = 0.401$

This is discussed in detail further down, but the purpose of this section was to confirm the correlation between light path travel distance difference and expected fringe shift.

Checking the light path length as stated by M&M.

$A_{stone} := 1.5\ m^2$ surface area of top surface of stone

$b_{stone} := \sqrt{A_{stone}} = 4.018\ ft$ length of sides (assuming square)

For measuring the light paths (based on the sketch provided in the paper) the measurements were calibrated using the length of the side of the stone.

$l_{1\_2} := 2.927\ ft$ $l_{2\_3} := 5.081\ ft$ $l_{3\_4} := 5.230\ ft$ $l_{4\_5} := 5.367\ ft$

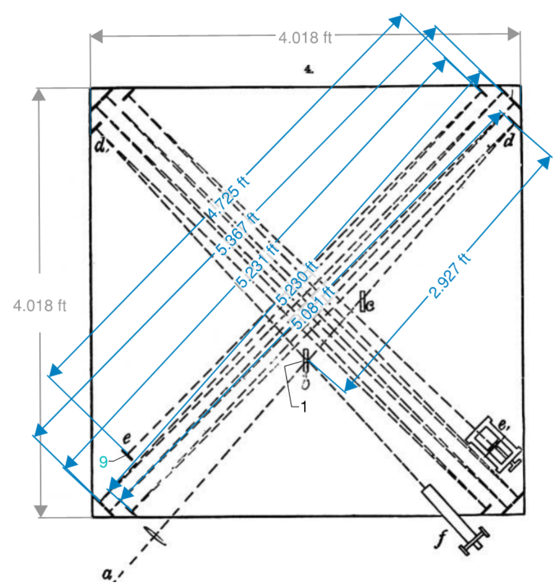
$l_{5\_6} := l_{4\_5} = 5.367\ ft$ $l_{6\_7} := l_{4\_5} = 5.367\ ft$ $l_{7\_8} := 5.231\ ft$

$l_{8\_9} := 4.725\ ft$

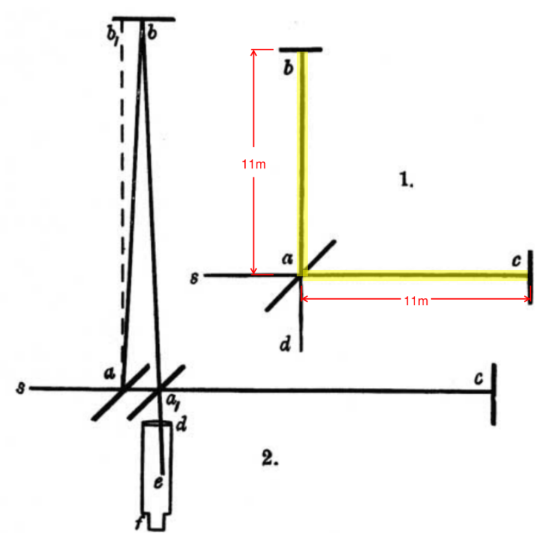
$$D_{measure} := l_{1\_2} + l_{2\_3} + l_{3\_4} + l_{4\_5} + l_{5\_6} + l_{6\_7} + l_{7\_8} + l_{8\_9} = 11.98\ m$$

measured 1-way light path distance(point 1 to point 9)

M&M stated that this distance is about 11 m.



This sketch is from the 1887 experiment.



This sketch is from the simpler 1881 experiment, but is shown to demonstrate the distance being calculated (the one-way path length).

The light path distance I measured (11.98 m) from the sketch (calibrated from the length of the side of the stone) is approximate because the sketch is not perfectly to scale nor are the measurements perfectly precise. The measured distance is close enough to the stated distance to show that the length of the light path of 11 m, as they stated, makes sense given the surface area of the stone (assumed to be square) and the number of mirrors.

In the sketch, if each mirror is moved in (towards the opposing mirrors) by 2 9/16 inches, then the measured distance aligns better with the stated distance of 11m. This is just showing that the about 0.98m difference between measured and stated can easily be made up by moving the mirrors slightly, demonstrating the difference between measured and stated light path is trivial.

$$D_{measure} - 15 \cdot \left(2 + \frac{9}{16}\right) in = 11.00\ m$$

Note: 2 9/16 inches is multiplied by 15, because the first measurement (l_{1_2}) is reduced once (only one interaction with mirror), the remaining measurements are reduced twice (two interactions with mirrors) and there are 7 of them, 1+2*7 = 15.

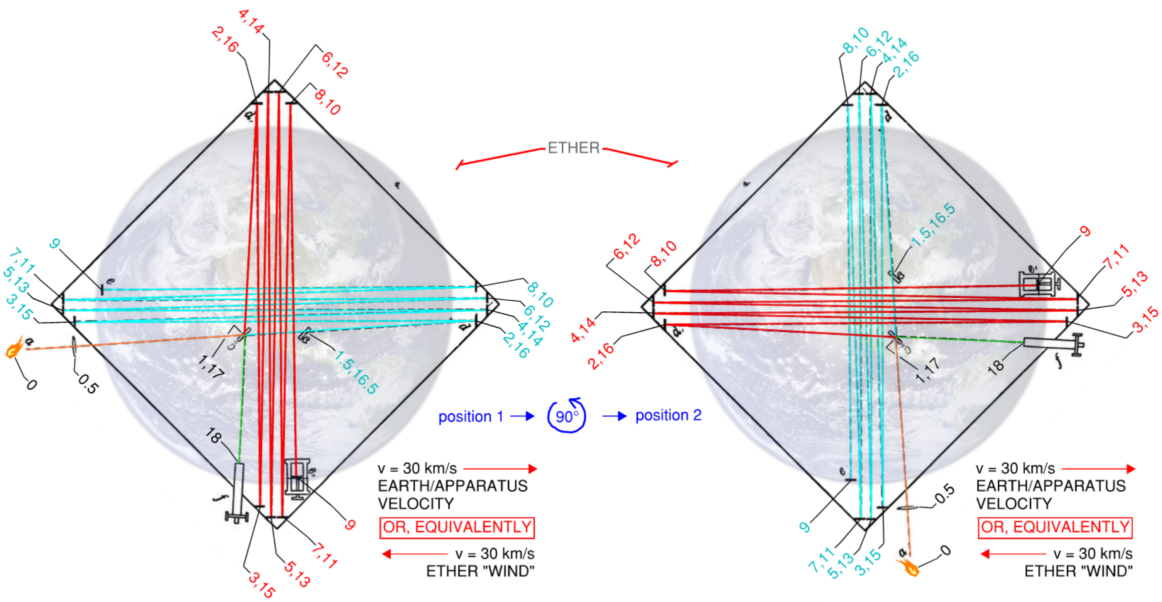
Discussion on the interferometer positions.

In M&M 1881 & 1887 paper, they state that the light path distance difference is directly related to something that can be measured on the interference fringe pattern (as just examined and confirmed), and since the light path distance difference is dependent on the 30 km/s earth velocity, we see that there is a measurement that they can make in the observable fringe patterns which would correspond to the assumed earth orbital velocity of 30 km/s. If their measurement matched what they calculated it should be, they would say that the relative motion between the earth and the ether is 30 km/s, thereby proving the existence of an ether (to them).

The fringe shift measurement that M&M are measuring is a measurement of how much one fringe pattern (corresponding to 1 apparatus position) has moved in relation to another fringe pattern based on a different apparatus position. Since earth is moving at 30 km/s, there is no "at-rest" reference point to measure from, so they need at least two fringes patterns from two apparatus positions.

As explained in the 1887 paper, "If now the whole apparatus be turned through 90°, the difference will be in the opposite direction, hence the displacement of the interreference fringes should be $2 \cdot D \cdot \frac{v^2}{V^2}$".

In position 1 (refer to diagrams below), there is the maximum positive fringe shift because this position created the maximum path length difference (based on the orientation of the paths with respect to v, we'll say when blue > red, shift is +). Position 2 is another position of maximum difference, just in the opposite direction (red > blue), so it is the maximum shift in the other direction.

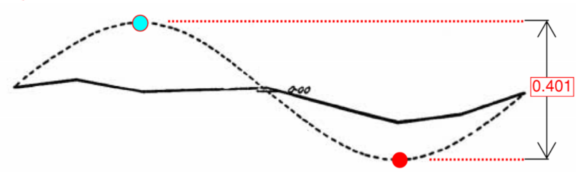


So they established what the maximum limits of shift should be, and given that this is a rotating device, they assumed a periodic trend of shift (like a sine wave).

That is why they multiplied by 2 to get the expected fringe displacement range.

$\Delta_{D\lambda.2} = 0.401$

blue point represents position 1 (maximum "positive" fringe shift)



red point represents position 2 (maximum "negative" fringe shift)

The following pages include calculations and derivations of equations in order to check Michelson & Morley's equations that they included in their papers (light travel distances, time for light to travel these distances, etc.). The calculation is the form of a moving light clock configuration, but the configuration is exactly equivalent to the moving interferometer.

Once the equations are examined, an analysis of the interferometer positions and the results of the experiment are discussed.

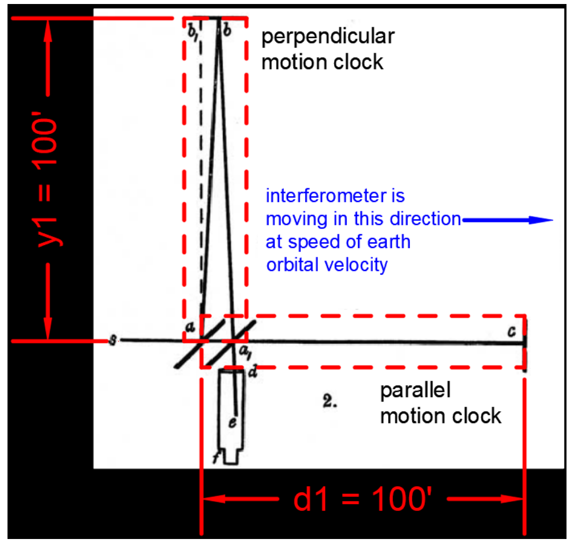
Moving Light clock example (not considering special relativity theory)

$c = 983571056 \frac{ft}{s}$ speed of "object" (light) used in clock mechanism

$v := 0.9 \cdot c = 885213951 \frac{ft}{s}$ velocity of interferometer

$y_1 := 100\ ft$ clock height (for perpendicular motion clock)

$d_1 := 100\ ft$ clock length (for parallel motion clock)



The perpendicular motion light clock and parallel motion light clock are combined to represent the two arms of the Michelson Morley interferometer.

Perpendicular motion clock analysis (refer to example 1 for equation derivations).

$$t_{1.perp} := \frac{y_1}{\sqrt{c^2 - v^2}} = 0.000000233247748\ s$$

$$t_{2.perp} := \frac{y_1}{\sqrt{c^2 - v^2}} = 0.000000233247748\ s$$

$$t_{perp} := t_{1.perp} + t_{2.perp} = 0.000000466495496\ s$$

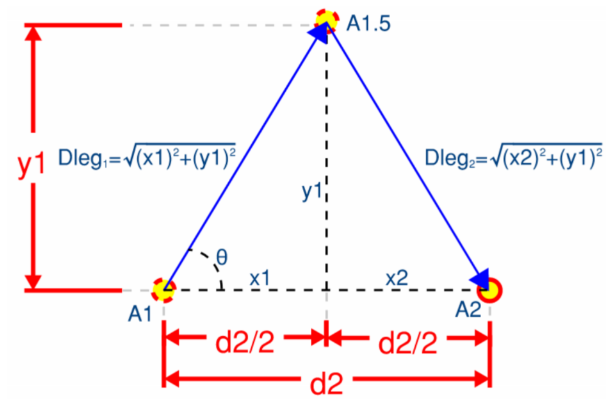
$$d_2 := v \cdot \left(t_{1.perp} + t_{2.perp}\right) = 412.948\ ft$$

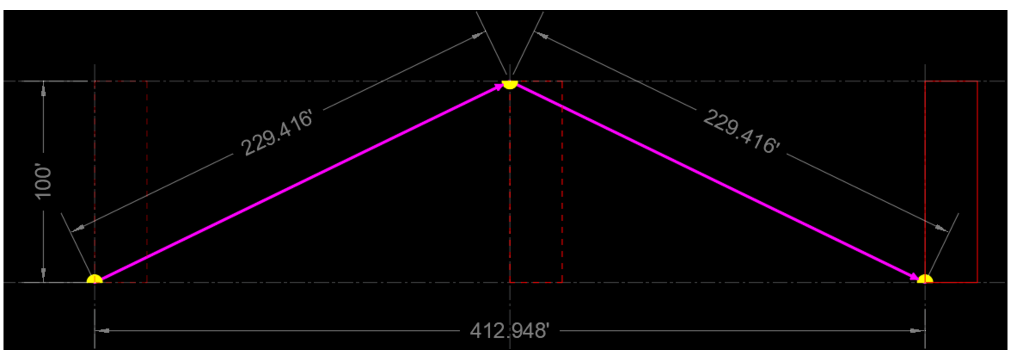
$$x_1 := 0.5 \cdot d_2 = 206.474\ ft$$

$$D_{leg1.perp} := \sqrt{\left(x_1\right)^2 + \left(y_1\right)^2} = 229.416\ ft$$

$$D_{leg2.perp} := D_{leg1.perp} = 229.416\ ft$$

$$D_{perp} := D_{leg1.perp} + D_{leg2.perp} = 458.831\ ft$$





Derivation of distance traveled (perp. clock)

$$D_{perp} = D_{leg1.perp} + D_{leg2.perp} = 1 \quad \text{-->} \quad D_{perp} = \sqrt{\left(x_1\right)^2 + \left(y_1\right)^2} + \sqrt{\left(x_1\right)^2 + \left(y_1\right)^2} = 1 \quad \text{-->} \quad D_{perp} = 2 \cdot \sqrt{\left(0.5 \cdot d_2\right)^2 + \left(y_1\right)^2} = 1$$

$$D_{perp} = 2 \cdot \sqrt{\left(0.5 \cdot v \cdot \left(t_{1.perp} + t_{2.perp}\right)\right)^2 + \left(y_1\right)^2} = 1 \quad \text{-->} \quad D_{perp} = 2 \cdot \sqrt{\left(0.5 \cdot v \cdot \left(2 \cdot \sqrt{\frac{y_1^{\ 2}}{\left(c^2 - v^2\right)}}\right)\right)^2 + \left(y_1\right)^2} = 1$$

$$D_{perp} = 2 \cdot \sqrt{\left(v \cdot \sqrt{\frac{y_1^{\ 2}}{\left(c^2 - v^2\right)}}\right)^2 + \left(y_1\right)^2} = 1 \quad \text{-->} \quad D_{perp} = 2 \cdot \sqrt{\left(v \cdot \frac{y_1}{\sqrt{\left(c^2 - v^2\right)}}\right)^2 + \left(y_1\right)^2} = 1 \quad \text{-->} \quad D_{perp} = 2 \cdot \sqrt{\left(v \cdot \frac{y_1}{\sqrt{\left(c^2 - v^2\right)}}\right)^2 + \left(y_1\right)^2} = 1$$

$$D_{perp} = 2 \cdot \sqrt{\frac{y_1^{\ 2} \cdot c^2 - y_1^{\ 2} \cdot v^2 + y_1^{\ 2} \cdot v^2}{c^2 - v^2}} = 1 \quad \text{-->} \quad D_{perp} = 2 \cdot \frac{\sqrt{y_1^{\ 2} \cdot c^2 - y_1^{\ 2} \cdot v^2 + y_1^{\ 2} \cdot v^2}}{\sqrt{c^2 - v^2}} = 1 \quad \text{-->} \quad D_{perp} = 2 \cdot \frac{\sqrt{y_1^{\ 2} \cdot c^2}}{\sqrt{c^2 - v^2}} = 1$$

$$D_{perp} = 2 \cdot y_1 \cdot \frac{c}{\sqrt{c^2 - v^2}} = 1 \quad \text{-->} \quad D_{perp} = 2 \cdot y_1 \cdot \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 1 \quad \text{-->} \quad D_{perp} = 2 \cdot y_1 \cdot \gamma = 1$$

Sidenote on Lorentz factor origination:

They had to make the two light travel paths equal, $D_{perp} = D_{par}$, to attempt to explain the lack of fringe shift due to earth's orbital velocity of 30 km/s. Rather than question the earth's orbital velocity which may seem logical, they chose to speculate that the dimensions of object must shorten, but only objects moving in the direction of earth's orbital velocity shorten.

