Special Relativity as an Emergent Structure in a Timeless Euclidean Model

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Abstract

We consider a model in which the observed structure of spacetime emerges from a real scalar field satisfying the Laplace equation in four-dimensional Euclidean space without time or distinguished directions. Using an operational definition of the observer and events, we show that the structure of inertial reference frames, Lorentz transformations, and elements of dynamics can be reconstructed without postulating a Minkowski metric. Elements of general relativity are obtained, including the emergence of foliation curvature and the derivation (rather than postulation) of the weak and strong equivalence principles. The results demonstrate that models without fundamental time and metric can be consistent with observed spacetime structures and admit a rigorous operational reconstruction of dynamics and geometry.

1 Introduction

1.1 The problem of time and causality

Modern fundamental physics continues to face three interconnected open questions: (1) Is it possible to formulate a physical theory in the complete *absence of time*? (2) How can causality and measurement be defined in such a context? (3) What accounts for the emergence of the observed pseudo-Riemannian spacetime signature $(-, +, +, +)$, despite the apparent naturalness of Euclidean symmetry?

These questions become particularly relevant in the search for a unified theory combining quantum theory and gravity, as many arguments [1] indicate that time in such a theory should be emergent rather than fundamental.

Closely related is the question of the nature of the observer. In standard formulations, the observer is treated as an abstract external agent, measuring what objectively exists independently of the act of observation.

Many frameworks - such as causal set theory [2, 3], loop quantum gravity [4], and relational approaches [5] - attempt to remove time from the equations, yet retain it in hidden form, through partial orders, evolution parameters, logical structures of histories, or functional dependencies between states. Even Euclidean path-integral formalisms rely on Wick rotation to recover time as a physical coordinate.

In contrast, the present model is formulated in four-dimensional Euclidean space $\mathbb{E}^4$ and eliminates not only coordinate time, but any fundamental structure prescribing order, evolution, or direction. The field $\Phi(x)$, which is the sole constituent of the model and is detailed in subsequent sections, satisfies the Laplace equation and is given as a single static configuration. Structures typically associated with time and causality in conventional theories are not postulated, but instead emerge through the interaction of the observer with the field. The observer is modeled not as an external device, but as a physically realizable structure within the model itself (see Section 3).

In standard quantum field theory, causality is imposed axiomatically in terms of light cones and constant-time hypersurfaces. Such a structure presupposes a Minkowski metric and loses its justification if time is not fundamental, but instead operationally emergent. In problems involving gravity, spacetime reconstruction, and timeless quantum formulations, these axioms become inapplicable and lack universal validity.

Moreover, the Minkowski metric in special and general relativity is always postulated. However, the signature theorem [6, 7] prohibits globally transforming a positive-definite form into a pseudo-Riemannian one, implying that if Lorentzian structure emerges, it must be effective and local. This is particularly important in models where spacetime and dynamics are emergent.

Several approaches attempt to eliminate time from fundamental theory. For instance, in *causal set theory*, time persists as a partial ordering on events, effectively defining an oriented causal structure. In *loop quantum gravity* and spin foams, evolution is implemented via transitions between boundary states, while the temporal axis appears as a parameter in external interpretation. In the *Page–Wootters mechanism* [8] and *relational quantum mechanics* [5], time is defined through quantum correlations between subsystems, yet the existence of a postulated Hilbert space and measurement act is assumed.

In *timeless* approaches, such as Barbour's model [9], time is eliminated as a parameter, but configuration space or spatiotemporal relations remain,

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allowing the recovery of dynamics. In *QBism* [10] and *observer-centric QFT* [11], the subject is introduced as an external interpreting structure rather than as a physical body embedded in the theory. In all such cases, either time is retained in a hidden form, or the observer is treated as external - see the comparative analysis in [12].

The proposed model differs radically: it eliminates not only coordinate time, but also any internal ordering or evolution parameter, while modeling the observer as a localized configuration of the same field. All causal relations, dynamics, and events emerge *operationally* from interaction with the field on a chosen foliation and are not postulated a priori.

Operationality in this work refers to the definition of physical structures based on observable interactions between a localized observer and the field configuration, without invoking external time, coordinate dynamics, or an a priori metric. This allows for the formalization of causality, measurement, and observable transformations (see Section 5) as internal operational structures.

In this sense, the model provides a rigorous realization of a timeless formalism with an internal observer, from which both special relativity (SR) and key elements of general relativity (GR) can be derived. Modeling the observer as a physical part of the configuration enables the reproduction of phenomena inaccessible in external-observer approaches - from causality reconstruction to the emergence of SR and GR.

1.2 Euclidean models and the role of the observer

Euclidean methods have proven powerful in statistical physics and quantum field theory (e.g., in the path-integral formalism), but in all known cases they are treated as auxiliary, with an essential return to time via Wick rotation. Attempts to construct physically meaningful Euclidean models face challenges: no mechanism exists for the emergence of causality, the structure of events and its link to observation remains unclear, and Lorentz transformations and the speed limit are not derived.

Furthermore, in most such approaches the observer is either absent or introduced post hoc as an external agent. This work explores the possibility of describing the observer as a physical configuration emergent within the same model: through localized decomposition of the field (see Section 3). Such an observer interacts with $\Phi(x)$ and, via the choice of foliation, determines which quantities acquire physical meaning - including the structure of events, their ordering, dynamics, and causal connectivity.

This approach allows causality and measurement to be treated not as external postulates but as operational structures that emerge within the model

and depend on the observer. At the same time, the model remains formally Euclidean, without introducing time or predefined dynamics.

1.3   Objective and structure of the work

The main goal of this work is to introduce timeless models into the discourse of theoretical physics. We adopt a model defined by a four-dimensional Euclidean space and a real scalar field with no distinguished directions or internal symmetries. It is regarded as minimally sufficient to demonstrate key properties of timeless models and to show that such models can be consistent with observed aspects of known physics. The objective is not to construct a complete physical theory, but to analyze structural consequences of the model.

To this end, we demonstrate that in a strictly Euclidean model governed by the Laplace equation, one can:

- formalize causality as a local operational structure, independently defined in each inertial reference frame (IRF);

- derive both postulates of special relativity;

- derive Lorentz transformations;

- indicate how the model can be extended to incorporate general relativity.

