The magnetic mass operator of electron

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Abstract

Using the knowledge of the Green function of electron in the homogenous magnetic field and the photon propagator which form together the mathematical object known as the mass operator, we derive it in the explicit form. The physical meaning of this object is the emission and absorption of a photon by the electron in magnetic field. It means that an electron is not the Hawking radiating black hole. The contact terms are also determined from the appropriate conditions. The zero magnetic limit gives the mass operator for the free electron.

1 Introduction

Quantum electrodynamics (QED) describes the interaction of matter and light. It was formulated quite early in the history of quantum mechanics, beginning by Dirac. One of the first calculations of QED effects was the effective Lagrangian by Heisenberg and Euler, and the first new effect predicted using QED was the scattering of light on light (Karplus et al., 1950; Akchiezer et al., 1965; Berestetskii et al., 1982). In 1948, Casimir predicted a vacuum interaction between neutral conducting plates caused by the quantized electromagnetic field confined between the plates. These two effects ware later interpreted as radiative correction to external influences.

There is a different interpretation for these effects saying that the vacuum of QED is filled with a fluctuating electromagnetic field, the interaction of which with the mentioned influences causes the effects (Bordag et al., 2024). However, in a more formal approach, there is no need to speak about fluctuating fields. Namely, the mentioned effects can be equivalently described as vacuum-to-vacuum transition amplitude in an external field, or, as the vacuum expectation value of the energy-momentum tensor in the presence of external influences discussed in the modern form by Schwinger (1969, 1970, 1973, 1989) and Dittrich (1978).

The integral part of this new theory is the Green function of electron in the homogenous magnetic field which forms the mathematical object known as the mass operator, which we derive in the explicit form. The contact terms are also determined from the appropriate conditions. The zero magnetic limit gives the mass operator for the free electron. We follow the text by Dittrich et al. (1985).

2 Mass operator of electron

The mass operator in the x-representation is usually denoted by the symbol $M(x', x'')$ and in the past literature as $\Sigma(x', x'')$. It is defined by the relation (Tsai, 1974):

$$M(x', x'') = ie^2 \gamma^\mu G_+(x', x'') D_+(x' - x'') \gamma_\mu + C.T., \tag{1}$$

where $D_+(x' - x'')$ is the photon propagator

$$D_+(x) = \int \frac{(dk)}{(2\pi)^4} \frac{e^{ikx}}{k^2 - i\varepsilon} \tag{2}$$

and $G_+(x', x'')$ is the electron propagator

$$G_+(x', x'') = \Phi(x', x'') \int \frac{(dp)}{(2\pi)^4} e^{ip(x'-x'')} G(p), \tag{3}$$

where $\Phi(x', x'')$ and $G(p)$ are function which bill be specified later.

The contact terms C.T. in eq. (1) will be determined later by the physical normalization condition such that for $\gamma\Pi \to -m$, both M and its first derivative with respect to $\gamma\Pi$ are zero. The motivation for such definition of the contact terms can be found in the monograph (Dittrich et al., 1985). In this chapter we derive the mas operator $M(x', x'')$ in the presence of the constant magnetic field following the treatment of Dittrich et al. (1985).

The substitution of eqs. (3) and (2) into eq. (1) gives:

$$M(x', x'') = \Phi(x', x'') \int \frac{(dp)}{(2\pi)^4} e^{ip(x'-x'')} M(p), \tag{4}$$

where

$$M(p) = ie^2 \gamma^\mu \int \frac{(dk)}{(2\pi)^4} \frac{1}{k^2 - i\varepsilon} G(p - k) \gamma_\mu + C.T.. \tag{5}$$

Using the identity

$$\frac{1}{k^2 - i\varepsilon} = i\int_0^\infty ds_2 e^{-is_2(k^2-i\varepsilon)} \tag{6}$$

and $G(k)$ in the proper-time form we get after insertion:

$$M(p) = -ie^2\int_0^\infty ds_1\int_0^\infty ds_2\int\frac{(dk)}{(2\pi)^4}e^{-is_2(k^2-i\varepsilon)} \quad \times$$

$$\exp\left\{-is_1[m^2+(p-k)^2_\parallel+\frac{\tan z}{z}(p-k)^2_\perp]\right\} \quad \times$$

$$\gamma^\mu\frac{e^{i\sigma^3 z}}{\cos z}\left[m-\gamma(p-k)_\parallel-\frac{e^{-i\sigma^3 z}}{\cos z}\gamma(p-k)_\perp]\right]\gamma_\mu + C.T.. \tag{7}$$