From further down in this calculation the parallel light path distance was determined to be: $D_{par} := 2 \cdot d_1 \cdot \frac{c^2}{c^2 - v^2} = 1052.632\ ft$

So I am assuming they said, how do we make $D_{perp} = D_{par}$? Let's set them equal and see what we need to multiply one by so it equals the other.

$D_{perp} = 458.831\ ft$ $\qquad D_{par} = 1052.632\ ft$

$$D_{perp} = 2 \cdot d_1 \cdot \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad D_{par} := 2 \cdot d_1 \cdot \frac{c^2}{c^2 - v^2}$$

(note:y1 = d1, y1 is swapped out so the same variable is used)

What can me multiply D_{par} by so it equals D_{perp}?

Let's multiply D_{par} by the reciprocal of how D_{perp} is affected by earth's motion.

$$D_{par.cont} := 2 \cdot d_1 \cdot \frac{c^2}{c^2 - v^2} \cdot \sqrt{1 - \frac{v^2}{c^2}} = 458.831\ ft$$

And this is how the Lorentz factor came to be. It is an imposed reduction in the parallel light path length in Michelson and Morley's interferometer. It was what had to be done to maintain the Copernican (heliocentric) model. The experiment showed that the corresponding fringe shift did not align with a 30 km/s earth velocity, so they invented distance and time distortion.

Derivation of time taken (perp. clock)

$$t_{perp} = t_{1.perp} + t_{2.perp} = 1 \quad \text{-->} \quad t_{perp} = \left(\frac{y_1}{\sqrt{c^2 - v^2}} + \frac{y_1}{\sqrt{c^2 - v^2}} \right) = 1 \quad \text{-->} \quad t_{perp} = 2 \cdot y_1 \cdot \frac{1}{\sqrt{c^2 - v^2}} = 1$$

Sidenote on Lorentz factor origination:

From further down in this calculation the parallel light path time was determined to be: $t_{par} := 2 \cdot d_1 \cdot \frac{c}{c^2 - v^2} = 0.000001070214065\ s$

And the perpendicular light path time is: $t_{perp} := 2 \cdot y_1 \cdot \frac{1}{\sqrt{c^2 - v^2}} = 0.000000466495496\ s$

The ratio of the parallel time to the perpendicular time (not considering special relativity theory) is equal to the Lorentz factor.

$$\frac{2 \cdot d_1 \cdot \frac{c}{c^2 - v^2}}{2 \cdot y_1 \cdot \frac{1}{\sqrt{c^2 - v^2}}} = 2.29415733870562 \qquad \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 2.29415733870562$$

Again, this reveals how the Lorentz factor came to be mathematically. The Lorentz factor equation was specifically created to address the Michelson & Morley experimental results.

Parallel motion clock analysis of the interferometer (refer to example 2 for equation derivations).

$t_{1.par} := \frac{d_1}{c - v} = 0.000001016703362\ s$ $\qquad t_{2.par} := \frac{d_1}{c + v} = 0.000000053510703\ s$

$t_{par} := t_{1.par} + t_{2.par} = 0.00000107021407\ s$

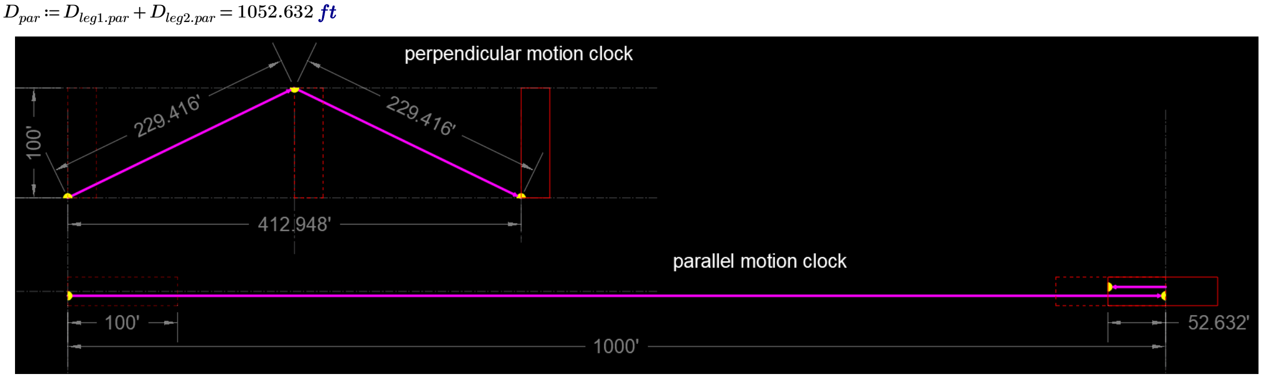
$d_2 := v \cdot t_{1.par} = 900\ ft$

$D_{leg1.par} := d_1 + d_2 = 1000\ ft$

$d_3 := v \cdot t_{2.par} = 47.368\ ft$

$D_{leg2.par} := d_1 - d_3 = 52.632\ ft$

$D_{par} := D_{leg1.par} + D_{leg2.par} = 1052.632\ ft$



Derivation of distance traveled (parallel clock)

$D_{par} = D_{leg1.par} + D_{leg2.par} = 1$ ---> $D_{par} = d_1 + d_2 + d_1 - d_3 = 1$ ---> $D_{par} = d_1 + v \cdot t_{1.par} + d_1 - v \cdot t_{2.par} = 1$

$D_{par} = d_1 + v \cdot \frac{d_1}{c-v} + d_1 - v \cdot \frac{d_1}{c+v} = 1$ ---> $D_{par} = 2 \cdot d_1 + v \cdot \left(\frac{d_1}{c-v} - \frac{d_1}{c+v}\right) = 1$ ---> $D_{par} = 2 \cdot d_1 + v \cdot \left(\frac{2 \cdot d_1 \cdot v}{(c+v) \cdot (c-v)}\right) = 1$

$D_{par} = 2 \cdot d_1 + \frac{2 \cdot d_1 \cdot v^2}{(c+v) \cdot (c-v)} = 1$ ---> $D_{par} = 2 \cdot d_1 \cdot \left(1 + \frac{v^2}{c^2 - v^2}\right) = 1$ ---> $D_{par} = 2 \cdot d_1 \cdot \frac{c^2}{c^2 - v^2} = 1$

Derivation of time taken (parallel clock)

$t_{par} = \frac{d_1}{c-v} + \frac{d_1}{c+v} = 1$ ---> $t_{par} = \frac{2 \cdot d_1 \cdot c}{(c+v) \cdot (c-v)} = 1$ ---> $t_{par} = 2 \cdot d_1 \cdot \frac{c}{c^2 - v^2} = 1$

Summary

Time for light to tick $t := \begin{bmatrix} t_{perp} \\ t_{par} \end{bmatrix} = \begin{bmatrix} 0.000000466495496 \\ 0.000001070214065 \end{bmatrix} s$ $\begin{bmatrix} \text{“perpendicular clock”} \\ \text{“parallel clock”} \end{bmatrix}$

Distance traveled by light to tick $D := \begin{bmatrix} D_{perp} \\ D_{par} \end{bmatrix} = \begin{bmatrix} 458.831 \\ 1052.632 \end{bmatrix} ft$ $\begin{bmatrix} \text{“perpendicular clock”} \\ \text{“parallel clock”} \end{bmatrix}$

The time it took for the parallel light ray to complete 1 tick is greater than the time it took the perpendicular light ray to complete 1 tick. The perpendicular light ray reaches the detector before the parallel light ray.

The difference in time is: $\Delta_t := t_{par} - t_{perp} = 0.00000060371857\ s$

$t_{par} - t_{perp} = \left(\frac{d_1}{c-v} + \frac{d_1}{c+v}\right) - \left(\sqrt{\frac{{y_1}^2}{(c^2 - v^2)}} + \sqrt{\frac{{y_1}^2}{(c^2 - v^2)}}\right) = 1$ ---> $t_{par} - t_{perp} = \left(\frac{d_1}{c-v} + \frac{d_1}{c+v}\right) - 2 \cdot \sqrt{\frac{{y_1}^2}{(c^2 - v^2)}} = 1$

$t_{par} - t_{perp} = \frac{2 \cdot d_1 \cdot c}{(c+v) \cdot (c-v)} - 2 \cdot \sqrt{\frac{{y_1}^2}{(c^2 - v^2)}} = 1$ ---> $t_{par} - t_{perp} = 2 \cdot \left(\frac{d_1 \cdot c}{(c+v) \cdot (c-v)} - \sqrt{\frac{{y_1}^2}{(c^2 - v^2)}}\right) = 1$

$t_{par} - t_{perp} = 2 \cdot \left(\frac{d_1 \cdot c}{(c+v) \cdot (c-v)} - \frac{y_1}{\sqrt{(c+v) \cdot (c-v)}}\right) = 1$ ---> $\Delta_t = 2 \cdot \left(\frac{d_1 \cdot c - y_1 \cdot \sqrt{(c+v) \cdot (c-v)}}{(c+v) \cdot (c-v)}\right) = 1$

The ratio of the times is: $r_t := \frac{t_{par}}{t_{perp}} = 2.294$ which looks familiar... $\gamma := \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 2.294$

$\frac{t_{1.par} + t_{2.par}}{t_{1.perp} + t_{2.perp}} = \frac{t_{par}}{t_{perp}} = 1$ --> $\frac{\frac{d_1}{c+v} + \frac{d_1}{c-v}}{\sqrt{\frac{{y_1}^2}{(c^2 - v^2)}} + \sqrt{\frac{{y_1}^2}{(c^2 - v^2)}}} = \frac{t_{par}}{t_{perp}} = 1$ --> $\frac{\frac{d_1}{c+v} + \frac{d_1}{c-v}}{2 \cdot \sqrt{\frac{{y_1}^2}{(c^2 - v^2)}}} = \frac{t_{par}}{t_{perp}} = 1$ --> $\frac{\frac{2 \cdot d_1 \cdot c}{(c+v) \cdot (c-v)}}{2 \cdot \sqrt{\frac{{y_1}^2}{(c^2 - v^2)}}} = \frac{t_{par}}{t_{perp}} = 1$

$\frac{\frac{d_1 \cdot c}{(c+v) \cdot (c-v)}}{\sqrt{\frac{{y_1}^2}{(c^2 - v^2)}}} = \frac{t_{par}}{t_{perp}} = 1$ --> $\frac{c \cdot d_1 \cdot \sqrt{(c+v) \cdot (c-v)}}{y_1 \cdot (c+v) \cdot (c-v)} = \frac{t_{par}}{t_{perp}} = 1$ and $d_1 = 100\ ft$ $y_1 = 100\ ft$ $d_1 = y_1 = 1$ so

$\frac{c \cdot \sqrt{(c+v) \cdot (c-v)}}{(c+v) \cdot (c-v)} = \frac{t_{par}}{t_{perp}} = 1$, γ has another form, $\gamma = \frac{c \cdot \sqrt{(c+v) \cdot (c-v)}}{(c+v) \cdot (c-v)} = 1$ --> $r_t = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 1$

The Lorentz factor is the ratio of light travel time in each arm of the interferometer (parallel arm / perpendicular arm).

The difference in distance is: $\Delta_D := D_{par} - D_{perp} = 593.800111206245\ ft$

$D_{par} - D_{perp} = 2 \cdot d_1 \cdot \frac{c^2}{c^2 - v^2} - 2 \cdot y_1 \cdot \frac{c}{\sqrt{c^2 - v^2}} = 1$ --> $D_{par} - D_{perp} = 2 \cdot d_1 \cdot \frac{c^2}{c^2 - v^2} - 2 \cdot d_1 \cdot \frac{c}{\sqrt{c^2 - v^2}} = 1$

$D_{par} - D_{perp} = 2 \cdot d_1 \cdot \left(\frac{c^2}{c^2 - v^2} - \frac{c}{\sqrt{c^2 - v^2}}\right) = 1$ --> $D_{par} - D_{perp} = 2 \cdot d_1 \cdot \left(\frac{c^2}{c^2 - v^2} - \frac{c}{\sqrt{c^2 - v^2}}\right) = 1$ --> $\Delta_D = \frac{2 \cdot d_1 \cdot c \cdot \left(c - \sqrt{(c+v) \cdot (c-v)}\right)}{(c+v) \cdot (c-v)} = 1$

Summary of times and distances of perpendicular and parallel paths of Michelson's interferometer.

$$\Delta_t = 2 \cdot \left(\frac{d_1 \cdot c - y_1 \cdot \sqrt{(c+v) \cdot (c-v)}}{(c+v) \cdot (c-v)} \right) = 1$$

parallel arm time - perpendicular arm time

$$\Delta_D = \frac{2 \cdot d_1 \cdot c \cdot \left(c - \sqrt{(c+v) \cdot (c-v)}\right)}{(c+v) \cdot (c-v)} = 1$$

parallel arm distance traveled- perpendicular arm distance traveled

$$\frac{\Delta_D}{\Delta_t} = 983571056 \, \frac{ft}{s} \qquad \frac{\Delta_D}{\Delta_t} = c = 1$$

the ratio of distance difference to time difference is equal to c (this is what special relativity theory revolves around, the constancy of the round trip velocity of the speed of light, c)

$$\frac{D_{par}}{D_{perp}} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 1$$

the ratio of the parallel arm distance to the perpendicular arm distance is the Lorentz factor

$$\frac{t_{par}}{t_{perp}} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 1$$

the ratio of the parallel arm time to the perpendicular arm time is the Lorentz factor

Comparison to Michelson & Morley's computations.