The construction is based on a scalar field $\Phi(x)$ in $\mathbb{E}^4$, satisfying:

$$\Delta_{\mathbb{E}^4}\Phi(x) = 0.$$

In this work, the field $\Phi(x)$ is not considered as an object for which an explicit solution to the Laplace equation is sought, but rather as a generalized configuration satisfying the equation in the weak (distributional) sense. The focus is not on solving the equation per se, but on deriving physical consequences arising from imposing constraints on admissible solutions - in particular, the derivation of special relativity transformations and operational structures arising from interaction with a localized observer.

The field contains no fundamental dynamics, temporal parameters, or internal symmetries. Its interaction with a localized observer - defined through foliation and mode decomposition - allows for the operational construction of events, evolution, and the structure of IRFs.

The main results of the work are:

- It is shown that through the operational definition of events and transitions between IRFs, the following structures emerge:

  - causality as a local operational structure, independently defined in each IRF;
  - both postulates of special relativity;
  - observable transformations with Lorentzian structure;
  - an invariant maximal speed v_{max} as the maximal speed of causal influence propagation within any IRF.

- A strict distinction is introduced between:

  - *direct transformations*, which describe mappings of events between IRFs as if a global event space existed;
  - *observable transformations*, which define how events and relations appear to an observer situated in a given IRF.

- It is established that special relativity corresponds to the structure of observable transformations - an apparent consistency of events across IRFs in the absence of a global event space.

- The causality principle is shown to be modified: it applies independently within each IRF and does not rely on a global event space. However, differences between causal structures in different IRFs vanish as their relative velocity approaches zero.

- A correspondence between the model's operational structure and principles of general relativity is outlined, to be further developed.

The work is organized as follows. Sections 2–3 describe the fundamental formulation of the model and the definition of the observer. Section 4 constructs IRFs and introduces the concept of relative velocity. Section 5 analyzes the types of transformations arising from changing IRFs and derives the postulates of special relativity. Section 6 derives Lorentz transformations as a specific type of transformation between IRFs. Section 7 discusses observer memory. Section 8 sketches the emergent dynamics and explores the possibility of recovering elements of general relativity within the model. Section 9 is devoted to model limitations. Section 10 summarizes conclusions and outlines future directions.

2 Fundamental Setup

2.1 Euclidean Space $\mathbb{E}^4$

We consider a four-dimensional real Euclidean space $\mathbb{E}^4$, equipped with the standard metric δ_{AB} of signature $(+,+,+,+)$, where Latin indices $A, B = 1, \ldots, 4$. The space contains no distinguished directions, coordinates, temporal axes, or causal structure.

The geometry of $\mathbb{E}^4$ is invariant under the orthogonal group $O(4)$. Any hyperplane defined by the equation $n_A x^A = s$, where n_A is a unit normal vector, plays an equivalent role. No foliation of space is a priori physically privileged.

2.2 The Basic Field and the Laplace Equation

A real scalar field $\Phi(x)$ is defined on $\mathbb{E}^4$, satisfying the equation

$$\Delta_{\mathbb{E}^4} \Phi(x) = 0, \tag{1}$$

where $\Delta_{\mathbb{E}^4} = \delta^{AB} \partial_A \partial_B$ is the Laplacian in Euclidean space.

This equation is regarded as the *only equation of the model*. It contains no designated time variable, imposes no intrinsic dynamics, possesses no internal symmetries or preferred directions, and includes no interactions - neither linear nor nonlinear. The solution $\Phi(x)$ is assumed to be uniquely fixed, including boundary conditions. This reflects the fact that in a timeless model, independent initial data cannot be specified: the entire content of the model is determined by a single field configuration, without recourse to evolution.

2.3 Absence of Time and Causal Structure

No additional structures defining a direction of evolution, ordering of events, or dynamical variables are introduced. The entire construction presupposes the complete absence of time - both as a coordinate and as a functional parameter. The field $\Phi(x)$ is interpreted as a *static configuration* on $\mathbb{E}^4$, not as the outcome of any evolution.

This means that neither the field nor the space explicitly contains causal relations. The only source of observed causality arises from the observer, localized in space and interacting with the field through the operational scheme described below.

2.4 Purpose of the Construction

The goal of this section is to define the stage on which all observable physics will emerge. No physical quantities, events, symmetries, or equations of motion are built into the model a priori. Anything that may be interpreted as spacetime, matter, or dynamics must arise as the result of the operational interaction of the observer - a functionally distinguished local structure within the field $\Phi(x)$ - with its global configuration.

3 Observer and Operational Definition of Events

3.1 Localization of the Observer

In this model, the observer is not treated as an external agent but is described using the same construction as all other elements: as a localized structure within the space $\mathbb{E}^4$, identified within the field $\Phi(x)$. To this end, we fix a hyperplane

$$\Sigma_s^3 = \left\{ x \in \mathbb{E}^4 \mid n_A x^A = s \right\}, \qquad s \in \mathbb{R}, \tag{2}$$

where n_A is a fixed unit vector specifying the foliation of space. A local region $\Omega \subset \Sigma_s^3$, compact in three directions, is then chosen.

Within this region, the field $\Phi(x)$ is orthonormally decomposed with respect to a basis $\{u_\alpha(x)\}$ constructed on the hyperplane. The specific choice of basis and domain Ω determines the specific observer:

$$\Phi(x)\Big|_\Omega = \sum_\alpha a_\alpha u_\alpha(x) \tag{3}$$

The coefficients a_α are interpreted as the internal variables of the observer - their collection constitutes the *observer's body*.

3.2 Foliation and Transport Direction

The foliation of Euclidean space $\mathbb{E}^4$, defined in (2), partitions space into a family of three-dimensional hyperplanes Σ_s^3, orthogonal to the chosen vector n_A and parametrized by a real scalar s, which is interpreted as *operational time* in the observer's frame. Each hyperplane is interpreted as a moment of time in the corresponding inertial reference frame (IRF). We will later show that different orientations of the hyperplanes, with the selected time direction, correspond to different IRFs.

The fixed orientation n_A determines the "transport" direction between slices, while the choice of foliation defines the structure of local temporal

ordering. Thus, the time direction in the model is not specified a priori but is defined operationally by the observer, through the orientation of hyperplanes used in reconstruction.