Introducing new variables by equations

$$s_1 \stackrel{d}{=} su, \tag{8}$$

$$s_2 \stackrel{d}{=} s(1-u), \tag{9}$$

$$z \stackrel{d}{=} eBs_1 \stackrel{d}{=} eBsu \stackrel{d}{=} y, \tag{10}$$

we have

$$\int_0^\infty ds_1 ... \int_0^\infty ds_2 ... = \int_0^\infty sds ... \int_0^1 du .... \tag{11}$$

The exponential function in eq. (7) can be suitable rearranged by the following way:

$$e^{-is_2k^2}\exp\left\{-is_1[m^2+(p-k)^2_\parallel+\frac{\tan z}{z}(p-k)^2_\perp]\right\} \quad =$$

$$\exp\left\{-is\left[(1-u)k^2+u[m^2+p^2_\parallel-2pk_\parallel+k^2_\parallel \quad +\right.\right.$$

$$\left.\left.\frac{\tan y}{y}(p^2_\perp-2pk_\perp \quad +k^2_\perp)]\right]\right\} = e^{-is\chi}, \tag{12}$$

where

$$\chi = um^2+\varphi+(k-up)^2_\parallel \quad +\left(1-u+u\frac{\tan y}{y}\right)\left(k-\frac{u\tan y/y}{1-u+u\tan y/y}p\right)^2_\perp \tag{13}$$

and

$$\varphi \stackrel{d}{=} u(1-u)p^2_\parallel+\frac{u}{y}\,\frac{(1-u)\sin y}{(1-u)\cos y+u\sin y/y}p^2_\perp. \tag{14}$$

Then, using $\exp -is\chi$ we get for $M(p)$ in eq. (7)

$$M(p) = -ie^2 \int_0^\infty sds \int_0^1 du \frac{1}{\cos y} \int \frac{(dk)}{(2\pi)^4} e^{-is\chi(u,k)} \quad \times$$

$$\gamma^\mu e^{i\sigma^3 y} \left[m - \gamma(p-k)_\parallel - \frac{e^{-i\sigma^3 y}}{\cos y} \gamma(p-k)_\perp \right] \gamma_\mu + C.T., \tag{15}$$

which gives after further simplifications

$$M(p) = -ie^2 \int_0^\infty sds \int_0^1 du \frac{1}{\cos y} \int \frac{(dk)}{(2\pi)^4} e^{-is\chi(u,k)} \quad \times$$

$$\gamma^\mu e^{i\sigma^3 y} \left[m - (1-u)\gamma(p-k)_\parallel + \frac{e^{-i\sigma^3 y}}{\cos y} \frac{1-u}{1-u+u\tan y/y} \gamma p_\perp \right] \gamma_\mu \quad + C.T.. \tag{16}$$

The next step in the evaluation of $M(p)$ is the k-integration . For this goal we exploit well-known formulas (Dittrich et al., 1985)

$$\int_{-\infty}^\infty dx \cos(ax^2) = \left(\frac{\pi}{2a} \right)^{1/2}; \quad a > 0 \tag{17}$$

$$\int_{-\infty}^\infty dx \sin(ax^2) = \left(\frac{\pi}{2a} \right)^{1/2}; \quad a > 0 \tag{18}$$

with the obvious consequences

$$\int_{-\infty}^\infty dx e^{\pm iax^2} = e^{\pm i\frac{\pi}{4}} \left(\frac{\pi}{a} \right)^{1/2}; \quad a > 0, \tag{19}$$

where in our case with

$$a \stackrel{d}{=} 1 - u - u\,\frac{\tan y}{y} \tag{20}$$

the k-integration gives:

$$\int \frac{(dk)}{(2\pi)^4} e^{-is\chi} = \frac{-i}{(4\pi)^2} \frac{1}{s^2} e^{-is(\varphi + um^2)} \frac{1}{1+u+u\tan y/y}. \tag{21}$$

