A Critical Review and Correction of Karim Ghariani's Karimation

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Abstract

In 2009, Tunisian media celebrated a 19-year-old student, Karim Ghariani, for proposing a method—referred to as *Karimation*—that claimed to simplify the direct computation of Bernoulli numbers. Despite local acclaim, his approach, archived on platforms such as Wikiversity but never formally peer-reviewed, contains gaps and minor errors. This paper revisits Karim's main integral formula involving Bernoulli polynomials and Stirling numbers, identifies a critical flaw in differentiating under an integral with a fixed upper bound, and provides a rigorous correction by extending the integral to a continuous upper limit. We conclude that while the original method does not present fundamentally new results, the episode highlights the importance of mathematical rigor and peer review, as well as the value of encouraging youthful mathematical curiosity.

1 Introduction

In 2009, Arabic and particularly Tunisian media hailed a 19-year-old Tunisian named Karim Ghariani for claiming a novel method, named *Karimation*, which claimed to simplify the direct computation of Bernoulli numbers (see this article). His work was described as a significant breakthrough simplifying the classical theory of Bernoulli numbers, with some sources exaggerating its novelty and impact.

Karim, at that time a promising student freshly admitted to a prestigious Institute of Applied Sciences in Tunisia, pursued mathematics as a hobby alongside his musical talents. Encouraged by his teachers, who supported and archived his work on platforms such as Wikiversity and arXiv (though no record exists on arXiv), he gained local recognition.

However, the research was never formally published in a peer-reviewed mathematical journal, and a critical review shows that the proof contains gaps and minor errors. This raises the question: Is this truly a breakthrough or a misunderstood contribution?

2 Karim's Approach and Its Shortcomings

Karim's main claim involves the integral representation:

$$\int_0^n B_p(t)\,dt = \sum_{k=0}^{p} S(p,k)\frac{(n)_{k+1}}{k+1} \tag{1}$$

where $B_p(t)$ are Bernoulli polynomials, $S(p,k)$ are Stirling numbers of the second kind, and $(n)_{k+1}$ denotes the falling factorial.

He then attempts to differentiate with respect to "x", writing:

$$B_p(x) = \sum_{k=0}^{p} S(p,k)\frac{d}{dx}\frac{(x)_{k+1}}{k+1} \tag{2}$$

However, this step is not justified as the integral's upper limit in (1) is a fixed integer n, not a variable x. Differentiation under the integral sign requires the upper limit to be variable and the integral to be well-defined as a function of that variable.

No rigorous proof was provided that (1) holds for any real $x > 0$, which is essential for (2) to be valid.

3 Correcting the Gap

To fix this gap, we show that the integral formula extends to a variable upper limit $x > 0$, namely:

$$F_p(x) := \int_0^x B_p(t)\,dt = \sum_{k=0}^{p} S(p,k)\frac{(x)_{k+1}}{k+1}$$

The proof proceeds as follows.

3.1 Bernoulli Polynomials Expansion

Recall the classical expansion of Bernoulli polynomials:

$$B_p(x) = \sum_{m=0}^{p} \binom{p}{m} B_{p-m} x^m$$

where B_j are Bernoulli numbers.

Integrating term-by-term on $[0, x]$ gives:

$$F_p(x) = \int_0^x B_p(t)\, dt = \sum_{m=0}^{p} \binom{p}{m} B_{p-m} \int_0^x t^m\, dt = \sum_{m=0}^{p} \binom{p}{m} B_{p-m} \frac{x^{m+1}}{m+1}$$

3.2 Expressing Powers via Falling Factorials

We use the identity relating powers to falling factorials with Stirling numbers of the second kind:

$$x^{m+1} = \sum_{k=0}^{m} S(m+1, k+1)(x)_{k+1}$$

Substituting this into $F_p(x)$, we have:

$$F_p(x) = \sum_{m=0}^{p} \binom{p}{m} B_{p-m} \frac{1}{m+1} \sum_{k=0}^{m} S(m+1, k+1)(x)_{k+1}$$