Requirement of Causal Reconstruction. Each operational inertial reference frame (IRF) is associated with a specific foliation Σ^3 and time direction $\mathbf{n}$. In order for the reconstruction of events in a given IRF to be consistent with the principle of causality, the following conditions must be satisfied:

(i) the modal decomposition of the field $\Phi(x)$ with respect to the foliation must be such that the individual modes u_a are localized within the hyperplane Σ^3 and allow for local propagation along $\mathbf{n}$ with a finite effective speed $v \leq v_{\mathrm{max}}$[1], as defined in the operational reconstruction of events;

(ii) the equation satisfied by the field $\Phi(x)$ must admit such local modes and preserve their consistent evolution along any admissible direction $\mathbf{n}$ compatible with the Euclidean structure;

(iii) a transition between nearby IRFs (i.e., small rotations of the foliation) must not disrupt the consistent structure of events: the symmetric difference between the reconstructed event sets $\mathcal{C}_{\mathbf{n}} \triangle \mathcal{C}_{\mathbf{n}'}$, i.e., the set of events present in only one of the reconstructed sets, must vanish as $\theta \to 0$.

These are operational conditions: they are not postulated externally but arise from the requirement of reproducibility of the event structure and consistency across different foliations. Thus, causality in the model is not given a priori but emerges as a condition for the admissibility of reconstructions and the observer's compliance with limitations on the propagation speed of interactions.

Admissibility of Decompositions and Configurations. It is important to note that not every solution of equation (1) allows for a causal reconstruction. The model imposes an additional constraint: only those solutions are considered that admit a decomposition with respect to the foliation Σ^3 into local modes $\{u_a\}$ satisfying causal consistency. These solutions form the physically admissible subset $\mathcal{S} \subset \ker \Delta$, within which:

(a) the modes u_a are localized within a region of the hyperplane Σ^3;

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[1]Here, v_{max} is the maximum interaction speed in the reconstructed spacetime, defined in Section 6.

(b) interactions between field modes and observer modes admit an event-based interpretation;

(c) small rotations of the direction **n** preserve a consistent event reconstruction.

This means that the admissibility of a field configuration is not determined solely by the satisfaction of the equation, but also by the operational feasibility of a modal decomposition with a causal structure. In particular, if a configuration $\Phi(x)$ does not admit any foliation with a consistent modal decomposition, it is excluded from the physical description.

Invariance of the Transport Operator. For any foliation direction n_A, define the coefficients

$$a_\alpha^{(n)}(s) \;=\; \int_{\Sigma_s^{(n)}} u_\alpha^{(n)}(x)\,\Phi(x)\,d^3x, \qquad \alpha \in \Lambda_O, \tag{4}$$

where $\{u_\alpha^{(n)}\}$ is an orthonormal basis of modes on the hyperplane $\Sigma_s^{(n)}$: $n_A x^A = s$. The operational reconstruction within a given IRF postulates a linear transport

$$a_\alpha^{(n)}(s + ds) \;=\; \sum_{\beta \in \Lambda_O} A_{\alpha\beta}[\Phi]\, a_\beta^{(n)}(s), \tag{5}$$

where $A_{\alpha\beta}[\Phi]$ depends only on *local* values of the field configuration Φ.

Since the original field satisfies the linear equation $\Delta_{\mathbb{E}^4}\Phi = 0$ and is invariant under the full orthogonal group $O(4)$, equation (5) must preserve its functional form under any rotation of the hyperplane $\Sigma_s^{(n)} \mapsto \Sigma_s^{(n')}$:

$$A_{\alpha\beta}[\Phi] \;=\; A'_{\alpha\beta}[\Phi] \quad \forall\, n_A, n'_A. \tag{6}$$

That is, the transport operator $A[\Phi]$ is *universal* across all IRFs. This universality expresses the equivalence of operational "laws of physics" for all observers, regardless of foliation orientation.

Emergence of Causality. Thus, the principle of causality in each IRF is realized not as a fundamental postulate, but as a condition of operational consistency: the observer can reconstruct a causal structure only if an appropriate field decomposition exists. This is particularly significant in the context of Euclidean symmetry: despite the absence of a preferred time direction at the fundamental level, temporal orientation and causality emerge as operational structures defined by the choice of foliation and the feasibility of reconstruction.

3.3   Definition of an Event

Two key requirements must be met in defining an *event*.

(i) **Operational origin.** An event must arise as a result of the interaction between the observer and the field, not as a pre-existing ontological entity. In the present context, an event is a configuration of interaction between the modes of the basic field and those of the observer that triggers a discrete update of the latter's internal state, which is recorded by the observer and added to its "operational history".

(ii) **Coherence across descriptive scales.** Since a physical observer has finite extent and a limited spectral range, its interaction with the field is restricted to the projection onto a finite-dimensional subspace. The definition of an event must remain consistent when transitioning to coarser or finer modal decompositions - that is, it must be independent of the specific level of "resolution" used by the observer.

As will be shown in subsequent sections, the operational model admits consistent reconstruction of event structures when transitioning between IRFs defined by different foliation directions. Although events are defined only relative to a given observer in its own IRF, agreement between different IRFs is ensured through compatibility of reconstructions. Transformations between descriptions of the event structure are then formally equivalent to Lorentz transformations.

We now state a definition of an event satisfying both requirements.

Let observer O fix the orientation of hyperplanes Σ_s and a finite-dimensional subspace $\mathcal{H}_{\text{field}}^{(O)} = \text{span}\{u_\alpha\} \subset L^2(\Sigma_s)$, along with a set of intrinsic modes $\{\chi_\beta\}$ representing its "body". The interaction is described by the detector functional

$$\mathcal{E}_O(s) = \sum_{\alpha,\beta} \rho_{\alpha\beta}^{(O)} \, a_\alpha(s) \, b_\beta(s), \qquad (7)$$

where $a_\alpha(s)$ and $b_\beta(s)$ are the field and observer decomposition coefficients on the slice Σ_s, and $\rho^{(O)}$ is a fixed symmetric (in the real case) or Hermitian (in the complex extension) matrix encoding the observer's sensitivity to different combinations of interacting modes. The functions $a_\alpha(s), b_\beta(s)$ are assumed to be smooth in s.