M&M performed two interferometer experiments, in 1881 and 1887. This section will compare the equations used for the distance light traveled and time taken in each arm of the interferometer between M&M's papers and these calculations.

clear (v)

$$v := \begin{bmatrix} 30 \, \frac{km}{s} \\ 0.9 \cdot c \end{bmatrix}$$

(earth orbital velocity)

(light clock example speed)

MM 1881 - distance traveled (perpendicular arm)

$D_{perp.MM.1881} := 2 \cdot y_1$ (M&M) $\qquad D_{perp} := 2 \cdot y_1 \cdot \frac{c}{\sqrt{c^2 - v^2}}$ (these calcs.)

$D_{perp.MM.1881} = 200.000000000000000 \, ft \qquad D_{perp} = \begin{bmatrix} 200.000001001385 \\ 458.831467741123 \end{bmatrix} ft$

The additional horizontal travel distances due to the orbital motion of the earth were not accounted for in M&M's equation. Their equation corresponds to a stationary interferometer, meaning earth orbital velocity = 0. They pointed this out in their 1887 paper. It was accounted for in 1887.

MM 1887 - distance traveled (perpendicular arm)

$D_{perp.MM.1887} := 2 \cdot y_1 \cdot \sqrt{1 + \frac{v^2}{c^2}} = \begin{bmatrix} 200.000001001385 \\ 269.072480941474 \end{bmatrix} ft \qquad D_{perp} := 2 \cdot y_1 \cdot \frac{c}{\sqrt{c^2 - v^2}}$ (these calcs.)

$2 \cdot y_1 \cdot \left(1 + \frac{v^2}{2 \cdot c^2}\right) = \begin{bmatrix} 200.000001001385 \\ 281 \end{bmatrix} ft \qquad D_{perp} = \begin{bmatrix} 200.000001001385 \\ 458.831467741123 \end{bmatrix} ft$

At speed of 30km/s, the equations are in alignment, but at speed of 0.9*c, the equations are not in alignment They used the second equation.

MM 1881 - distance traveled (parallel arm)

An equation for the distance taken to traverse the parallel arm is not provided in the 1881 paper.

MM 1887 - distance traveled (parallel arm)

$D_{par.MM.1887} := 2 \cdot d_1 \cdot \left(\frac{c^2}{c^2 - v^2}\right) = \begin{bmatrix} 200.00000200277 \\ 1052.63157894737 \end{bmatrix} ft \qquad D_{par} := 2 \cdot d_1 \cdot \frac{c^2}{c^2 - v^2}$ (these calcs.)

$2 \cdot d_1 \cdot \left(1 + \frac{v^2}{c^2}\right) = \begin{bmatrix} 200.00000200277 \\ 362 \end{bmatrix} ft \qquad D_{par} = \begin{bmatrix} 200.00000200277 \\ 1052.63157894737 \end{bmatrix} ft$

At speed of 30km/s, the equations are in alignment, but at speed of 0.9*c, the equations are not in alignment They used the second equation.

MM 1881 - time taken (perpendicular arm)

$$t_{perp.MM.1881} := 2 \cdot \frac{y_1}{c} = 0.000000203340672 \; s$$

$$t_{perp} := 2 \cdot y_1 \cdot \frac{1}{\sqrt{c^2 - v^2}}$$ (these calcs.)

$$t_{perp} = \begin{bmatrix} 0.000000203340673 \\ 0.000000466495496 \end{bmatrix} s$$

For this calculation, M&M incorrectly computed the time light takes to traverse the perpendicular arm as if earth were stationary.

MM 1887 - time taken (perpendicular arm)

An equation for the time taken to traverse the perpendicular arm is not provided in the 1887 paper.

MM 1881 - time taken (parallel arm)

$$t_{par.MM.1881} := 2 \cdot d_1 \cdot \frac{c}{c^2 - v^2} = \begin{bmatrix} 0.000000203340674 \\ 0.000001070214065 \end{bmatrix} s$$

$$t_{par} := 2 \cdot d_1 \cdot \frac{c}{c^2 - v^2}$$ (these calcs.)

$$t_{par} = \begin{bmatrix} 0.000000203340674 \\ 0.000001070214065 \end{bmatrix} s$$

Their equation aligns with these calculations.

MM 1887 - time taken (parallel arm)

$$t_{par.MM.1887} := 2 \cdot d_1 \cdot \frac{c}{c^2 - v^2} = \begin{bmatrix} 0.000000203340674 \\ 0.000001070214065 \end{bmatrix} s$$ (same as 1881 paper)

Their equation aligns with these calculations.

Summary of comparison.

Distance traveled by light in perpendicular arm:

$$D_{perp.MM.1881} := 2 \cdot y_1 = 200.000000000000000 \; ft$$

$$D_{perp.MM.1887} := 2 \cdot y_1 \cdot \left(1 + \frac{v^2}{2 \cdot c^2}\right) = \begin{bmatrix} 200.000001001385 \\ 281 \end{bmatrix} ft$$

$$D_{perp} := 2 \cdot y_1 \cdot \frac{c}{\sqrt{c^2 - v^2}} = \begin{bmatrix} 200.000001001385 \\ 458.831467741123 \end{bmatrix} ft$$

at speed of 30km/s, the equations are in alignment, but at speed of 0.9*c, the equations are not in alignment

Time taken for light to travel perpendicular arm:

$$t_{perp.MM.1881} := 2 \cdot \frac{y_1}{c} = 0.000000203340672 \; s$$

$$t_{perp} := 2 \cdot y_1 \cdot \frac{1}{\sqrt{c^2 - v^2}} = \begin{bmatrix} 0.000000203340673 \\ 0.000000466495496 \end{bmatrix} s$$

this is not expected to align due to the mistake in the 1881 equation, in 1887 an equation for this was not provided

Distance traveled by light in parallel arm:

$$D_{par.MM.1887} := 2 \cdot d_1 \cdot \left(1 + \frac{v^2}{c^2}\right) = \begin{bmatrix} 200.00000200277 \\ 362 \end{bmatrix} ft$$

$$D_{par} := 2 \cdot d_1 \cdot \frac{c^2}{c^2 - v^2} = \begin{bmatrix} 200.00000200277 \\ 1052.63157894737 \end{bmatrix} ft$$

at speed of 30km/s, the equations are in alignment, but at speed of 0.9*c, the equations are not in alignment

Time taken for light to travel parallel arm:

$$t_{par.MM.1881} := 2 \cdot d_1 \cdot \frac{c}{c^2 - v^2} = \begin{bmatrix} 0.000000203340674 \\ 0.000001070214065 \end{bmatrix} s$$

$$t_{par} := 2 \cdot d_1 \cdot \frac{c}{c^2 - v^2} = \begin{bmatrix} 0.000000203340674 \\ 0.000001070214065 \end{bmatrix} s$$

their equation aligns with these calculations, in 1887 they did not provide an equation for this

Time and distance difference between perpendicular and parallel arms: M&M 1881

$v := \begin{bmatrix} 30 \frac{km}{s} \\ 0.9 \cdot c \end{bmatrix}$ (earth orbital velocity) (light clock example speed) $d_1 = 100\ ft$ $y_1 = 100\ ft$

$\tau_1 := 2 \cdot d_1 \cdot c \cdot \left(\frac{1}{c^2 - v^2} - \frac{1}{c^2}\right) = \begin{bmatrix} 0.000000000000002 \\ 0.000000866873393 \end{bmatrix} s$

$\tau_2 := 2 \cdot d_1 \cdot c \cdot \left(\frac{v^2}{c^2 \cdot (c^2 - v^2)}\right) = \begin{bmatrix} 0.000000000000002 \\ 0.000000866873393 \end{bmatrix} s$

these two equations they provided are equivalent

$T_o := \frac{y_1}{c} = 0.000000101670336\ s$

$\tau_3 := 2 \cdot T_o \cdot \frac{v^2}{c^2} = \begin{bmatrix} 0.000000000000002 \\ 0.000000164705945 \end{bmatrix} s$

as they noted, the first two equations are nearly equivalent to this, as can be seen differences become apparent numerically the higher v gets

$\tau_1 - \tau_3 = \begin{bmatrix} 0.000000000000000 \\ 0.000000702167448 \end{bmatrix} s$

they used this third equation, so $\Delta_{t.MM.1881} := 2 \cdot T_o \cdot \frac{v^2}{c^2}$

$\Delta_{D.MM.1881} := c \cdot \Delta_{t.MM.1881}$ ---> $\Delta_{D.MM.1881} = c \cdot 2 \cdot T_o \cdot \frac{v^2}{c^2} = 1$

$\Delta_{D.MM.1881} = c \cdot 2 \cdot \frac{d_1}{c} \cdot \frac{v^2}{c^2} = 1$ ---> $\Delta_{D.MM.1881} = 2 \cdot d_1 \cdot \frac{v^2}{c^2} = 1$

in summary, difference between perpendicular and parallel arms:

$\Delta_{t.MM.1881} = 2 \cdot d_1 \cdot \frac{v^2}{c^3}$ $\Delta_{D.MM.1881} = 2 \cdot d_1 \cdot \frac{v^2}{c^2}$

Distance difference between perpendicular and parallel arms: M&M 1887

$\Delta_{D.MM.1887} := 2 \cdot d_1 \cdot \left(1 + \frac{v^2}{c^2}\right) - 2 \cdot y_1 \cdot \left(1 + \frac{v^2}{2 \cdot c^2}\right)$

note: time difference is left out, we are concerned with distance difference because this is directly correlated to expected fringe displacement

$\Delta_{D.MM.1887} = 2 \cdot d_1 \cdot \left(1 + \frac{v^2}{c^2} - 1 - \frac{v^2}{2 \cdot c^2}\right) = 1$ ---> $\Delta_{D.MM.1887} = 2 \cdot d_1 \cdot \left(1 + \frac{v^2}{c^2} - 1 - \frac{v^2}{2 \cdot c^2}\right) = 1$

$\Delta_{D.MM.1887} = 2 \cdot d_1 \cdot \left(\frac{v^2}{c^2} - \frac{v^2}{2 \cdot c^2}\right) = 1$ ---> $\Delta_{D.MM.1887} = 2 \cdot d_1 \cdot \left(\frac{1}{2} \cdot \frac{v^2}{c^2}\right) = 1$ ---> $\Delta_{D.MM.1887} = d_1 \cdot \frac{v^2}{c^2} = 1$

Distance difference between perpendicular and parallel arms: these calculations

$\Delta_t = 2 \cdot \left(\frac{d_1 \cdot c - y_1 \cdot \sqrt{(c+v) \cdot (c-v)}}{(c+v) \cdot (c-v)}\right) = 1$

parallel arm time minus perpendicular arm time

$\Delta_D = \frac{2 \cdot d_1 \cdot c \cdot \left(c - \sqrt{(c+v) \cdot (c-v)}\right)}{(c+v) \cdot (c-v)} = 1$

parallel arm distance traveled minus perpendicular arm distance traveled

Michelson & Morley's expected fringe displacements.

$v := 30 \frac{km}{s}$ earth orbital velocity

1881.

$\lambda_{1881} := \left(1200\ mm \cdot \frac{1}{2000000}\right) = 600\ nm$ light wave length

$d_1 := 1200\ mm = 3.937\ ft$ 1-way light travel distance (at rest) or length of each interferometer arm

$d_\lambda := \frac{d_1}{\lambda_{1881}} = 2000000$ 1-way light travel distance (at rest) or length of each interferometer arm (in terms of light wavelengths)

$\Delta_{D.MM.1881} := 2 \cdot d_1 \cdot \frac{v^2}{c^2} = 0.000000078849217\ ft$ parallel light travel distance minus perpendicular light travel distance

$\Delta_{D\lambda.MM.1881} := 2 \cdot d_\lambda \cdot \frac{v^2}{c^2} = 0.040$ parallel light travel distance minus perpendicular light travel distance (in terms of light wavelengths)

$2 \cdot \Delta_{D\lambda.MM.1881} = 0.080$ expected fringe displacement - multiply by 2 (comparison of two interferometer positions as discussed previously)

1887.

$\lambda_{1887} := \left(11\ m \cdot \frac{1}{20000000}\right) = 550\ nm$ light wave length

$d_1 := 11\ m = 36.089\ ft$ 1-way light travel distance (at rest) or length of each interferometer arm

$d_\lambda := \frac{d_1}{\lambda_{1887}} = 20000000$ 1-way light travel distance (at rest) or length of each interferometer arm (in terms of light wavelengths)

$\Delta_{D.MM.1887} := d_1 \cdot \frac{v^2}{c^2} = 0.000000361392243\ ft$ parallel light travel distance minus perpendicular light travel distance

$\Delta_{D\lambda.MM.1887} := d_\lambda \cdot \frac{v^2}{c^2} = 0.2$ parallel light travel distance minus perpendicular light travel distance (in terms of light wavelengths)

$2 \cdot d_\lambda \cdot \frac{v^2}{c^2} = 0.401$ expected fringe displacement - multiply by 2 (comparison of two interferometer positions as discussed previously)

These calculations.

$d_1 := \begin{bmatrix} 1.2 \\ 11 \end{bmatrix} m = \begin{bmatrix} 3.937 \\ 36.089 \end{bmatrix} ft \quad \begin{bmatrix} 1881 \\ 1887 \end{bmatrix}$ $\quad d_\lambda := \frac{d_1}{\begin{bmatrix} \lambda_{1881} \\ \lambda_{1887} \end{bmatrix}} = \begin{bmatrix} 2000000 \\ 20000000 \end{bmatrix} \quad \begin{bmatrix} 1881 \\ 1887 \end{bmatrix}$

$\Delta_D := \frac{2 \cdot d_1 \cdot c \cdot \left(c - \sqrt{(c+v) \cdot (c-v)}\right)}{(c+v) \cdot (c-v)} = \begin{bmatrix} 0.000000039424609 \\ 0.000000361392247 \end{bmatrix} ft$ parallel light travel distance minus perpendicular light travel distance

$\Delta_{D\lambda} := \frac{2 \cdot d_\lambda \cdot c \cdot \left(c - \sqrt{(c+v) \cdot (c-v)}\right)}{(c+v) \cdot (c-v)} = \begin{bmatrix} 0.020 \\ 0.200 \end{bmatrix} \quad \begin{bmatrix} 1881 \\ 1887 \end{bmatrix}$ parallel light travel distance minus perpendicular light travel distance (in terms of light wavelengths)

$2 \cdot \Delta_{D\lambda} = \begin{bmatrix} 0.040 \\ 0.401 \end{bmatrix} \quad \begin{bmatrix} 1881 \\ 1887 \end{bmatrix}$ expected fringe displacement - multiply by 2 (comparison of two interferometer positions as discussed previously)

Expected (calculated) fringe displacement summary.