So, we have obtained the mass operator in the following form:

$$M(p) = \frac{(-i)^2 e^2 m}{(4\pi)^2} \times$$

$$\int_0^\infty \frac{ds}{s} \int_0^1 du \frac{e^{-is(um^2+\varphi)}}{(1-u)\cos y + u\sin y/y} [A_1 + A_2 + A_3] + C.T., \tag{22}$$

where

$$A_1 = \gamma^\mu e^{i\sigma^3 y} \gamma_\mu \tag{23}$$

$$A_2 = \gamma^\mu e^{i\sigma^3 y}\left(-(1-u)\frac{\gamma p_\parallel}{m}\right)\gamma_\mu \tag{24}$$

$$A_3 = \frac{1-u}{(1-u)\cos y + u\sin y/y}\gamma^\mu \frac{\gamma p_\perp}{m}\gamma_\mu, \tag{25}$$

or, after some modification

$$A_1 = -2e^{i\sigma^3 y}\left(1 + e^{-2i\sigma^3 y}\right) \tag{26}$$

$$A_2 = -2e^{i\sigma^3 y}\left((1-u)\frac{\gamma p_\parallel}{m}e^{-2i\sigma^3 y}\right) \tag{27}$$

$$A_3 = -2e^{i\sigma^3 y}\left(\frac{(1-u)}{(1-u)\cos y + u\sin y/y}\frac{\gamma p_\perp}{m}e^{-2i\sigma^3 y}\right). \tag{28}$$

After insertion of A_i from eqs. (26)–(28) we get

$$M(p) = \frac{\alpha m}{2\pi}\int_0^\infty \frac{ds}{s}\int_0^1 du \frac{e^{-is(um^2+\varphi)}}{(1-u)\cos y + u\sin y/y}e^{i\sigma^3 y} \quad \times$$
$$\left(1 + e^{-2i\sigma^3 y} + (1-u)e^{-2i\sigma^3 y}\frac{\gamma p_\parallel}{m} \quad + \right.$$
$$\left.(1-u)\frac{e^{-i\sigma^3 y}}{(1-u)\cos y + u\sin y/y}\frac{\gamma p_\perp}{m}\right) \quad + \quad C.T. \tag{29}$$

with the fine structure constant $\alpha = e^2/4\pi$ in the $\hbar = c = 1$ system.

The remaining integration cannot be carried out in a close form. That is why will return to the x-representation. Using the following auxiliary equations

$$\Phi(x', x'')\int \frac{(dp)}{(2\pi)^4}e^{ip(x'-x'')}e^{-is\varphi} \equiv \Phi(x', x'')\int \frac{(dp)}{(2\pi)^4}e^{ip(x'-x'')} \quad \times$$
$$e^{-isu(1-u)p_\parallel^2}\exp\left\{-\frac{i}{eB}\,\frac{(1-u)\sin y}{(1-u)\cos y + u\sin y/y}p_\perp^2\right\} \quad =$$
$$\cos\beta\langle x'|e^{-isu(1-u)p_\parallel^2}e^{-i\frac{\beta}{eH}\Pi_\perp^2}|x''\rangle \tag{30}$$

and

$$\Phi(x', x'')\int \frac{(dp)}{(2\pi)^4}e^{ip(x'-x'')}e^{-is\varphi}(a\gamma p_\parallel + b\gamma p_\perp) \quad =$$
$$\cos\beta\langle x'|e^{-isu(1-u)p_\parallel^2}e^{-i\frac{\beta}{eB}\Pi_\perp^2}\left(a\gamma p_\parallel + b\cos\beta e^{i\sigma^3}\gamma\Pi_\perp\right)|x''\rangle, \tag{31}$$

where we have put

$$\tan\beta \stackrel{d}{=} \frac{(1-u)\sin y}{(1-u)\cos y + u\sin y/y}, \tag{32}$$

which gives

$$\cos\beta = (1+\tan^2\beta)^{-1/2} = \left[1+\frac{(1-u)^2\sin^2 y}{[(1-u)\cos y + u\sin y/y]^2}\right]^{1/2} =$$

$$[(1-u)\cos y + u\sin y/y]\Delta^{-1/2}, \tag{33}$$

where

$$\Delta \stackrel{d}{=} (1-u)^2 + 2u(1-u)\cos y\sin y/y + u^2\sin^2 y/y^2. \tag{34}$$