An **event** $\mathsf{E}_O(s_0)$ is defined as a value s_0 such that

$$\mathcal{E}_O(s_0) \geq I_{\text{thr}}^{(O)}, \qquad \partial_s \mathcal{E}_O\big|_{s_0} = 0, \qquad \partial_s^2 \mathcal{E}_O\big|_{s_0} < 0, \qquad (8)$$

where $I_{\text{thr}}^{(O)} > 0$ is the sensitivity threshold. Once the conditions in (8) are met, an internal "flag" $P_O \subset \mathcal{H}_{\text{obs}}$ is discretely switched $0 \to 1$, recording the event in the observer's memory.

The decomposition in (7) defines the *primary* level of event structure, based on the local coincidence of field and observer modes. If the observer later transitions to a coarser description (e.g., by grouping nearby modes into effective combinations), the functional $\mathcal{E}_O$ can be re-expressed in terms of new coefficients without altering the criterion (8). Thus, the definition of an event is independent of the level of detail and remains operationally stable.

Each event is recorded *within* a single IRF; the model does not assume a global space of events. Causality is realized as an ordered discrete history $\{\mathsf{E}_O\}$ recorded by the observer in its own reference frame.

4 Inertial Frames of Reference and Relative Velocity

4.1 Foliations as Inertial Frames

In the absence of time and dynamics, each foliation direction in $\mathbb{E}^4$, specified by a unit vector n_A, defines a local event structure emerging from the observer's interaction with the field. This structure is entirely determined by the choice of hyperplanes $n_A x^A = s$, orthogonal to n_A, and the operational interpretation of s as emergent operational time resulting from this interaction.

In this model, an *inertial frame of reference* (IFR) is understood as a foliation direction n_A with respect to which events, causality, and observable quantities can be consistently defined through the localized decomposition of the field and its interaction with the observer's body. All IFRs in this work are interpreted in this strictly operational sense.

No IFR is physically distinguished: the model is invariant under the full orthogonal group $O(4)$, and the differences between IFRs arise solely from the choice of foliation direction for reconstruction. As will be shown below, transitions between foliation directions give rise to consistent transformations of observables that are formally equivalent to Lorentz transformations.

Operational Principle of Inertia. Each physical body in the model is represented as a localized collection of modes of the basic field, defined on the hyperplanes of a given foliation. Its *motion* in a fixed IFR is described as a sequence of events arising from the interaction of the body with the global configuration of the field $\Phi(x)$.

If the sequence of events along the parameter s changes its geometry within the hyperplanes Σ_s^3 - for instance, if a shift or curvature of the tra-

jectory is observed - such behavior is interpreted as *acceleration*. According to the operational approach, acceleration requires the presence of a *cause*, i.e., an additional interaction of the body with field modes outside its local decomposition. Such interaction is interpreted as *external* to the body and leads to a deviation of the event trajectory from inertial motion.

Thus, if a body exhibits uniform and rectilinear motion (in terms of a consistent sequence of events) in an IFR, this indicates the absence of external influence and, therefore, of any cause for change. In this sense, causality in the model is realized through deviations from inertiality: every acceleration is operationally linked to an additional interaction.

Consequently, in the absence of external influence, the event trajectory of a body remains straight and uniform in the chosen IFR. This corresponds to an operational formulation of the principle of inertia: *if no cause acts upon a body, its reconstructed behavior in a given IFR remains unchanged.* In this way, inertiality is understood as the stability of the event structure of the body under a fixed field configuration and chosen foliation direction.

4.2 Transition Between IFRs and Definition of Relative Velocity

Let two inertial frames of reference (IFRs) be given, corresponding to foliations along directions n_A and n'_A. These directions are related by an orthogonal transformation of Euclidean space:

$$n'_A = R_A{}^B n_B, \qquad R \in O(4). \tag{9}$$

Each foliation direction n_A defines a family of hyperplanes $n_A x^A = s$, interpreted as "moments of time" in the given IFR. Events are defined as local interactions within these hyperplanes. However, the projections of the same point $x \in \mathbb{E}^4$ onto the hyperplanes of two different foliations - e.g., $n_A x^A = s$ and $n'_A x^A = s'$ - generally differ. This leads to an operationally observable shift of events during sequential transport along the direction n_A, if n'_A is tilted relative to n_A.

Such discrepancy in the reconstruction of events is interpreted as the *relative velocity* between IFRs. Let θ denote the angle between the directions n_A and n'_A. Then, at each unit transport step along n_A, the observer registers a transverse shift of the hyperplane n'_A by an amount proportional to $\sin\theta$. This shift is interpreted as the observable relative velocity:

$$v = v_t \cdot \sin\theta, \tag{10}$$

where v_t is a scaling coefficient relating the transport step to the unit of observable time (to be fixed in Section 6).

Thus, the relative velocity between IFRs is determined entirely by the angle between their foliations. In the limit $\theta \to 0$, the corresponding spacetimes become locally consistent.

4.3 Consequence: A Multiplicity of Spacetimes

Since foliations n_A and n'_A lead to different field decompositions and, hence, to different sets of events, each direction n_A defines its own spacetime. There exists no global mapping of events between these spacetimes. The model contains no global event space: only local projections exist, specific to each IFR.

The observed velocity arises as the relative displacement of events between these spacetimes under perpendicular shifts of hyperplanes. In the limit $\theta \to 0$, the spacetimes become locally consistent, a fact that will be used in deriving observable transformations formally equivalent to Lorentz transformations.

5 Observable Transformations and the Postulates of Special Relativity

5.1 Two Types of Transformations

In the absence of a global event space, the transition between different inertial frames of reference (IFRs), corresponding to foliation directions n_A and n'_A, can be understood in two distinct ways:

- **Direct transformations** - mathematical mappings of field configurations and events between different foliations in $\mathbb{E}^4$, as if a global event space existed. They reflect differences in the definition of events and causal relations between IFRs, unconstrained by the information available to the observer.

- **Observable transformations** - operational reconstructions of the event structure by the observer during the transition from one IFR to another. Since the observer has no access to the field outside the current foliation, the reconstruction in the new IFR is performed solely on the basis of the localized configuration of modes retained in the observer's body. This reconstruction is carried out to ensure a consistent and continuous description of observable interactions in the new frame.

Unlike direct transformations, observable transformations do not constitute an objective mapping between two field structures. Rather, they implement an internal reconstruction of event structure consistent with the prior state of operational memory. In this process, events may disappear or emerge, without compromising the consistency of the observed history.