Michelson & Morley

$\begin{bmatrix} \Delta_{D\lambda.MM.1881} \\ \Delta_{D\lambda.MM.1887} \end{bmatrix} = \begin{bmatrix} 0.040 \\ 0.200 \end{bmatrix} \quad \begin{bmatrix} 1881 \\ 1887 \end{bmatrix}$

$\begin{bmatrix} 2 \cdot \Delta_{D\lambda.MM.1881} \\ 2 \cdot \Delta_{D\lambda.MM.1887} \end{bmatrix} = \begin{bmatrix} 0.080 \\ 0.401 \end{bmatrix} \quad \begin{bmatrix} 1881 \\ 1887 \end{bmatrix}$

These calculations

$\Delta_{D\lambda} = \begin{bmatrix} 0.020 \\ 0.200 \end{bmatrix} \quad \begin{bmatrix} 1881 \\ 1887 \end{bmatrix}$ parallel light travel distance minus perpendicular light travel distance (in terms of light wavelengths)

$2 \cdot \Delta_{D\lambda} = \begin{bmatrix} 0.040 \\ 0.401 \end{bmatrix} \quad \begin{bmatrix} 1881 \\ 1887 \end{bmatrix}$

$\lambda_{1887} = 550\ nm$ (light wavelength)

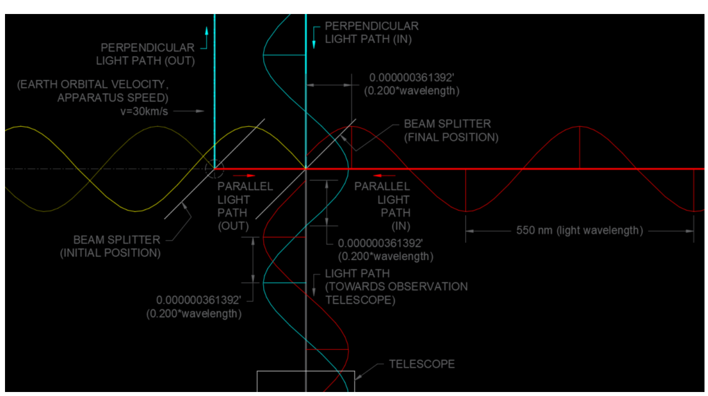
$\Delta_{D\lambda.MM.1887} = 0.200$

These numbers are lengths in terms of light wavelengths. For example, $\Delta_{D\lambda.MM.1887} \cdot 550\ nm = 110.152\ nm$

$\Delta_{D\lambda.MM.1887} \cdot 550\ nm = 0.000000361392\ ft$

These numbers represent the difference in the path lengths that the light had to take (parallel light path minus perpendicular light path).

The diagram to the right is representative of the 1887 Michelson & Morley experiment (what was expected to occur). The blue light path traveled along the perpendicular path and reached the beam splitter first. The red light path traveled along the parallel path and reached the beam splitter behind the first one, because it had to travel an additional distance of 110.152 nm, or 0.000000361392 ft, or 0.200*1 light wavelength, than the blue perpendicular path light had to travel.

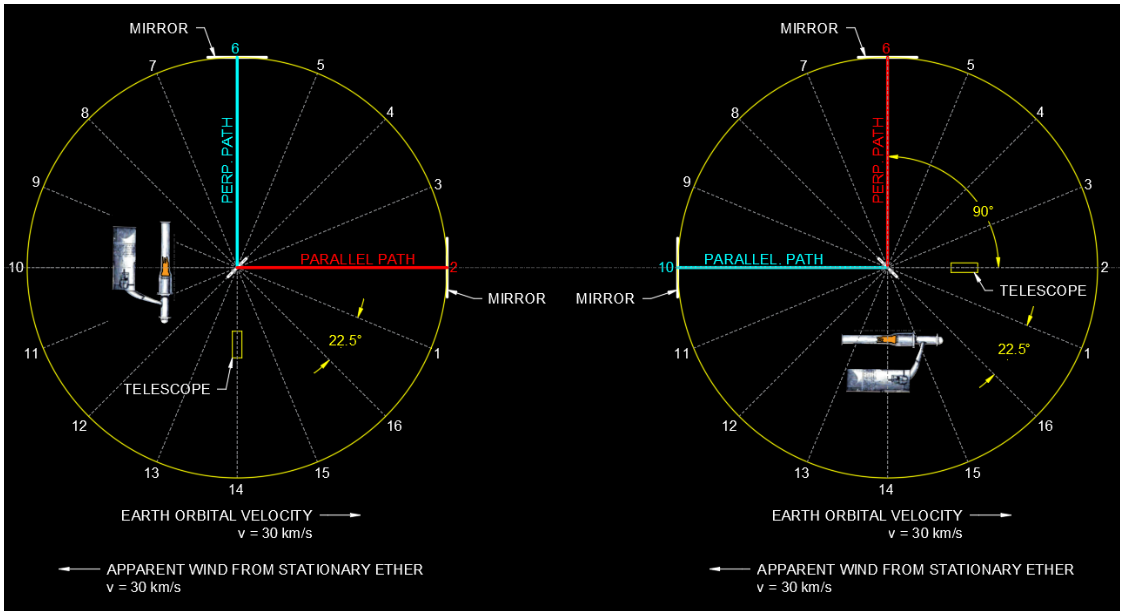


The distances calculated were the distances between the parallel path and perpendicular path. The parallel path is in the direction of the earth's velocity (30 km/s).

In both the 1881 and 1887 papers, this distance is multiplied by 2 to get the expected fringe displacement. Michelson & Morley touched on this in each paper. This is discussed later on.

This diagram shows the angles in which the apparatus was rotated. They chose to divide the circle into 22.5 degree segments, resulting in 16 possible position of the apparatus. At all of these positions they took fringe displacement readings.

In this diagram, for position naming convention, the position that is across from the light source is the position number (diagram on left is position 2, on right is position 6).

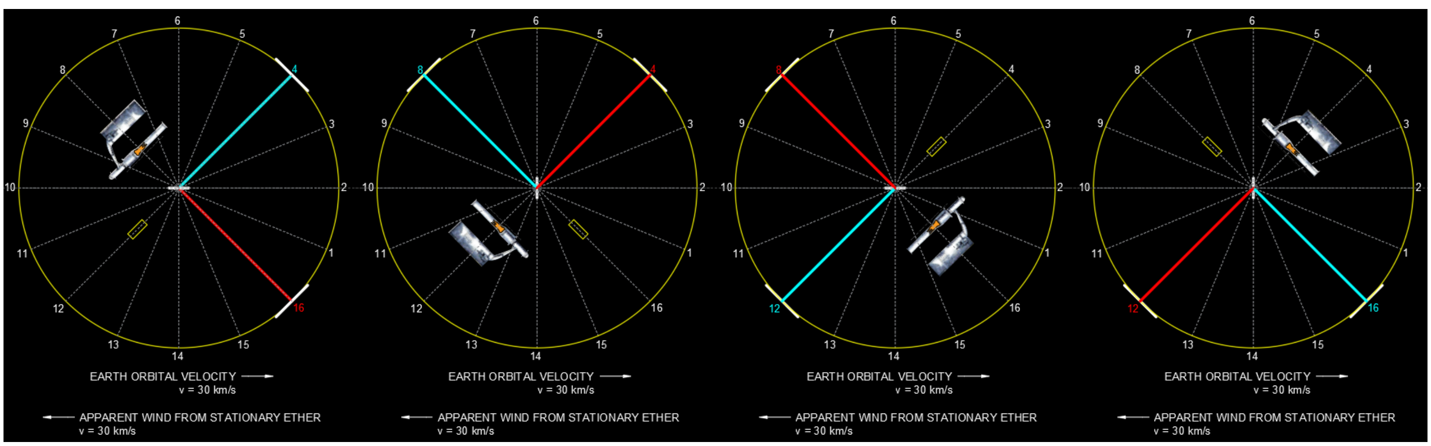


In position 2 (left diagram), this configuration produces that maximum difference between the path lengths (0.200 wavelengths).

In position 6 (right diagram), this configuration also produces that maximum difference between the path lengths (0.200 wavelengths).

There are 4 positions in which the difference between path lengths is maximized: positions 2, 6, 10, and 14.
There are also 4 positions in which the differences between the path lengths are 0: positions 16, 4, 8, and 12 (see diagram below).

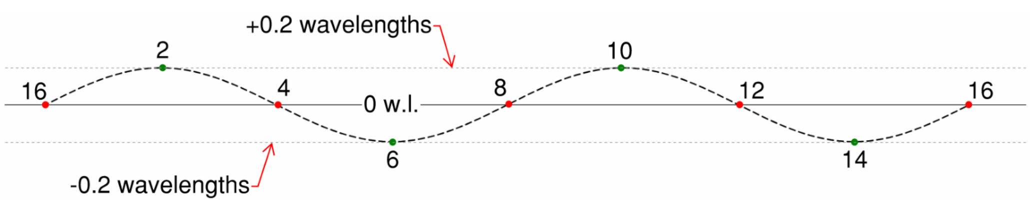
So let's say the apparatus starts at position 16. The expected fringe displacement would equal 0 (since path lengths are the same). They are the same due to their orientation with respect to earth's orbital velocity, v (to the right). Then as it is rotated counterclockwise, the path length difference grows to a maximum at position 2, and back to 0 at position 4. Continuing to rotate would follow the same trend, growing from 0, reaching a max, then back to 0.



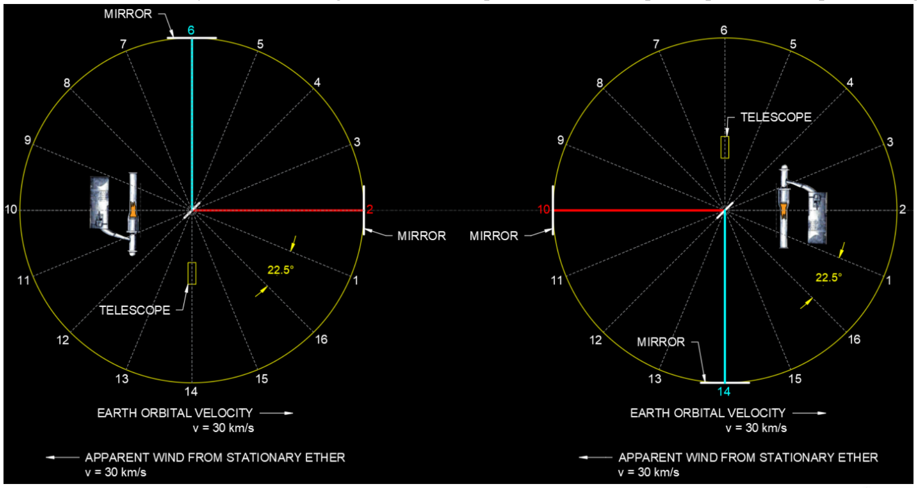
For sign convention: when red path is longer (red path - blue path > 0), difference will be positive +

when red path is shorter (red path - blue path < 0), difference will be positive -

The below graph would be the expected graph of fringe displacement as the apparatus is rotated 360 degrees.



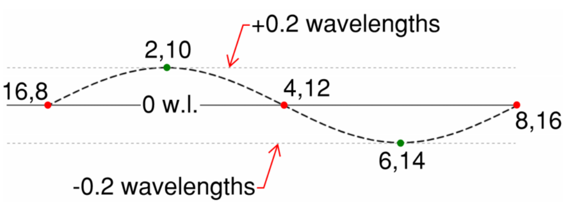
Each position of the apparatus has a sister position, in which the path length difference will be the same in magnitude and sign.
For example, looking at position 2 and 10, in these two positions, the path difference is the same (+0.2 wavelengths), as shown in the diagram below. Due to this, Michelson and Morley took the average of the sistered positions to develop their plot of the expected fringe displacement.

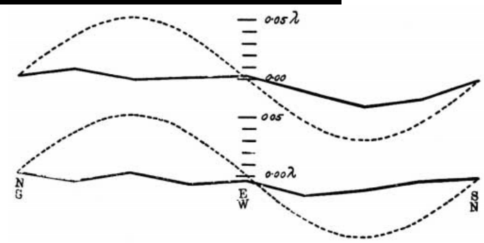


The sistered positions are:

16	&	8	The resulting graph is on the bottom left.
2	&	10	
4	&	12	
6	&	14	
8	&	16	

The Michelson & Morley expected fringe (dotted lines) and recorded fringe displacements (solid lines) are shown in the graphs on the bottom right. The top is from the "noon" observations and the bottom is from the "PM" observations.





MM 1887

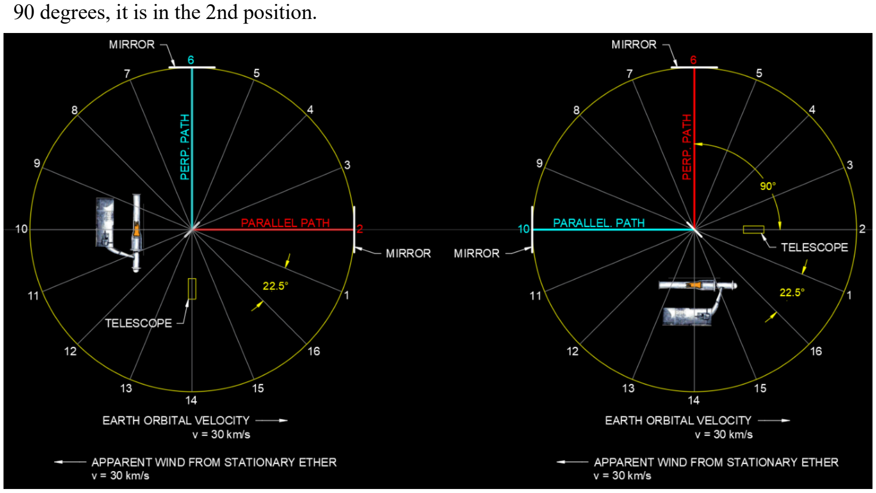
Note: In M&M's plots, the dotted lines (representing expected fringe displacement) are 1/8th of the expected displacement (0.4 λ *1/8 = 0.05* λ). In my plot, the maxima and minima are 0.2* λ , where M&M would have put 0.4* λ . I do not know why they multiplied it by two again, but it really does not matter because their recorded fringes shifts are so small.

Discussion on multiplying the expected fringe displacement by 2 again.

Unfortunately, M&M did not tell us in their paper which readings correspond to which positions in their table of fringe shifts.

As explained in the 1887 paper, "The difference is therefore $D \cdot \frac{v^2}{V^2}$. If now the whole apparatus be turned through 90°, the difference will be in the opposite direction, hence the displacement of the interreference fringes should be $2 \cdot D \cdot \frac{v^2}{V^2}$".

The two positions shown below, correspond to the situation M&M describe in the above quote from their paper. The first position is on the left, and after rotating 90 degrees, it is in the 2nd position.