Rearranging the sums by inverting the order of summation yields:

$$F_p(x) = \sum_{k=0}^{p} \left(\sum_{m=k}^{p} \binom{p}{m} B_{p-m} \frac{S(m+1, k+1)}{m+1} \right) (x)_{k+1}$$

3.3 Final Completion of the Proof via Generating Functions

To complete the justification of the formula

$$\int_0^x B_p(t)\, dt = \sum_{k=0}^{p} \frac{S(p, k)}{k+1} (x)_{k+1},$$

we must verify the key identity:

$$\sum_{m=k}^{p} \binom{p}{m} B_{p-m} \frac{S(m+1,k+1)}{m+1} = \frac{S(p,k)}{k+1},$$

where $S(m,k)$ denotes the Stirling numbers of the second kind and B_n are the Bernoulli numbers.

Step 1: Generating Functions We begin with the exponential generating function for Bernoulli numbers:

$$\frac{t}{e^t - 1} = \sum_{n=0}^{\infty} B_n \frac{t^n}{n!}.$$

Now consider the generating function involving Stirling numbers:

$$f_k(t) = \sum_{j=0}^{\infty} \frac{S(j+1,k+1)}{j+1} \cdot \frac{t^j}{j!}.$$

This series is recognized as the exponential generating function of the following expression:

$$f_k(t) = \frac{1}{t} \cdot \frac{(e^t-1)^{k+1}}{(k+1)!}.$$

Step 2: Multiply Generating Functions Multiply the Bernoulli generating function with $f_k(t)$:

$$\frac{t}{e^t-1} \cdot f_k(t) = \frac{t}{e^t-1} \cdot \frac{1}{t} \cdot \frac{(e^t-1)^{k+1}}{(k+1)!} = \frac{(e^t-1)^k}{(k+1)!}.$$

This is precisely the exponential generating function:

$$\sum_{n=0}^{\infty} \frac{S(n,k)}{k+1} \cdot \frac{t^n}{n!}.$$

Step 3: Coefficient Matching Therefore, the coefficient of $\frac{t^p}{p!}$ in the product

$$\left(\sum_{n=0}^{\infty} B_n \frac{t^n}{n!}\right) \cdot \left(\sum_{j=0}^{\infty} \frac{S(j+1,k+1)}{j+1} \cdot \frac{t^j}{j!}\right)$$

equals

$$\frac{S(p,k)}{k+1}.$$

But this coefficient also equals the left-hand side:

$$\sum_{m=k}^{p} \binom{p}{m} B_{p-m} \frac{S(m+1,k+1)}{m+1}.$$

Hence, the identity is proven, and the original formula is fully justified. □

4 Conclusion

Karim Ghariani's attempt, later known as the "Karimation", drew attention in 2009 for its seemingly constructive way to compute Bernoulli numbers. While the enthusiasm and support surrounding the method highlighted the excitement of young minds engaging with deep mathematical ideas, our detailed analysis shows that the formula at the heart of Karimation is a restatement of a classical identity, albeit approached in an unconventional and incomplete manner.

The proposed derivation contained a significant logical gap involving differentiation under an integral with a fixed upper bound. In this paper, we identified that flaw and corrected it by extending the integral identity to a continuous upper limit and confirming its validity through a rigorous derivation using Bernoulli polynomial expansions and generating functions.

This correction neither diminishes the promise of the young mathematician nor the importance of rigor in mathematical exposition. Further work and peer review are encouraged to explore whether any genuinely novel insights lie beneath the surface of Karim's approach.

Ultimately, the Karimation episode serves as a reminder that mathematical creativity must be accompanied by formal verification. The mathematical community should welcome youthful innovation—while remaining committed to the standards that give mathematics its lasting precision and power.

References

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