Since the reconstruction is based only on operationally available information, an observer transitioning to a new IFR operates with an *observationally equivalent* event structure. This equivalence does not imply physical identity of events but only their compatibility with the observer's limited accessible information. Thus, observable transformations realize a continuous transition between descriptions without presupposing the existence of a global event space.

This distinction between the two types of transformations underlies the derivation of the observed invariance of physical laws and the postulates of special relativity, as will be shown in the next section.

5.2   Operational Equivalence of Events

An observer moving from one IFR to another has no information about events that do not belong to their current foliation. All memory related to events is retained in the form of a configuration of field modes localized in the observer's body. The operational reconstruction of the event structure in the new IFR is carried out to be consistent with the accessible part of this memory. During this process, some previously recorded events may become operationally inaccessible, while new events may be added to ensure the consistency of causal relations in the new IFR. This leads to an agreement between observed events across IFRs, despite possible discrepancies under direct comparison of field configurations.

Thus, despite differences in causal relations and events between IFRs, the observer operates with *operationally equivalent* events in different IFRs. This equivalence does not reflect physical identity: when comparing field decompositions directly, the events may differ. However, since the observer has no access to events outside their foliation and operates solely on the retained operational memory, the reconstruction in the new IFR is performed as if the event structure were preserved. A global event space is not formed.

5.3   Invariance of the Operational Law of Interaction

The Laplace equation is invariant under the rotations of the group $O(4)$. This implies that the rule of evolution of the field decomposition coefficients under perpendicular transport (5) is the same for all foliation directions.

Consequently, all observers, regardless of the chosen direction n_A, use the same operational scheme for reconstructing the sequence of events.

This corresponds to the *first postulate of special relativity* in its operational formulation:

> The operational structure of physical interactions is identical in all inertial frames of reference.

5.4   Limitation on the Maximum Speed

In this model, causality is defined as the operationally consistent reconstruction of events within a single inertial frame of reference (IFR). From this operational definition of causality, a natural limitation arises on the maximum speed at which observable causal influence can propagate without violating reconstruction consistency.

If such a maximum speed $v_{\max}$ exists, then the $O(4)$ symmetry of the scalar field implies that its value is the same in all IFRs. It is important to emphasize that $v_{\max}$ limits the speed of causal connections within a single IFR; it does not restrict the relative velocities between different IFRs, as such velocities have no operational meaning for a single observer.

For instance, if the foliations of two IFRs are orthogonal, their relative speed in the reconstructed spacetime would be formally infinite. This is acceptable in the model, since causality is defined locally - within each IFR - and does not require global consistency between all foliations.

The maximum speed $v_{\max}$ is determined by the structure of the observer's modes and by the constraints on the consistent projection of the field configuration onto the chosen foliation. It is strictly operational in nature: the observer cannot interpret two events as causally related if their reconstruction would require exceeding $v_{\max}$ within their own coordinate structure.

In the next section, $v_{\max}$ will be associated with a scaling parameter of temporal normalization, which is determined by the consistency of reconstructions under small transitions between IFRs. Since consistency is possible only for a finite value of this parameter, it follows that $v_{\max}$ must also be finite.

Thus, $v_{\max}$ represents an internal bound of operational causality within a given IFR. It does not depend on the choice of coordinates, detection procedures, or other observers, and its existence corresponds to the *second postulate of special relativity* in the following form:

> There exists a finite maximum speed $v_{\max}$, identical for all observers within their respective inertial frames of reference, which

defines the highest speed at which causal interaction can propagate without violating the consistency of operational event reconstruction.

5.5 Observable Transformations with Lorentz-like Structure

The requirement of preserving event structure under transitions between IFRs implies that observable transformations must preserve the operationally reproducible structure of event causality. Geometrically, this is expressed by preserving a form analogous to the invariance of the quadratic quantity

$$s^2 - v^2 r^2 = \text{const}, \tag{11}$$

where s is the transport parameter along the foliation direction (the temporal coordinate in the reconstruction), r is the Euclidean norm of the coordinate in the hyperplane Σ^3, and v is the relative velocity between IFRs in the reconstructed description. These transformations preserve operational eventhood and realize consistent observable descriptions.

The expression (11) is used in this section as a heuristic representation of event structure preservation; its rigorous derivation from operational requirements will be provided in the next section.

Thus, the structure of special relativity emerges as a consequence of operationally admissible transformations between IFRs, ensuring consistent event reconstruction in the absence of a global event space. This derivation does not require the Minkowski metric or any fundamental time coordinate, and it is based solely on the operational consistency of observations within a local foliation.

6 Derivation of Lorentz Transformations from the Operational Structure

6.1 Constraints on the Class of Reconstructions

This section considers only those cases of operational reconstruction for which:

- the direction of operational time of the observer is defined by a unit vector $\mathbf{n} \subset \mathbb{R}^4$;

- the operational space is aligned with the hyperplane $\Sigma^3 \perp \mathbf{n}$;

- the reconstructed distance between events in Σ^3 equals the Euclidean length $\lambda = \ell$.

This is a particular but physically meaningful case, which allows for a rigorous derivation of the Lorentz transformations based on geometry and operational constraints. A more general setting, involving curvature and nonlinearity, leads to elements of general relativity and is considered separately.

Remark on the Limitation of Causal Connections. In this model, event reconstruction must preserve causal consistency. This requires that two spatially separated events at distance λ can only be interpreted as causally related if at least a time $t = \lambda/v_{\mathrm{max}}$ elapses between them, where v_{max} is a certain finite limiting speed of interaction. This constraint is not postulated but arises as a consequence of the requirement of operational consistency in the reconstruction of events by the observer.

The invariance of the quantity v_{max} across all admissible reconstructions follows from the full $O(4)$-symmetry of the Laplace equation: since all directions in $\mathbb{E}^4$ are physically equivalent, the limiting speed of interactions defined within a foliation cannot depend on the orientation of the hyperplane.

This fundamental property is complemented by the operational requirement that when transitioning between two IFRs, the difference in causal structure must vanish in the limit of vanishing relative velocity. Together, these conditions guarantee that the reconstructed event structure remains consistent under infinitesimal foliation transitions, without the appearance or disappearance of events.