They are saying that the path difference in the first position is $D \cdot \frac{v^2}{V^2}$ and the path difference in the 2nd position is also of this magnitude, just the opposite direction. So if you are measuring the fringe displacement between these two positions, the total difference would be the difference between $+ D \cdot \frac{v^2}{V^2}$ and $- D \cdot \frac{v^2}{V^2}$, or $2 \cdot D \cdot \frac{v^2}{V^2}$.

first position displacement $\Delta_{D\lambda.1st.pos} := \Delta_{D\lambda.MM.1887} = 0.200$

second position displacement $\Delta_{D\lambda.2nd.pos} := \Delta_{D\lambda.MM.1887} \cdot -1 = -0.200$

displacement difference between these two positions $\Delta_{D\lambda.diff} := \Delta_{D\lambda.1st.pos} - \Delta_{D\lambda.2nd.pos} = 0.401$

So 0.401 would be the range of shifting, not the maxima and minima of the plots

Attempt to reproduce the plots shown in the M&M 1887 paper.

Noon observations

$$div_{noon} := \begin{bmatrix} 44.7 & 44.0 & 43.5 & 39.7 & 35.2 & 34.7 & 34.3 & 32.5 & 28.2 & 26.2 & 23.8 & 23.2 & 20.3 & 18.7 & 17.5 & 16.8 & 13.7 \\ 57.4 & 57.3 & 58.2 & 59.2 & 58.7 & 60.2 & 60.8 & 62.0 & 61.5 & 63.3 & 65.8 & 67.3 & 69.7 & 70.7 & 73.0 & 70.2 & 72.2 \\ 27.3 & 23.5 & 22.0 & 19.3 & 19.2 & 19.3 & 18.7 & 18.8 & 16.2 & 14.3 & 13.3 & 12.8 & 13.3 & 12.3 & 10.2 & 7.3 & 6.5 \end{bmatrix}$$

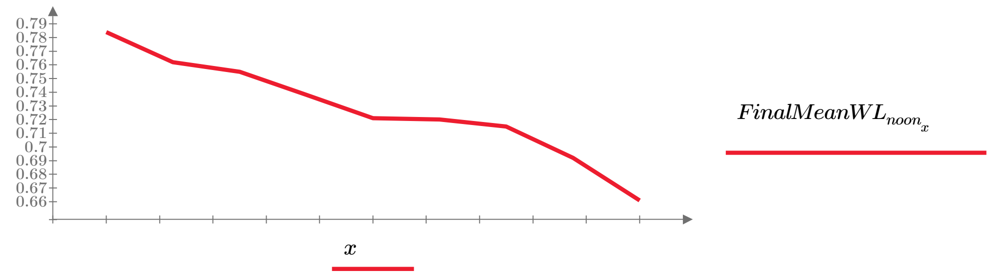
$j := 1 .. \frac{\text{cols}(div_{noon})}{2}$ $k := \frac{\text{cols}(div_{noon})+1}{2} .. \text{cols}(div_{noon})$ $Mean_j := \sum \frac{div_{noon}^{\langle j \rangle}}{\text{rows}(div_{noon})}$ $MeanWL_j := \text{Round}(Mean_j, 0.1) \cdot 0.02$ $Mean_k := \sum \frac{div_{noon}^{\langle k \rangle}}{\text{rows}(div_{noon})}$ $MeanWL_k := \text{Round}(Mean_k, 0.1) \cdot 0.02$

$$Mean^{T} = [43.1\ \ 41.6\ \ 41.2\ \ 39.4\ \ 37.7\ \ 38.1\ \ 37.9\ \ 37.8\ \ 35.3\ \ 34.6\ \ 34.3\ \ 34.4\ \ 34.4\ \ 33.9\ \ 33.6\ \ 31.4\ \ 30.8]$$

$$MeanWL^{T} = [0.862\ \ 0.832\ \ 0.824\ \ 0.788\ \ 0.754\ \ 0.762\ \ 0.758\ \ 0.756\ \ 0.706\ \ 0.692\ \ 0.686\ \ 0.688\ \ 0.688\ \ 0.678\ \ 0.672\ \ 0.628\ \ 0.616]$$

$\text{submatrix}(MeanWL, 1, 9, 1, 1)$ $\text{submatrix}(MeanWL, 9, 17, 1, 1)$ $FinalMeanWL_{noon} := \frac{\text{submatrix}(MeanWL, 1, 9, 1, 1) + \text{submatrix}(MeanWL, 9, 17, 1, 1)}{2}$ $x := 1 .. 9$

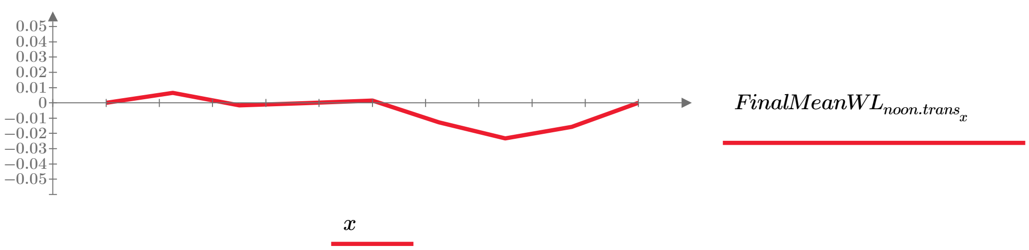
$$FinalMeanWL_{noon}^{T} = [0.784\ \ 0.762\ \ 0.755\ \ 0.738\ \ 0.721\ \ 0.72\ \ 0.715\ \ 0.692\ \ 0.661]$$



This plot shows raw data from the table in the paper.

$m_{PM} := \frac{(FinalMeanWL_{noon}^{T})_{1,9} - (FinalMeanWL_{noon}^{T})_{1,1}}{8} = -0.015$ $\theta := \text{atan}\left(\frac{(FinalMeanWL_{noon}^{T})_{1,9} - (FinalMeanWL_{noon}^{T})_{1,1}}{8}\right) = -0.881\,deg$

$A_j := \left((FinalMeanWL_{noon}^{T})_{1,j+1} - (FinalMeanWL_{noon}^{T})_{1,1} - j \cdot m_{PM}\right) \cdot \cos(\theta)$ $FinalMeanWL_{noon.trans} := \text{stack}(0, A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8) \cdot -1$



This plot shows the transformed data from the table in the paper.

$$FinalMeanWL_{noon.trans}^{T} = [0.000\ \ 0.007\ \ -0.002\ \ 0.000\ \ 0.001\ \ -0.013\ \ -0.023\ \ -0.016\ \ 0.000]$$

PM observations

$$div_{PM} := \begin{bmatrix} 61.2 & 63.3 & 63.3 & 68.2 & 67.7 & 69.3 & 70.3 & 69.8 & 69.0 & 71.3 & 71.3 & 70.5 & 71.2 & 71.2 & 70.5 & 72.5 & 75.7 \\ 26.0 & 26.0 & 28.2 & 29.2 & 31.5 & 32.0 & 31.3 & 31.7 & 33.0 & 35.8 & 36.5 & 37.3 & 38.8 & 41.0 & 42.7 & 43.7 & 44.0 \\ 66.8 & 66.5 & 66.0 & 64.3 & 62.2 & 61.0 & 61.3 & 59.7 & 58.2 & 55.7 & 53.7 & 54.7 & 55.0 & 58.2 & 58.5 & 57.0 & 56.0 \end{bmatrix}$$

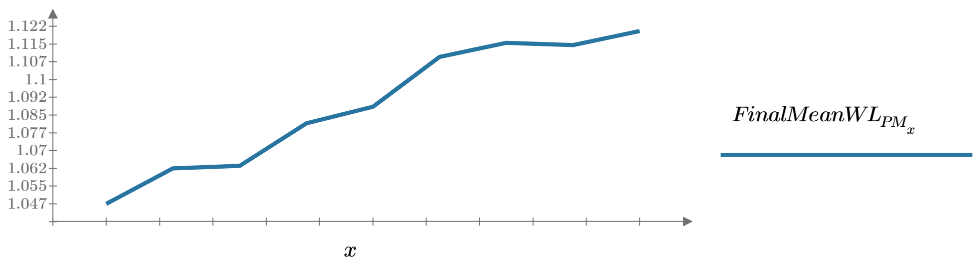
$k := \frac{\text{cols}(div_{PM}) + 1}{2} .. \text{cols}(div_{PM})$ $\quad Mean_j := \sum \frac{div_{PM}^{\langle j \rangle}}{\text{rows}(div_{PM})}$ $\quad MeanWL_j := \text{Round}(Mean_j, 0.1) \cdot 0.02$ $\quad Mean_k := \sum \frac{div_{PM}^{\langle k \rangle}}{\text{rows}(div_{PM})}$ $\quad MeanWL_k := \text{Round}(Mean_k, 0.1) \cdot 0.02$

$$Mean^{T} = \begin{bmatrix} 51.3 & 51.9 & 52.5 & 53.9 & 53.8 & 54.1 & 54.3 & 53.7 & 53.4 & 54.3 & 53.8 & 54.2 & 55 & 56.8 & 57.2 & 57.7 & 58.6 \end{bmatrix}$$

$$MeanWL^{T} = \begin{bmatrix} 1.026 & 1.038 & 1.05 & 1.078 & 1.076 & 1.082 & 1.086 & 1.074 & 1.068 & 1.086 & 1.076 & 1.084 & 1.1 & 1.136 & 1.144 & 1.154 & 1.172 \end{bmatrix}$$

$submatrix(MeanWL, 1, 9, 1, 1)$ $\quad submatrix(MeanWL, 9, 17, 1, 1)$ $\quad FinalMeanWL_{PM} := \frac{\text{submatrix}(MeanWL, 1, 9, 1, 1) + \text{submatrix}(MeanWL, 9, 17, 1, 1)}{2}$

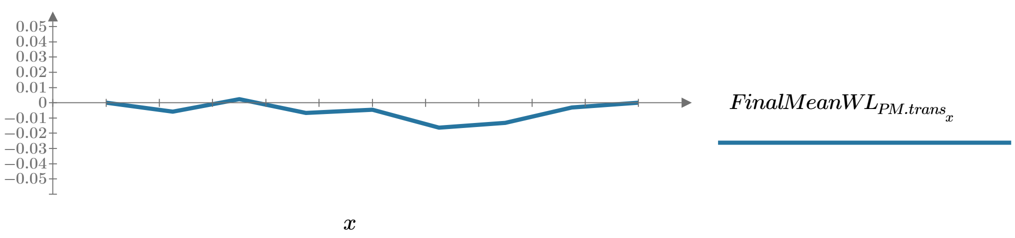
$$FinalMeanWL_{PM}{}^{T} = \begin{bmatrix} 1.047 & 1.062 & 1.063 & 1.081 & 1.088 & 1.109 & 1.115 & 1.114 & 1.120 \end{bmatrix}$$



This plot shows raw data from the table in the paper.

$m_{PM} := \frac{\left(FinalMeanWL_{PM}{}^{T}\right)_{1,9} - \left(FinalMeanWL_{PM}{}^{T}\right)_{1,1}}{8} = 0.009$ $\quad \theta := \text{atan}\left(\frac{\left(FinalMeanWL_{PM}{}^{T}\right)_{1,9} - \left(FinalMeanWL_{PM}{}^{T}\right)_{1,1}}{8}\right) = 0.523\,\textbf{deg}$

$A_j := \left(\left(FinalMeanWL_{PM}{}^{T}\right)_{1,j+1} - \left(FinalMeanWL_{PM}{}^{T}\right)_{1,1} - j \cdot m_{PM}\right) \cdot \cos(\theta)$ $\quad FinalMeanWL_{PM.trans} := \text{stack}(0, A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8) \cdot -1$



This plot shows the transformed data from the table in the paper.

$$FinalMeanWL_{PM.trans}{}^{T} = \begin{bmatrix} 0.000 & -0.006 & 0.002 & -0.007 & -0.004 & -0.016 & -0.013 & -0.003 & 0.000 \end{bmatrix}$$

Notes on transforming the raw data:

1. This took some guesswork, as the 1887 paper provided no information of how they went from the raw data presented in the table to the plotted data.
2. To visualize the transformation, draw a line from the 1st point of the raw data plot to the last point. This line becomes the new x-axis in the transformed plot. I assume since the first and last readings are the same interferometer position and these positions are mathematically 0 fringe displacement positions (because they are the same position, one is the starting position and then after the interferometer has rotated 360°, readings are taken again at this same position), they put these readings to a value of 0 fringe shift (which is why this line becomes the transformed x-axis). This is what lead to my decision to put the positions (1 through 16) where they are in the interferometer diagrams above.
3. Then the data was mirrored about the x-axis (multiplied by -1). I assume this is for sign convention or presentation reasons.

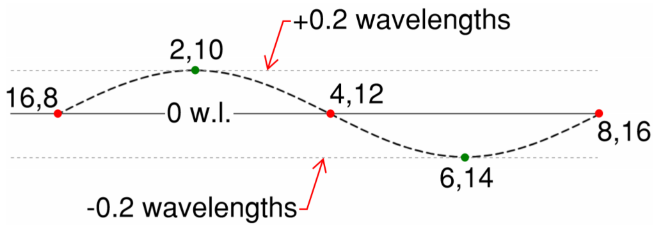
These are the final fringe displacements numbers from the MM 1887 paper:

Expected fringe displacement $\Delta_{D\lambda.MM.1887} \cdot 2 = 0.401$

Experimentally measured fringe displacements (transformed)

16,8 2,10 4,12 6,14 8,16 <--Interferometer positions

$$\begin{bmatrix} FinalMeanWL_{noon.trans}{}^{T} \\ FinalMeanWL_{PM.trans}{}^{T} \end{bmatrix} = \begin{bmatrix} [0.000 & 0.007 & -0.002 & 0.000 & 0.001 & -0.013 & -0.023 & -0.016 & 0.000] \\ [0.000 & -0.006 & 0.002 & -0.007 & -0.004 & -0.016 & -0.013 & -0.003 & 0.000] \end{bmatrix}$$



At the maxima the fringe shift is: $b_{shift.max} := 0.002$ (we will just use the positive value here from the PM observations, as a negative value in this position does not make sense)

At the minima the fringe shift is: $b_{shift.min} := \text{mean}(-0.023, -0.013) = -0.018$

Average fringe shift: $b_{avg} := \text{mean}(b_{shift.max}, |b_{shift.min}|) = 0.010$

$\Delta_{D\lambda} := b_{avg} = 0.010$

Since we know the fringe shift, we can solve for the orbital velocity of earth that corresponds to this shift.

Using the equation from my calculations:

$$\Delta_D = \frac{2 \cdot d_1 \cdot c \cdot \left(c - \sqrt{(c+v) \cdot (c-v)}\right)}{(c+v) \cdot (c-v)}$$ (path length difference)

$$\Delta_{D\lambda} = \frac{\Delta_D}{\lambda}$$ (path length difference, in terms of wavelength)

$\text{clear}(v)$ $\lambda = 550\ nm$ $d_1 := 11\ m$

Guess Values

$v := 1 \frac{km}{s}$

Constraints

$$\Delta_{D\lambda} = \frac{\dfrac{2 \cdot d_1 \cdot c \cdot \left(c - \sqrt{(c+v) \cdot (c-v)}\right)}{(c+v) \cdot (c-v)}}{\lambda}$$

Solver

$\mathbf{v} := \text{find}(v)$

Note: To solve for an unknown variable in an equation (v in this case) a solve block is used in Mathcad. A guess at the value of v is required for this function to work. The guess value is arbitrary and can be ignored.

$\mathbf{v} = 6.7 \frac{km}{s}$

Corresponding relative motion between the earth and the light propagation medium (ether or the nothingness of empty space depending on your choice) which corresponds to the fringe shifts that Michelson & Morley measured

They concluded that the relative motion between the earth and the medium in which light propagates (which was assumed to be stationary in this experiment) must be "small, quite small".

The fringe shifts did not shift the distance that they had calculated based on an assumed velocity of earth of 30 km/s. Their mathematics have nothing to do with the ether (there are not any calculations regarding the medium). Their mathematics is trigonometry and distance = velocity*time. So you can completely remove the ether from the experiment and look at this experiment as just simply trying to calculate earth's orbital velocity with respect to the light propagation medium.

If you believe the medium doesn't exist (it is nothingness) as mainstream teaches (and nothing is of course motionless, or stationary), then you can view Michelson's experiment as attempting to calculate the speed of the earth moving through the stationary nothing. We are told earth is moving absurdly fast through the nothingness. We are told earth is spinning at ~1000 mph, revolving around the sun at ~67,000 mph (30 km/s), and also revolving around the galactic center (along with everything else in our galaxy) at ~500,000 mph (which is not even considered in these experiments because this alleged speed had not been fabricated yet). So if light propagated through a stationary nothing, then fringe shifts should have been measured corresponding to what the light path travel distance difference (in the the interferometer) is when considering an interferometer velocity of 500,000 mph, relative to stationary nothing.

$$v := \begin{bmatrix} 30 \dfrac{km}{s} \\ 500000\ mph \end{bmatrix} = \begin{bmatrix} 30 \\ 224 \end{bmatrix} \frac{km}{s} \quad \begin{bmatrix} \text{“earth orbital velocity through stationary nothingness”} \\ \text{“earth velocity around galaxy through stationary nothingness”} \end{bmatrix}$$

$$d_\lambda := \frac{11\ m}{550\ nm} = 20000000$$ interferometer 1-way distance in terms of wavelengths of light

$$\Delta_{D\lambda} := \overrightarrow{\frac{2 \cdot d_\lambda \cdot c \cdot \left(c - \sqrt{(c+v) \cdot (c-v)}\right)}{(c+v) \cdot (c-v)}} = \begin{bmatrix} 0.200 \\ 11.118 \end{bmatrix}$$ (expected fringe shift in + or - direction, wavelengths of light)

$$2 \cdot \Delta_{D\lambda} = \begin{bmatrix} 0.401 \\ 22.236 \end{bmatrix} \quad \begin{bmatrix} \text{“considering earth solar orbital velocity only”} \\ \text{“considering earth galactic orbital velocity only”} \end{bmatrix}$$ (expected fringe shift range, wavelengths of light)

Summary of Michelson & Morley's results and progression in physics thereafter.

Assumptions of the experiment:

1. Earth is revolving around the sun at 30 km/s.
2. The medium (ether) in which light propagates is stationary, relative to the earth.

Calculated (expected) results:

The observed fringe shifts should have been equal to the calculated fringe shifts and should correspond to the light path distance difference between the perpendicular and parallel paths of light due to a relative motion of the earth and the stationary ether of 30 km/s.

Observed results:

The observed fringe shifts did not equal the calculated fringe shifts and corresponded to a relative motion of the earth and the stationary ether of 7.5 km/s (upper limit). The accepted interpretation is a 0 km/s relative motion (which is what the mathematics of Lorentz's ether theory and Einstein's special relativity theory uses).

If the experimental results corresponded to an earth orbital velocity of 7.5 km/s or less, one might conclude that the assumption of an earth orbital velocity of 30 km/s may need another look, rather than hypothesizing that solid objects begin to shrink and time begins to distort. For the heliocentric model to be correct, and if you are alleging to know with reasonable certainty what the distance between the sun and the earth is, then based off of that distance there is only one earth orbital velocity that will get the earth around a circumference based on that known distance (radius) in one year, and that is 30 km/s.

Michelson stated that the upper limit of the relative motion of the earth and the ether was 7.5 km/s. This result was given the label "null" or "negative", meaning that the observed results did not match the expected result. It is important to note that Michelson (and Dayton Miller) found a non-zero relative motion, however the mathematics that ensued to explain the results of the Michelson experiment "null" results treats the relative motion as 0 km/s.

I am assuming that investigating the Copernican model was out of the question, and the first person who set forth to find an alternative solution was Hendrik Lorentz. Lorentz (and George Fitzgerald) began hypothesizing that the ether was somehow influencing solid objects in such a way as to physically alter their dimensions if these objects were in motion in a particular orientation through the ether. Lorentz argued that since molecular forces in solid objects are transmitted through the ether (since the ether permeates everything), then if the solid object is moving through the ether, perhaps the molecular forces in the solid object are influenced resulting in dimensional change. Lorentz's spatial transformation (change in dimensions of solid bodies) is entirely dependent on the existence of an ether. The motion of bodies through the ether was responsible for their "change in dimension". Besides the spatial transformation, Lorentz also introduced a new time concept, called "local time". He discussed this in his paper which attempted to show the spatial and temporal distortions through electromagnetic means[8].

There is no problem with attempting to come up with new theories to explain observations, but the issue with Lorentz's work is that he introduces the factor for his coordinate system transform, $\beta^2 = \dfrac{c^2}{c^2 - v^2}$ (which is equivalent to the modern day notation, $\gamma = \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}$) without explanation in his paper in 1904[9], it is only introduced by means of "we shall".

So in 1904, Lorentz puts forth a theory which maintains the Copernican model, maintains the ether medium in which light propagates, is consistent with the interpretation of the Michelson experiment, and results in length contraction and time distortion. This did not last long as Einstein was fast at work having daydreams about light clocks and in 1905 he was ready to change physics and put it on the path it is on to this day.

In Einstein's 1905 special relativity theory (to explain Michelson's results), he redefined time and proposed that light waves propagate in empty space. Einstein's baseline definition of time (before considering effects of relative motion) is "the position of the small hand of my watch". In other words his definition is - time is what a clock says, and that clocks measure time. But that is not what time is. Time, for quite some years, has been defined by the relative movements of the earth and sun (solar time), or the relative movements of the earth and stars (sidereal time). Humans use solar time as their practical working understanding of time. Einstein removed this foundational underpinning in his definition of time. 1 rotation of the earth around its axis = 1 day = 24 hours = 1,440 minutes = 86,400 seconds. So 1 second is really 1/86,400th of 1 earth rotation. Time, relating to clocks, is a comparison of motions. In the case of an hourglass, the motion of the sand falling into the lower half (motion 1) = 1/24th of 1 earth rotation (motion 2). In the case of a wrist watch, the motion of the second hand ticking (motion 1) = 1/86,400th of 1 earth rotation (motion 2).

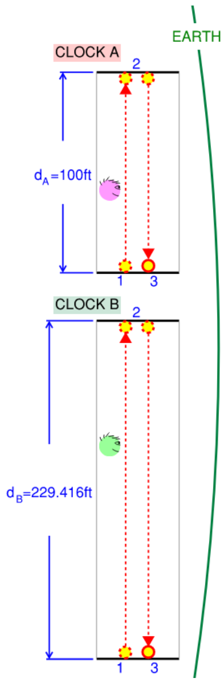
All clocks (spring-driven clocks, quartz crystal clocks, or atomic clocks) are all mechanisms that are able to perform an event (tick) in a cyclical nature, so that they will continuously "tick" at a duration of 1/86,400th of 1 earth rotation. So they are all initially calibrated based on the relative motion of the earth and sun. That is why leap seconds are added to our clocks. Because the man-made clocks need to be re-calibrated to be in sync with the true (in practice) timekeeping system - the earth and the sun. **Clocks count seconds, they do not measure time.** A clock only counts what is was made to count, it is not tapped into some universal river of flowing time. If a clockmaker doesn't configure the gears properly in the clock he is making, then it will not count seconds properly. If a clockmaker does not configure the components of a quartz crystal clock properly, then it will not count seconds properly. A clock counts seconds dependent on how the clockmaker makes the clock, not dependent on some fictitious dimension of our reality.

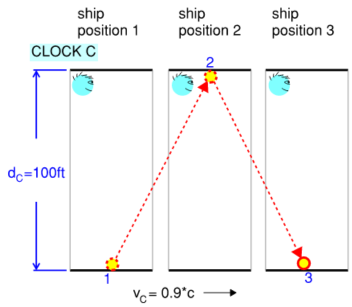
Let's say you make a light clock. The light clocks ticks every time light goes from point 1, bounces off point 2, and then returns to point 3. In order for you to have this light clock count seconds at the correct rate (thus displaying the correct time), you are going to have ensure that the distance that the light travels corresponds to the tick rate you want. Let's say you do this correctly and you built your light clock to the dimensions that allow it to tick at the correct rate. Your clock is only telling you the correct time because you built it to certain dimensions that would allow that. If you place points 1/3 too far away or too close to point 2, then the light would not travel the appropriate distance corresponding to the correct tick rate, and your clock would not tell you the right time. That does not mean time is dilating or contracting, that means your clock is useless.

The light clock thought experiment only shows that the mechanism of the clock is altered (as observed from another frame of reference, unless you are Michelson) when the clock is moving. This has nothing to do with time. You now just have a clock that is not calibrated to count seconds at the same rate as an equivalent stationary clock that is properly calibrated.

Another way to view this is shown in the example below.

$c = 299792458 \frac{m}{s}$ speed of object (light) used in clock mechanism





Stationary clock A

Clockmaker A built his clock to be 100' tall.

$v_A := 0 \frac{m}{s}$ (clock A velocity)

$d_A := 100\ ft$ (clock A height)

$t_A := \frac{2 \cdot d_A}{c} = 0.000000203340672\ s$ (clock A tick rate)

Moving clock C

Clockmaker C built his clock to be 100' tall and he is moving at 0.9*the speed of light.

$v_C := 0.9 \cdot c = 269813212 \frac{m}{s}$ (clock C velocity)

$d_C := 100\ ft$ (clock C height)

$t_C := 2 \cdot \frac{d_A}{\sqrt{c^2 - v_C^2}} = 0.000000466495496\ s$ (clock C tick rate)

Stationary clock B

Clockmaker B wants to build his clock so it ticks at the same rate as clock C. Clockmaker B has to increase his clock height so the light has to travel a longer path.

$v_B := 0 \frac{m}{s}$ (clock B velocity)

$\gamma := \frac{1}{\sqrt{1 - \frac{v_C^2}{c^2}}} = 2.294$ (factor that Clockmaker B has to increase his clock height by)

$d_B := d_A \cdot \gamma = 229.416\ ft$ (clock B height)

$t_B := \frac{2 \cdot d_B}{c} = 0.000000466495496\ s$ (clock B tick rate)

tick rate x
1 trillion ticks

After 1 trillion ticks have occurred, each clockmaker would say the following amount of time has passed:

$time_{passed} := \begin{bmatrix} t_A \\ t_B \\ t_C \end{bmatrix} \cdot 10^{12} = \begin{bmatrix} 56 \\ 130 \\ 130 \end{bmatrix} hr$ $\begin{bmatrix} \text{“clockmaker A”} \\ \text{“clockmaker B”} \\ \text{“clockmaker C”} \end{bmatrix}$

The motion of Clockmaker C has the same affect on the clock's tick rate as Clockmaker B increasing his clock's height. In essence, by moving, Clockmaker C is increasing his clock's height (increasing the travel path that light has to go to tick). Clockmaker C altered his clock configuration by moving, in the same way Clockmaker B altered his clock configuration by increasing his clock's height.

None of these clocks are plugged into a "river of flowing time". The were built by clockmakers. The clock is ticking at a rate entirely dependent on how the clockmaker made it. Time (the clock's tick rate) is not some mysterious entity that exists and permeates the universe.

Now who's clock is telling the "correct" time. What is "correct" time? When a clockmaker makes a clock, do they just randomly choose the rate to have their clocks tick at? As previously discussed, humans chose to use the relative motion of the earth and the sun to use as the recurring event to model clock's tick rates after. So, "correct" time, for all practical purposes of humanity, is produced when a clock has a tick rate that will have displayed the passing of 24 hours when the sun is back in the same position in the sky as it was on the previous day.

Obviously, we would not find it useful to have a clock that only ticks every time the sun is back in the same spot as it was the previous day. That would be like having a watch that only changes when it is a new day, not very useful. If you looked at your watch at 9 in the morning and at 10 at night, it would still display the same thing (whatever day it was). So we decided to arbitrarily subdivide the day into smaller units (hours, minutes, and seconds), which are useful units for us to keep track of each day.

If someone makes a clock that is not in sync with the celestial motions in which all clocks are modeled after, then they have a meaningless clock. In the society in which they live, their clock will be different than everyone else's so it will be of no use.

Stationary Clockmaker B has a meaningless clock. He altered his clock tick rate by altering the distance light has to travel to tick by increasing his clock height. Moving Clockmaker C has a meaningless clock. He altered his clock tick rate by altering the distance light has to travel to tick by putting his clock in motion. It does not matter by what means they have altered the light's travel distance and thus the clock tick rate, they are both meaningless clocks. Time is not slowing down.

This is essentially what we are taught time is today: A clockmaker makes a clock. This clock is now connected to or "plugged" into the river of time or spacetime or whatever people want you to think time is. The clock is now connected to some fundamental natural phenomena that is going on and is "measuring time".

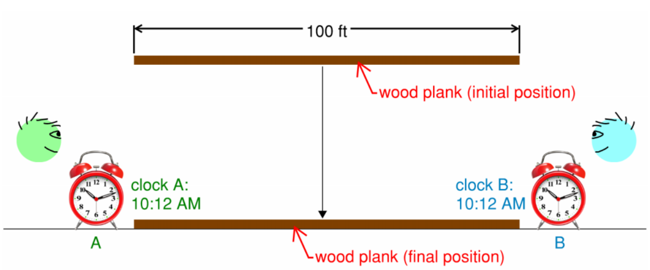
What is happening in reality? What is making clocks tick at the rate they tick? In the case of a spring-driven clock, it is ticking based on how all of the components inside the clock are functioning together to produce the end result of the clock, which is the movement of the hands. The hands on the clock are moving depending on the gear configuration and the speed at which the gears are turning. The gears are set in motion by a prestressed spring. The spring is wound up (stressed) and contained in such a way that the motion of the spring attempting to unstress itself is directed and used as the motion turning all of the shafts, gears, and eventually the hands of the clock. It is a human-made contraption, ticking at the rate it was built to tick at, which is hopefully in-sync with the established human-conceived earth/sun clock standard. I would love to see a time dilation / Lorentz factor calculation using a spring-driven clock. Chances are it has not been performed because it will result in a different mathematical equation for time dilation, because the Lorentz factor equation is dependent on clock mechanism motion in straight paths and how these straight paths are affected by translational motion. In the perpendicular motion light clock's case, Pythagoras is needed. In the spring clock's case, Pythagoras cannot mathematically describe all of the rotating motion of the gears which are also undergoing translational motion (ignoring the fact that light is not involved in the spring clock's mechanism, which is an issue, unless you feel like including the decreed Einsteinian velocity composition "rule"). Instead the very specific perpendicular motion light clock will be continued to be used to explain time dilation and it will be claimed that this applies to all types of clocks and is only being used for simplicities sake.