6.2 Operational Derivation of Lorentz Transformations

Let the observer $\mathcal{O}$ perform an operational reconstruction of events based on the hyperplane Σ^3 and the normal vector $\mathbf{n}$. The temporal parameter is defined as:

$$t = \frac{\ell}{v_t} \tag{12}$$

where ℓ is the distance along $\mathbf{n}$, and v_t is the scaling parameter connecting distance and time in the given reconstruction.

To ensure consistency of the event structure when transitioning to a new IFR (a different foliation $\mathbf{n}'$), the observer requires that the reconstruction provides continuity and operational consistency of observable event structure. In this context, it is natural to impose linearity of transformations between coordinates: while this condition is not directly derived from the fundamental

equation of the model, it is necessary for invertibility and local consistency of reconstruction in the limit of infinitesimal foliation rotations ($v \to 0$), which is analyzed in the following subsection.

Thus, the reconstruction must satisfy the following:

- the transformations are linear, ensuring consistent event matching under small foliation transitions;

- the limiting speed $v_{\max}$ is invariant, defining the maximum speed of causal propagation between events within a single IFR.

These assumptions uniquely yield the Lorentz transformations with parameter $v_{\max}$:

$$t' = \gamma \left(t - \frac{vx}{v_{\max}^2} \right), \tag{13}$$

$$x' = \gamma(x - vt), \tag{14}$$

where $\gamma = 1/\sqrt{1 - v^2/v_{\max}^2}$.

Geometric Interpretation. Changing the IFR corresponds to a rotation of the hyperplane Σ^3 by an angle θ in $\mathbb{R}^4$, such that:

$$\tan \theta = \frac{v}{v_t} \tag{15}$$

The maximal permissible value $\theta = \pi/2$ corresponds to the situation where the normal to one foliation becomes parallel to another, and reconstruction becomes impossible. However, this limit does not determine the value of $v_{\max}$: the angle between foliations is a geometric property of direct transformations, whereas $v_{\max}$ constrains interaction speeds within a single foliation.

Thus, the limiting observable speed of interaction $v_{\max}$ is not defined by the geometry of foliation rotation, but requires a separate operational analysis presented in the next subsection.

6.3 Time Normalization and Derivation of Maximum Speed

We employ the previously introduced time normalization via the scaling parameter v_t, equation (12).

This normalization alone does not specify the value of v_t. However, the requirement of consistency between reconstructions in closely related IFRs, connected by small foliation rotations, imposes a strict constraint on admissible values of v_t.

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Limit of Small Velocities. Consider two foliations defined by directions $\mathbf{n}$ and $\mathbf{n}'$, deviating by a small angle θ. In the model, this deviation is approximated by:

$$\theta \approx \frac{v}{v_t} \tag{16}$$

where v is the observable relative velocity between IFRs, emerging from the comparison of reconstructions.

Since all observable events are projected onto the chosen foliation, operational consistency requires that as $v \to 0$, i.e., under an infinitesimal rotation, the event structures in both IFRs become indistinguishable:

$$\lim_{v \to 0} (\mathcal{C}_{\mathbf{n}} \triangle \mathcal{C}_{\mathbf{n}'}) = 0 \tag{17}$$

where $\mathcal{C}_{\mathbf{n}}$ is the causally admissible event structure, and $\triangle$ denotes the symmetric difference of event sets.

This is only possible if the time scale v_t used in the reconstruction matches the maximal admissible speed of causal interaction $v_{\max}$, which, due to the $O(4)$-symmetry of the model, is the same in all IFRs.

Physical Interpretation. If $v_t < v_{\max}$, the reconstructed time would be excessive for registering causal connections: the observer could miss a valid interaction in the event structure. If $v_t > v_{\max}$, the observer would include inadmissible connections, inconsistent with reconstructions in other IFRs. In either case, even an infinitesimal foliation rotation would lead to inconsistencies in memory and event structure, which is operationally unacceptable.

Therefore, consistency of reconstructions in the limit $v \to 0$ requires:

$$v_t = v_{\max} \tag{18}$$

6.4   Conclusion

Thus, Lorentz transformations, the two postulates of special relativity, and the existence of a limiting speed do not follow from a postulated spacetime structure, but from three operational foundations:

- the Euclidean symmetry of the fundamental field,

- the operational definition of events via the observer,

- the independent application of causality within each IFR.

The resulting transformations are formally identical to the Lorentz transformations under the substitution $c \mapsto v_{\max}$, and describe the observable event structure in each IFR as emergent, without invoking an a priori metric or time coordinate.

This demonstrates that special relativity emerges as an operationally consistent structure within a timeless model governed solely by the Laplace equation.

7 Operational Memory and Event Consistency under IFR Transitions

7.1 Modal State and Operational Recording of Events

In this model, the observer is not external to the field: their body is formed as a localized structure of modal coefficients in a chosen foliation of the field $\Phi(x)$. All events are operationally defined coincidences between the observer's modes and the modes of the field, localized on hypersurfaces orthogonal to the foliation direction, as specified by the definition of an event and equation (8).

It is essential to distinguish between:

- **Local modal state** - the set of decomposition coefficients of the field $\Phi(x)$ describing a stable configuration of the observer's body within a fixed foliation. This state represents a physically defined localized structure in $\mathbb{E}^4$, which does not possess an absolute event interpretation outside the context of the selected foliation;

- **Operational recording of events** - the set of interactions interpreted by the observer as events, arising during the reconstruction of causal-event structure based on the modal state with respect to the chosen foliation. This record depends on the direction of foliation and may change under transitions between IFRs.

Thus, the observer's memory is not defined as an external global structure but is formed as the result of interpreting their own modal state in a given operational frame. The operational event record is not preserved under IFR transitions but must remain consistent with the observer's modes as reconstructed in the new foliation. In this sense, events do not possess absolute ontological stability: they are functions of the operational context of reconstruction.

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7.2 Reconstruction of Event Structure under Foliation Transition

Let the observer transition from one inertial frame of reference (IFR), associated with a foliation direction n_A, to another IFR with direction n'_A, differing by a finite angle. While the global field $\Phi(x)$ remains unchanged, the modal structure of the observer's body - defined via localized decomposition in the new foliation - is transformed.

Since event reconstruction is based on comparing the body's modes to those of the surrounding field, the change of basis results in a shift in the set of operationally significant coincidences. This leads to the following consequences:

- *Partial disappearance of events*: some events reconstructed in the previous IFR no longer satisfy the coincidence conditions in the new basis;

- *Appearance of new events*: the modal configuration in the new IFR may yield additional operationally admissible coincidences that were not previously registered.

Hence, a transition between IFRs does not produce a transferable absolute memory but necessitates a reconstruction of event structure based on the transformation of the localized decomposition. This reconstruction does not imply any violation of consistency, as it is carried out in accordance with the internal structure of the field and the direction of operational reconstruction.

In general, particularly in the presence of nonlocal correlations or mode aggregation, the reconstruction may proceed not at the level of individual coefficients but based on effective modal generalizations. In such cases, events are defined not as isolated points but as structural elements in the space of admissible coherent field projections.

7.3 Conclusion

In this model, operational memory is neither absolute nor invariant. It arises as the local interpretation of the observer's internal modal state and is therefore restructured upon foliation change. This reflects the fundamental irreducibility of event structure to a global set and emphasizes the emergent character of causality.

8   From Euclidean to Minkowski Dynamics (Sketch)

8.1 $O(4)$-Symmetric Averaging and the Effective Action

Based on a single solution of the Laplace equation $\Phi(x)$, the observer selects a foliation by hypersurfaces $\Sigma^3(s)$, orthogonal to a chosen direction $\mathbf{n}$. On each hypersurface Σ^3, the field is decomposed into an orthonormal basis of functions:

$$\Phi(x)\big|_{\Sigma^3} = \sum_n a_n(s) u_n(x) \tag{19}$$

where $u_n(x)$ is a fixed basis on the hypersurface, and $a_n(s)$ are the corresponding coefficients.

Operational reconstruction assumes the existence of a transport operator along $\mathbf{n}$, which governs the evolution of the coefficients under a shift of the foliation:

$$\frac{\mathrm{d}a_n(s)}{\mathrm{d}s} = H_{nm} a_m(s) \tag{20}$$

where H_{nm} is a (generally non-scalar) operator whose effective form depends on the local structure of $\Phi(x)$ and the choice of basis. This expresses the local validity of the causality principle: within each IFR, the observer can consistently reconstruct an ordered sequence of events via the change in modal content under foliation shift.

Similarly, one can introduce an effective action S_{eff}, governing the dynamics of the coefficients $a_n(s)$ in the transition from Euclidean to Minkowskian structure. Such reconstruction requires a justification of the transition from a static field to parameterized dynamics, which is achieved through coarse-graining.

Assume the observer is insensitive to the full modal structure of the field, and interacts only with a restricted subset $\{a_n\}$, where $n \in \mathcal{N}_{\mathrm{eff}}$. Then the effective dynamics is given by integrating out the suppressed modes, leading to the action:

$$\mathrm{e}^{-S_{\mathrm{eff}}[a]} = \int \mathcal{D}\Phi_\perp \, \delta(\Phi - \Phi_{\mathrm{full}}[a, \Phi_\perp]) \, \mathrm{e}^{-\int (\nabla\Phi)^2 d^4 x}. \tag{21}$$

Thus, S_{eff} defines the operationally observable evolution and specifies the structure of interactions between modes within the reconstructed dynamics.

8.2   OS-Positivity and Wick Rotation

Due to the linearity of the Laplace equation and the $O(4)$ symmetry of the model (see Appendix B for details), the correlation functions of the effective

decomposition coefficients possess reflection symmetry and positive definiteness. These properties correspond to the conditions of Osterwalder–Schrader (OS) positivity [13], necessary for a valid Wick rotation and the emergence of unitary Minkowski dynamics.

Under these conditions, the transformation

$$s \mapsto \mathrm{i}t \tag{22}$$

enables the reconstruction of dynamical equations where the parameter t serves as the physical time in the reconstructed Minkowskian structure.

8.3   Emergence of the Light Cone and the Scale v_t

The constraint on the maximal operational interaction speed $v_t = v_{\max}$ sets the scale of the reconstructed causal cone, analogous to the light cone under the assumption $v_{\max} = c$. Within this limit, one can define an analog of the constant-time hypersurface and the structure of the causal cone:

$$(x - x')^2 - v_t^2(t - t')^2 = 0 \tag{23}$$

which determines the observable boundary between causally connected and disconnected events in the reconstructed dynamics.

The emergence of v_t as an operationally distinguished scale in the reconstruction allows the reconstructed equations of motion to be compatible with Lorentz invariance.

Therefore, even in the absence of fundamental time and Minkowski metric, the model permits emergent dynamics locally described by Minkowskian effective field equations.

8.4   Gravitational Reconstruction and the Equivalence Principles

In this model, spacetime arises as an emergent structure defined by operational foliation and spectral averaging of the underlying scalar field $\Phi(x)$, satisfying the Laplace equation in $\mathbb{E}^4$. Earlier sections demonstrated how an effective metric emerges, consistent with Lorentz transformations and locally described by Minkowski structure.

The model can be generalized to the case where flat hypersurfaces are replaced by curved ones. This leads to curvature in the reconstructed emergent metric $g_{\mu\nu}$, which is operationally interpreted as a gravitational field. Since all physical objects are represented as localized configurations of $\Phi(x)$, their event structure depends solely on the local geometry of the foliation.

Hence, the *weak equivalence principle* holds: the trajectories of all objects are determined exclusively by the geometry of $g_{\mu\nu}$, regardless of their internal structure.

Gravitational effects in the model arise as a consequence of operational description from an accelerated observer's perspective. A change in foliation corresponding to acceleration leads to curvature in the reconstructed metric without any change in the fundamental field $\Phi(x)$, thus ensuring the equivalence of gravity and acceleration.

The transfer operator $A_{\alpha\beta}[\Phi]$ (5), which describes the local evolution of modes in event reconstruction, retains its universality under foliation curvature, as it depends only on the configuration $\Phi(x)$. This enables, in each sufficiently small region, the selection of a foliation that renders the metric locally Minkowskian and preserves operational laws. In this way, the *strong equivalence principle* is realized: local physical laws in a freely falling frame are indistinguishable from those in an inertial frame without gravity.

Finally, in the case of nonlinear reconstruction, involving mode aggregates and foliation variations, the effective action may include terms dependent on the emergent curvature R. This opens perspectives for studying Einstein's equations within a geometric reconstruction framework, which requires further analysis.