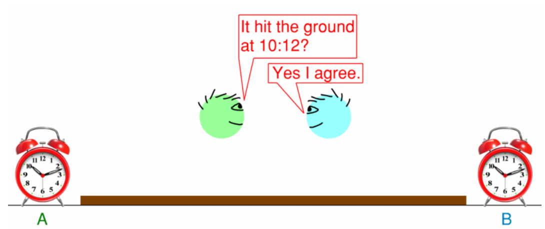
As shown in these calculations, the Lorentz factor was specifically calculated based on the Michelson & Morley 1887 interferometer experiment involving straight travel paths of light. The light clock is equivalent to the interferometer as the same motions are carried out and the same clock mechanism (light) is being used.

In Einstein's 1905 paper, he wrote, "If at the point A of space there is a clock, an observer at A can determine the time values of events in the immediate proximity of A by finding the positions of the hands which are simultaneous with these events. If there is at the point B of space another clock in all respects resembling the one at A, it is possible for an observer at B to determine the time values of events in the immediate neighborhood of B. But it is not possible without further assumption to compare, in respect of time, an event at A with an event at B".

So Einstein thinks that the following scenario is impossible:

A 100' long wood plank is floating in the air and then it falls to the ground. The event occurring at A is the left side of the plank hitting the ground. The event occurring at B is the right side of the plank hitting the ground. Person A and person B look at each of their clocks exactly when the plank hits the ground. They both say it happened at 10:12 AM. What is impossible about this?





What assumptions did we have to make for this to be possible?

1. The wood plank falls uniformly to the ground (the left side and right side fall at the same rates). Pretty reasonable.
2. Each clock is identical. Pretty reasonable - clocks made today all tick at the same rate.
3. Each clock has been identically calibrated to be in-sync with the established human-conceived earth/sun clock standard. This means that the times displayed on the clock have useful meaning (they tell the correct time). Pretty reasonable - in fact, this is necessary if you want a clock that displays the correct time.

There is no problem here. No impossibilities. No reason to redefine time.

The term "practical time" will be used as shorthand to represent the practical time that everyone in the world uses (based on the position of the sun in the sky each day, aka the relative motion of the earth and the sun). As previously discussed, there is great effort to hide this obvious fact of life of what time actually is, involving focusing your attention to the atomic clock definition of the second and completely removing the connection to the celestial timekeeping motions and making it seem like this is a thing of the past (leap seconds prove that wrong and prove that all man-made clocks are still calibrated to be in sync with practical time). This helps them make it seem like the atomic clock is measuring some intrinsic time property of the universe. It isn't. It is just a complex process which results in the counting of seconds, no different than a spring-driven clock, just with the added bonus of being more complex and thus mystifying everybody.

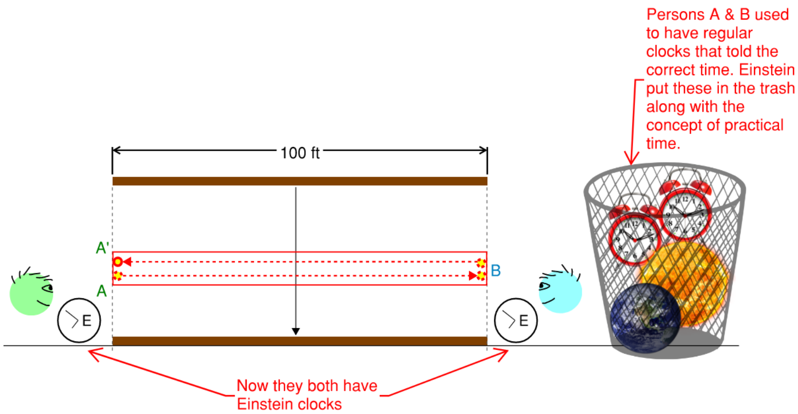
In my example, clocks A and B "resemble each other in all respects" (are identical) and are calibrated to practical time, which is the common time for everyone.

Einstein is most likely pretending to be unaware of what time actually is. He says that time is just what the clock says. It goes no further than that. There is nothing common between clocks A and B, other than they are identical. Essentially he is saying that the relative motion of the earth and the sun play no role in determining how a clockmaker configures a clock's tick rate, even though all clocks are constructed to count seconds in alignment with those motions.

Continuing with Einstein's quote on the previous page, he continues, "We have so far defined only an “A time” and a “B time.” We have not defined a common “time” for A and B, for the latter cannot be defined at all unless we establish by definition that the “time” required by light to travel from A to B equals the “time” it requires to travel from B to A. Let a ray of light start at the “A time” t_A from A towards B, let it at the “B time” t_B be reflected at B in the direction of A, and arrive again at A at the “A time” t'_A. In accordance with definition the two clocks synchronize if $t_B - t_A = t'_A - t_B$".

Einstein is pretending that there already isn't a clock synchronization process in place that every clock is calibrated to (the relative motion of the earth and sun). He apparently thinks that two regular individual clocks can not be used to compare time. He makes up a new synchronization process and connects the points in space where the two regular clocks once were. They are connected by a mutual light clock.

In the diagram below, persons A & B have been given their new Einstein clocks. These clocks are "synchronous" if the time it takes light to travel from A to B is the same time it takes light to travel from B back to A (this return position at A is called A').



So he took it upon himself to remove the celestial motions which define our practical concept of time, and replace it with the motion of light between two points, which nobody has or will ever use to calibrate or synchronize clocks.

He then assumes a new universal constant, the speed of light, and links this constant to his new definition of time in the form of: $\frac{2 \cdot AB}{t'_A - t_A} = c$.

It is important to note by the way he defines it, that he is defining the round trip speed of light to be constant, not the one way speed.

For example, the one way speed of light from A to B could be greater than c and the one way speed from B to A' could be less than c, but the average round trip speed is constant. This is exactly what is found in a parallel motion light clock showing variable time dilation (as shown in these calculations further up).

Let's look at what his equations are saying. So far, he is just discussing a stationary system.

$d_{AB} := 100\ ft \qquad d_{BA} := 100\ ft$ distance between A and B

$t_A := 0\ s$ starting time at A

$t_{ABA'} = t'_A - t_A = \frac{2 \cdot d_{AB}}{c}$ round trip time (A to B to A') so: $t_{ABA'} := \frac{2 \cdot d_{AB}}{c} = 0.000000203340672\ s$

$t_{A'} := t_{ABA'} + t_A = 0.000000203340672\ s$ end time (same as round trip travel time since $t_A := 0\ s$)

$t_B - t_A = t_{A'} - t_B$ --> $2 \cdot t_B = t'_A + t_A$ --> $t_B := \frac{t_{A'} + t_A}{2} = 0.000000101670336\ s$ (time at B)

$t_A = 0.000000000000000\ s$ time at A

$t_B = 0.000000101670336\ s$ time at B

$t_{A'} = 0.000000203340672\ s$ time at A'

$t_{AB} := t_B - t_A = 0.000000101670336\ s$ time it took light to go from A to B

$t_{BA'} := t_{A'} - t_B = 0.000000101670336\ s$ time it took light to go B to A'

$t_{ABA'} = 0.000000203340672\ s$ time of round trip

So in a stationary system, the time it takes light to complete the 1st leg of the journey is the same as the time it takes light to complete the return leg of the journey.

He then describes a scenario where this same light clock system is now moving, and it is being viewed from a stationary observer. Due to the movement of the system, the stationary observer would calculate the time of each leg differently as compared to the stationary system scenario, due to the affect of the translational motion on the light cock system. This is shown in his paper as: $t_B - t_A = \frac{r_{AB}}{c - v}$ and $t'_A - t_B = \frac{r_{AB}}{c + v}$.

$v := 0.6 \cdot c = 590142634\ \frac{ft}{s}$ (note: these times are from the stationary person's perspective of the moving person's light clock)

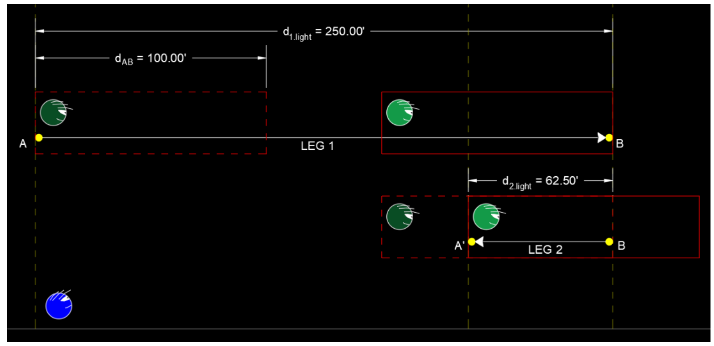
$t_{AB.moving} := \frac{d_{AB}}{c - v} = 0.000000254175841\ s$ time it took light to go from A to B

$t_{BA'.moving} := \frac{d_{AB}}{c + v} = 0.00000006354396\ s$ time it took light to go from B to A'

$t_{ABA'.moving} := t_{AB.moving} + t_{BA'.moving} = 0.000000317719801\ s$ round trip time

Let's see what the ratio of the times is: $\frac{t_{ABA'.moving}}{t_{ABA'}} = 1.563$ This does not equal the time dilation factor because length contraction has not been considered yet. $\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 1.25$

Since light propagates in nothing (empty space) according to this theory, and nothing is obviously stationary because you cannot ascribe motion to nothing (because it is not an object and you cannot tell me where nothing is in this reality and show that nothing is moving), then it can be said that the light is propagating with respect to the stationary empty space.



From the green guy's perspective (which he can consider to be the stationary reference and the blue guy is the one that is moving), he should see light travel a distance of 100 ft in leg 1 and 100 ft in leg 2. But from the light's perspective (which is propagating with respect to the stationary empty space), the light had to travel 250 feet in leg 1 and 62.50 ft in leg 2.

The motion of the light is independent of the motion of green guy's ship even if the light source is moving with the ship, so the light is going to travel whatever distance it needs to (with respect to stationary empty space) to get to point B, which is 250 feet. Green guy's spaceship can slide back and forth at different velocities along the travel axis and essentially move the goalposts back and forth along the axis of which light is propagating. This really means nothing. The travel distance that light has to travel is just being changed by virtue of green guy (and points A and B) being in motion. This only means something if you are trying to redefine time and you want your clock to tick at a rate dependent on the light bouncing off walls which are moving. It is necessary if you want to manipulate reality in order to explain 0 fringe shift in Michelson's experiment.

Sliding green guys ship back and forth at different velocities along the axis of light propagation is the same thing as making a bunch of different clocks that were made to tick at a bunch of different rates and then giving all of these different clocks to people and these people then start claiming time is running slow or fast for them. They all just have differently calibrated clocks. All that the Lorentz factor really is, is how much a clock needs its tick rate factored down by (to account for the affects of motion) so that it is ticking at a rate that is back in accordance with practical time that everybody uses for every clock. But this has been flipped on its head and we are supposed to believe that this factor is slowing down time.

I could make a theory that redefined time based on the affect that temperature has on a clock's mechanism. When my clock is really cold, for whatever reason the ticking mechanism of the clock starts to slow down. When the clock is really hot, the clock starts to tick faster. Then I would claim that time runs slower in colder locations and since I was allowed to redefine time and my theory has been thrust into international headlines and I am Time Magazine's best dude ever, you cannot argue. You must accept my definition of time. Rather than the affect that temperature has on a clock's mechanism, the affect that motion has on a clock's mechanism was what was used to redefine time, because this is what Michelson's experiment involved, and again, the whole purpose of these theories is to explain how the 30 km/s earth orbital velocity (and interferometer velocity) can have no influence on two different light paths that should be affected differently by this orbital velocity.

Same scenario as above, just re-written using green guy's perspective and the perspective of the light itself.

$$d_{1.gg} := d_{AB} = 100\ ft \qquad d_{1.light} := d_{AB} + v \cdot \frac{d_{AB}}{c - v} = 250\ ft$$

$$t_{1.gg} := \frac{d_{AB}}{c} = 0.000000101670336\ s \qquad t_{1.light} := \frac{d_{1.light}}{c} = 0.000000254175841\ s$$

$$d_{2.gg} := d_{BA} = 100\ ft \qquad d_{2.light} := d_{BA} - v \cdot \frac{d_{BA}}{c + v} = 62.5\ ft$$

$$t_{2.gg} := \frac{d_{BA}}{c} = 0.000000101670336\ s \qquad t_{2.light} := \frac{d_{2.light}}{c} = 0.00000006354396\ s$$

$$t_{total.gg} := t_{1.gg} + t_{2.gg} = 0.000000203340672\ s \qquad t_{total.light} := t_{1.light} + t_{2.light} = 0.000000317719801\ s$$

$$\frac{t_{total.light}}{t_{total.gg}} = 1.563$$

The stationary observer's perspective is the same as the perspective of the light.

The light is propagating with respect to the stationary nothing. It is not propagating with respect to the moving spaceship. In this sense, the light propagation medium (nothing) is the absolute reference frame.