9 Limitations and Discussion

9.1 Finite Memory and Modification of Event Structure

In the present model, the observer's memory is not represented as an absolute list of events, but rather as a spectrally localized configuration of the field modes constituting the observer's body. When transitioning between IFRs - that is, between foliations - the mode decomposition coefficients $\{a_\alpha(x)\}$ and the basis functions $\{\phi_\alpha(x)\}$ transform. Consequently, the event structure interpreted by the observer also undergoes transformation.

In particular:

- **Some events disappear** if the corresponding mode combination is no longer projected onto a structure that qualifies as an event in the new IFR;

- **New events emerge** if the new foliation gives rise to configurations interpreted as detector-like signatures, which had no counterpart in the previous structure.

This is an inevitable consequence of the finiteness of the observer's operational sensitivity: the observer can reconstruct only a limited number of modes with finite spectral support. As the foliation is rotated, the geometry of projection changes, and event reconstruction must be realigned with the new basis.

Thus:

> When the observer transitions to a new foliation (i.e., changes IFR), the list of events is modified: memory is not preserved in an absolute sense but is reconstructed in accordance with the new operational structure.

This mechanism enables operationally continuous event reconstruction despite the absence of a global event space. The model supports a strictly operational definition of causality within each IFR but excludes the existence of a universal event structure shared among all observers.

10 Conclusion and Outlook

This work considered a model in which the fundamental structure is a real scalar field $\Phi(x)$ satisfying the Laplace equation in four-dimensional Euclidean space $\mathbb{E}^4$, with no time, no distinguished directions, and no dynamics. From an analysis of the operational interaction between an observer and this field, the following main results were obtained:

- It was shown that spatial foliation by the observer and modal decomposition of the field give rise to a structure interpretable as an inertial reference frame (IRF), equipped with its own notions of events, causality, and inertia.

- Two types of transformations between IRFs were distinguished: direct transformations (mapping global field configurations) and observable transformations (performed by the observer based on their own memory).

- It was demonstrated that both postulates of special relativity can be operationally recovered from Euclidean geometry and the reconstruction of events:
  - the equivalence of all IRFs as observational foliations;
  - the existence of a maximal relative velocity $v_{\max}$, invariant for all observers.

- Observable transformations between IRFs preserving the operationally defined event structure were derived and shown to take the form of Lorentz transformations.

- It was established that observer memory in the model is not absolute: when transitioning between IRFs, events may disappear or appear, and reconstruction is performed to maintain operational consistency.

- A sketch of an emergent Minkowskian dynamics was constructed.

- Certain elements of general relativity were recovered, including a derivation - rather than a postulation - of the weak and strong equivalence principles.

Thus, starting solely from Euclidean geometry and a linear field equation, the model yields an observable spacetime structure of Minkowski type, consistent causality, the foundations of dynamics, and elements of general relativity.

The results demonstrate that models without fundamental time can be made strictly consistent with observable spacetime structures and may be incorporated into contemporary theoretical physics discourse as coherent and promising frameworks. This opens the possibility for further development of timeless models in a broader context, including the reconstruction of dynamics, metrics, and interactions solely from geometric and operational principles.

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A Absence of Bijection Between Individual Slices

Consider two foliation directions in $\mathbb{E}^4$, defined by unit vectors n^A and n'^A, which determine level hypersurfaces $n_A x^A = s$ and $n'_A x^A = s'$, respectively.

Let $\Phi(x)$ be a fixed solution of the Laplace equation:

$$\Delta_{\mathbb{E}^4}\Phi(x) = 0.$$

Decompose $\Phi(x)$ on the hypersurfaces Σ_s and $\Sigma'_{s'}$ using orthonormal bases $\{\phi_\alpha\}$ and $\{\phi'_\beta\}$:

$$\Phi(x)\big|_{\Sigma_s} = \sum_\alpha a_\alpha\,\phi_\alpha(x), \qquad \Phi(x)\big|_{\Sigma'_{s'}} = \sum_\beta a'_\beta\,\phi'_\beta(x).$$

If $n^A \neq n'^A$, the bases $\{\phi_\alpha\}$ and $\{\phi'_\beta\}$ correspond to different coordinate subspaces, and in general, there exists no bijective mapping between the coefficients a_α and a'_β.

The reasons are as follows:

- Different foliations induce different spectra of projection operators. Even with complete knowledge of $\{a_\alpha\}$, the projection onto $\{\phi'_\beta\}$ cannot be recovered without access to the global configuration $\Phi(x)$.

- An orthogonal rotation of the hypersurface corresponds to a non-local transformation between bases in $L^2(\mathbb{R}^3)$; no finite-dimensional or local operator implements such a mapping.

- For a generic field $\Phi(x)$, the values on one hypersurface do not determine the values on another unless they belong to the same foliation. This is characteristic of elliptic equations.

Consequently, there exists neither a bijection nor a surjection between modal decompositions across different foliations. This makes the transfer of information between IRFs in terms of event matching impossible and motivates the necessity of observable transformations, which operate solely on operationally accessible coefficients.

B   OS Positivity for the Linear Laplace Field

To construct a Minkowskian effective theory from the Euclidean model, it is necessary that the correlation functions obtained from the functional integral with the Euclidean action satisfy the Osterwalder–Schrader (OS) positivity conditions [13], which permit analytic continuation to causal Minkowski functions.

Consider the action

$$S[\Phi] = \frac{1}{2} \int_{\mathbb{E}^4} \left(\partial_A \Phi(x) \right)^2 d^4x,$$

which corresponds to a solution of the Laplace equation:

$$\Delta_{\mathbb{E}^4} \Phi(x) = 0.$$

The correlation functions, defined as functional integrals

$$\langle \Phi(x_1) \cdots \Phi(x_n) \rangle = \frac{1}{Z} \int \mathcal{D}\Phi \, \Phi(x_1) \cdots \Phi(x_n) \, e^{-S[\Phi]},$$

exist because the action is positive-definite and Gaussian. This ensures:

- the well-definedness of the functional measure;

- exponential decay of correlators at large distances;

- satisfaction of the OS-positivity conditions for all linear observables.

It follows that the field-theoretic object obtained as a result of coarse-graining from $\Phi(x)$ admits analytic continuation to Minkowski space via a standard Wick rotation.

Thus, the linearity of the Laplace equation and the positivity of the Euclidean action provide a fundamental justification for the transition to an effective Minkowskian dynamics.