Derivation of current time dilation factors (prior to considering length contraction).

overall time

$$\frac{\frac{d_{1.light}}{c} + \frac{d_{2.light}}{c}}{\frac{d_{AB}}{c} + \frac{d_{BA}}{c}} = \frac{t_{total.light}}{t_{total.gg}} = 1 \quad \text{-->} \quad \frac{\frac{d_{AB} + v \cdot \frac{d_{AB}}{c-v}}{c} + \frac{d_{BA} - v \cdot \frac{d_{BA}}{c+v}}{c}}{\frac{d_{AB}}{c} + \frac{d_{BA}}{c}} = \frac{t_{total.light}}{t_{total.gg}} = 1 \quad \text{-->} \quad \frac{d_{AB} + v \cdot \frac{d_{AB}}{c-v} + d_{BA} - v \cdot \frac{d_{BA}}{c+v}}{2 \cdot d_{AB}} = \frac{t_{total.light}}{t_{total.gg}} = 1$$

$$\frac{2 \cdot d_{AB} + v \cdot \frac{d_{AB}}{c-v} - v \cdot \frac{d_{AB}}{c+v}}{2 \cdot d_{AB}} = \frac{t_{total.light}}{t_{total.gg}} = 1 \quad \text{-->} \quad \frac{2 \cdot d_{AB} + v \cdot \left(\frac{d_{AB}}{c-v} - \frac{d_{AB}}{c+v}\right)}{2 \cdot d_{AB}} = \frac{t_{total.light}}{t_{total.gg}} = 1 \quad \text{-->} \quad \frac{2 \cdot d_{AB}}{2 \cdot d_{AB}} + \frac{v}{2 \cdot d_{AB}} \cdot \left(\frac{d_{AB}}{c-v} - \frac{d_{AB}}{c+v}\right) = \frac{t_{total.light}}{t_{total.gg}} = 1$$

$$1 + \frac{v}{2 \cdot d_{AB}} \cdot \left(\frac{d_{AB}}{c-v} - \frac{d_{AB}}{c+v}\right) = \frac{t_{total.light}}{t_{total.gg}} = 1 \quad \text{-->} \quad 1 + \frac{v}{2 \cdot d_{AB}} \cdot \frac{2 \cdot d_{AB} \cdot v}{(c+v) \cdot (c-v)} = \frac{t_{total.light}}{t_{total.gg}} = 1$$

$$1 + \frac{v^2}{(c+v) \cdot (c-v)} = \frac{t_{total.light}}{t_{total.gg}} = 1 \quad \text{-->} \quad \frac{c^2}{(c+v) \cdot (c-v)} = \frac{t_{total.light}}{t_{total.gg}} = 1 \quad \text{-->} \quad \frac{1}{1 - \frac{v^2}{c^2}} = \frac{t_{total.light}}{t_{total.gg}} = 1 \qquad \frac{1}{1 - \frac{v^2}{c^2}} = 1.563$$

leg 1 time

$$\frac{d_{AB} + v \cdot \frac{d_{AB}}{c - v}}{d_{AB}} = \frac{t_{1.light}}{t_{1.gg}} = 1 \quad \text{-->} \quad 1 + \frac{v}{c - v} = \frac{t_{1.light}}{t_{1.gg}} = 1 \qquad 1 + \frac{v}{c - v} = 2.5$$

leg 2 time

$$\frac{d_{AB} - v \cdot \frac{d_{AB}}{c + v}}{d_{AB}} = \frac{t_{2.light}}{t_{2.gg}} = 1 \quad \text{-->} \quad 1 - \frac{v}{c + v} = \frac{t_{2.light}}{t_{2.gg}} = 1 \qquad 1 - \frac{v}{c + v} = 0.625$$

average of the two legs = total dilation factor

$$\text{mean}\left(1 + \frac{v}{c - v}, 1 - \frac{v}{c + v}\right) = \frac{1}{1 - \frac{v^2}{c^2}} = 1$$

$$\text{mean}\left(1 + \frac{v}{c - v}, 1 - \frac{v}{c + v}\right) = 1.563$$

Derivation of current light path distance ratio (total shown, legs not shown as they will be the same as the time legs)

$d_{total.gg} := d_{1.gg} + d_{2.gg} = 200\ ft$ $\qquad$ $d_{light.total} := d_{1.light} + d_{2.light} = 312.5\ ft$ $\qquad$ $\frac{d_{light.total}}{d_{total.gg}} = 1.563$

$$\frac{d_{AB} + v \cdot \frac{d_{AB}}{c - v} + d_{BA} - v \cdot \frac{d_{BA}}{c + v}}{d_{AB} + d_{BA}} = \frac{d_{light.total}}{d_{total.gg}} = 1 \quad \text{-->} \quad \frac{1}{1 - \frac{v^2}{c^2}} = \frac{d_{light.total}}{d_{total.gg}} = 1 \qquad \frac{1}{1 - \frac{v^2}{c^2}} = 1.563$$

(skipped steps, you can see it is very similar to the previous page derivation and the conclusion is obvious)

Resulting transformations (putting terms in notation similar to Einstein's paper):

Total light path (leg 1 & leg 2):

$\beta := \frac{1}{1 - \frac{v^2}{c^2}} = 1.563$ $\qquad$ $\tau := t_{total.light} = 0.000000317719801\ s$ $\qquad$ $\xi := d_{light.total} = 312.5\ ft$

$t := t_{total.gg} = 0.000000203340672\ s$ $\qquad$ $x := d_{total.gg} = 200\ ft$

$\tau = \beta \cdot \frac{x}{c} = 1$ $\qquad$ $\xi = \beta \cdot x = 1$

(note: this is what the transformations would look like if the length contraction was not introduced)

Leg 1

$\beta_1 := 1 + \frac{v}{c - v} = 2.5$ $\qquad$ $\tau_1 := t_{1.light} = 0.000000254175841\ s$ $\qquad$ $\xi_1 := d_{1.light} = 250\ ft$

$t_1 := t_{1.gg} = 0.000000101670336\ s$ $\qquad$ $x_1 := d_{1.gg} = 100\ ft$

$\tau_1 = \beta_1 \cdot \frac{x_1}{c} = 1$ $\qquad$ $\xi_1 = \beta_1 \cdot x_1 = 1$

Leg 2

$\beta_2 := 1 - \frac{v}{c + v} = 0.625$ $\qquad$ $\tau_2 := t_{2.light} = 0.00000006354396\ s$ $\qquad$ $\xi_2 := d_{2.light} = 62.5\ ft$

$t_2 := t_{2.gg} = 0.000000101670336\ s$ $\qquad$ $x_2 := d_{2.gg} = 100\ ft$

$\tau_2 = \beta_2 \cdot \frac{x_2}{c} = 1$ $\qquad$ $\xi_2 = \beta_2 \cdot x_2 = 1$

Check total $\qquad$ $\tau = \tau_1 + \tau_2 = 1$ $\qquad$ $\xi = \xi_1 + \xi_2 = 1$

This calculation so far, without considering length contraction, does not solve the problem of the Michelson 1887 interferometer.

$t_{total.light} = 0.000000317719801\ s$ $\qquad$ this represents the parallel light path in the interferometer

$t_{perp} := \frac{2 \cdot d_{AB}}{\sqrt{c^2 - v^2}} = 0.000000254175841\ s$ $\qquad$ this represents the perpendicular path of the interferometer (equation for perpendicular path provided further up in document)

$\frac{t_{total.light}}{t_{perp}} = 1.25$ $\qquad$ this ratio needs to be 1, the times need to be equal for the 0 fringe shift of the Michelson interferometer to be explained

This final step is "achieved" in section 3 of Einstein's paper, which apparently can by summed up as a kinematical consequence of his rather unclear coordinate transformation. Once we introduce the magic of length contraction to the problem at hand, then we will arrive at the modern day Lorentz factor and the light path distances and times of the perpendicular and parallel light paths of the Michelson interferometer will be equal, thus producing 0 fringe shift.

$$d_{1.light.LC} := d_{AB} \cdot \sqrt{1 - \frac{v^2}{c^2}} + v \cdot \frac{d_{AB} \cdot \sqrt{1 - \frac{v^2}{c^2}}}{c - v} = 200\ ft$$

leg 1 distance (from stationary, or light's, perspective) (refer to example 2 for derivation of equations)

$$d_{2.light.LC} := d_{AB} \cdot \sqrt{1 - \frac{v^2}{c^2}} - v \cdot \frac{d_{AB} \cdot \sqrt{1 - \frac{v^2}{c^2}}}{c + v} = 50\ ft$$

leg 2 distance (from stationary, or light's, perspective)

$$d_{total.light.LC} := d_{1.light.LC} + d_{2.light.LC} = 250\ ft$$

total light path distance (parallel motion light path)

$$d_{perp} := 2 \cdot d_{AB} \cdot \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 250\ ft$$

total light path distance (perpendicular motion light path)

$$\frac{d_{total.light.LC}}{d_{perp}} = 1$$

now the light path distance in each light path of the interferometer is the same, resulting in a 0 fringe shift

$$t_{1.light.LC} := \frac{d_{1.light.LC}}{c} = 0.000000203340672\ s$$

time for light to travel leg 1

$$t_{2.light.LC} := \frac{d_{2.light.LC}}{c} = 0.000000050835168\ s$$

time for light to travel leg 2

$$t_{total.light.LC} := t_{1.light.LC} + t_{2.light.LC} = 0.000000254175841\ s$$

total light travel time (parallel motion light path)

$$t_{perp} = 0.000000254175841\ s$$

total light travel time (perpendicular motion light path)

$$\frac{t_{total.light.LC}}{t_{perp}} = 1$$

now the light travel time in each light path of the interferometer is the same, resulting in a 0 fringe shift

Derivations of the Lorentz factor and transformations are shown in example 2.

The derivation (prior to introducing length contraction), which lead to this being the dilation factor: $\frac{1}{1 - \frac{v^2}{c^2}} = 1.563$

sense if you redefine time to be measured by clocks in motion (in which their tick counting mechanism is now altered due to the motion). It is ridiculous, but it makes sense in the context of Einstein's kinematical framework. Each observer has a clock that ticks at different rates, so the stationary clock will be off from the moving clock by a factor that is equal to how much that moving clock's light path has been affected by the motion.

It is no different than a clockmaker making two different light clocks of different heights and then seeing that those clocks count seconds at different rates.

Closing remarks.

The concept of time requires a memory. Since the concept of time is defined by motions, a memory is required to perceive motion. When an object is moving, we remember it's original position and compare its original position to its current position, and thus perceive motion. If you did not have a memory, you would continuously see the world for the first time over and over and over and over with no concept of time.

So to conceptualize time, a being with memory capability is required. They say time is a dimension of the fabric of the universe. If all beings in the universe with memory capability suddenly disappeared, then the concept of time would disappear too. What would happen to everything else in the universe if time was no longer a concept for the remaining objects in it? How could gravity be described without spacetime? Nothing would happen. Because time is not a dimension of the universe, it is just a useful way for humans to organize.

Time is not something that exists and is permeating the universe and it does not run slow for a person in motion, relative to a stationary person. The clocks that everyone uses to display correct practical time, which are all models which represent the motion of the sun relative to earth, are only telling the correct time if they are in sync with this celestial motion. Once you set your clock in motion and affect its tick rate, you just have an impractical useless clock. But if you redefine time to be measured by light clocks with moving walls, then you can start saying time is dilating when clocks are moving around if you'd like to. You can also configure the mechanism of a clock to run slow in the cold and run fast in the warm and then redefine time based on your new temperature dependent clock and start saying that time is running slow in the north pole, relative to someone at the equator. It just depends on by which means you want to affect your meaningless clock.

Einstein did not provide a physical explanation for length contraction in his 1905 paper, nor did he comment about whether or not it is an actual physical contraction of an object. Apparently in 1911 he stated "the question as to whether length contraction really exists or not is misleading. It doesn't really exist for a comoving observer; though it really exists for a non-comoving observer" (could not locate exact source of this quote).

I think the answer lies in the context of the Michelson Morley experiment since that is what this theory is addressing. In the Michelson experiment, length contraction (and time dilation) need to be occurring (from somebodies reference frame) to the physical interferometer for those two light rays to meet back at the same time and produce 0 fringe shift (if they want to claim that earth is traveling at 30 km/s and light is self-propagating through nothing).

Michelson and Morley (and everyone else on earth) are co-moving with the interferometer as they are the ones performing the experiment and measuring fringe shifts, so the effects of length contraction and time dilation are apparent in the co-moving frame, otherwise Michelson could not measure 0 fringe shift. So Michelson can definitely observe the effects of time dilation and length contraction from his moving reference frame, but time dilation and length contraction can only be directly observed from a hypothetical observer floating around in stationary empty space, according to Einstein's comment? (see sidenote below).

Sidenote on reference frames and Einstein's comment:

The typical light clock experiment that we are all taught to learn about time dilation is the perpendicular motion light clock. This experiment is typically arranged as shown in example 1 in this document. The observer is stationary on earth and the person with a light clock is in a spaceship flying away. In this example, the person in the spaceship (moving observer) sees no indication of the effects of time dilation or length contraction occurring at all in their own reference frame. It is only the stationary observer from earth who would observe these strange phenomena.

But in the case of the Michelson interferometer, Michelson is in the moving reference frame with earth and his interferometer (or light clock), but he is able to measure things (0 fringe shift) which give an indication of time dilation and length contraction? Michelson should not be able to detect any indication of relativistic effects from within his own reference frame. It is only when an observer from another reference frame observes what Michelson is up to when such funny business will become apparent according to special relativity theory. That means that in Michelson's frame and from Michelson's perspective, which he can consider stationary, light is traveling at different speeds in each direction (dependent on the velocity of Michelson's reference frame, which is earth).

So since Michelson observed the effects of time dilation and length contraction and observed 0 fringe shift, what is the stationary reference frame that is required for special relativity theory to have the rights to invoke length contraction and time dilation to provide a solution to the 0 fringe shift? It would have to be the medium in which light propagates, whether you want to call it ether or nothingness (empty space).

So empty space is the stationary reference frame in the context of explaining Michelson's 0 fringe shift using special relativity theory. Is empty space moving with respect to the other objects in the solar system? If you perform this experiment on Mars, will a 0 fringe shift be observed and thus empty space be concluded to be the stationary reference frame again? Or will a fringe shift be detected on Mars and thus it is concluded that there is an empty space wind blowing across Mars? So empty space is stationary with respect to Mars and earth? I realize these are stupid questions, asking if nothing is moving or not with respect to things. It is nothing, you cannot say what it is doing because it does not exist, but if you want nothing to be used as your stationary reference frame to explain 0 fringe shift, then that nothing is the absolute stationary frame of this universe (stationary with respect to everything), because you cannot say that nothing is in motion. In Einstein's special relativity theory, there cannot be absolute reference frames as all frames are relative to each other, there is not a preferred (absolute) frame.

After looking into this, it is difficult to not walk away with the conclusion that both the theories of Lorentz and Einstein have been put forth as justifications to maintain a Copernican (heliocentric) model of our solar system. I do not know what revolves around what, what is moving with respect to what, if there is only slight relative motion between the earth and the ether, or if both the earth and the ether are absolutely stationary. I do not know if these interferometer results can be relied upon (however I went through Michelson's experiment thoroughly and did not find any alarming issues on a level equivalent to Einstein's definition of time), I do not know whether there is another ether theory out there that is harmonious with the heliocentric model, but it is odd (to put it mildly) that investigation of the 30km/s earth orbital velocity was never seriously undertaken (that I found).

To add further confusion to this ordeal, I will end this with a quote from Einstein in which he says that his own theory of general relativity is unthinkable without an ether. "Recapitulating, we may say that according to the general theory of relativity space is endowed with physical qualities; in this sense, therefore, there exists an ether. According to the general theory of relativity space without ether is unthinkable; for in such space there not only would be no propagation of light, but also no possibility of existence for standards of space and time (measuring-rods and clocks), nor therefore any spacetime intervals in the physical sense. But this ether may not be thought of as endowed with the quality characteristic of ponderable media, as consisting of parts which may be tracked through time. The idea of motion may not be applied to it"[10].

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