Using eqs. (30 and (31), the mass operator is of the form:

$$M(x',x'') = \Phi(x',x'')\int \frac{(dp)}{(2\pi)^4} e^{-ip(x'-x'')} M(p) = \langle x'|M(\Pi)|x''\rangle \tag{35}$$

with

$$M(p) = \frac{\alpha m}{2\pi}\int_0^\infty \frac{ds}{s}\int_0^1 du e^{-isum^2u}[C_1+C_2+C_3], \tag{36}$$

where C_1 denotes the contribution of the first two term in of eq. (29), or,

$$C_1 = \left\{\Phi(x',x'')\int \frac{(dp)}{(2\pi)^4} e^{-ip(x'-x'')} e^{-is\varphi}\right\} e^{i\sigma^3 y}\frac{1+e^{-2i\sigma^3 y}}{(1-u)\cos y + u\sin y/y}. \tag{37}$$

If we further use eq. (30) with eq. (33), we have instead of eq. (37):

$$C_1 = \{(1-u)\cos y + u\sin y/y\}\,\Delta^{-1/2} e^{-isu(1-u)p_\parallel^2} e^{-i\frac{\beta}{eB}\Pi_\perp^2} \quad\times$$

$$e^{i\sigma^3 y}\frac{1+e^{-2i\sigma^3 y}}{(1-u)\cos y + u\sin y/y} \quad =$$

$$\Delta^{-1/2} e^{-is\varphi} e^{isum^2 - isu^2m^2}\left(1+e^{-2i\sigma^3 y}\right) \tag{38}$$

with

$$\varphi \stackrel{d}{=} u(1-u)(m^2+\Pi^2-\frac{e}{2}\sigma_{\mu\nu}F^{\mu\nu}) + \frac{u}{y}(\beta-(1-u)y)\Pi_\perp^2 - u^2\frac{e}{2}\sigma_{\mu\nu}F^{\mu\nu}, \tag{39}$$

where can use obviously the identity

$$\frac{1}{2}\sigma_{\mu\nu}F^{\mu\nu} = B\sigma^3. \tag{40}$$

Similarly, we get for the contribution of the second term of eq. (29)

$$C_2 = \Delta^{-1/2} e^{-is\varphi} e^{isum^2 - isu^2m^2}(1-u)e^{-2i\sigma^3 y}\frac{\gamma\Pi_\parallel}{m} \tag{41}$$

and for the C_3 we have:

$$C_3 = (1-u)e^{i\sigma^3 y}\frac{e^{i\sigma^3\beta}}{(1-u)\cos y + u\sin y/y}e^{i\sigma^3} \quad \times$$
$$\left\{(1-u)\cos y + u\sin y/y\right\}\Delta^{-1/2}\frac{\gamma\Pi_\perp}{m} \quad \times$$
$$\Delta^{-1/2} e^{-is\varphi} e^{isum^2 - isu^2m^2}. \tag{42}$$

First, we evaluate $\sin\beta$ using the relation

$$\sin\beta = (1-\cos^2\beta)^{1/2} \quad =$$
$$\left[1 - \Delta^{-1}\left((1-u)^2\cos^2 y + 2u(1-u)\sin y\cos y/y + u^2\sin^2 y/y^2\right)\right]^{1/2} \quad =$$
$$\Delta^{-1/2}(1-u)\sin y, \tag{43}$$

from which we get

$$e^{i\sigma^3\beta} = \cos\beta + i\sigma^3\sin\beta \quad =$$
$$\Delta^{-1/2}\left\{(1-u)\cos y + u\sin y/y\right\} + i\sigma^3\Delta^{-1/2}(1-u)\sin y \quad =$$
$$\Delta^{-1/2}(1-u)e^{i\sigma^3 y} + \Delta^{-1/2}u\sin y/y. \tag{44}$$

After substitution of eq. (44) into C_3 we have:

$$C_3 = \Delta^{-1/2}(1-u)e^{-i\sigma^3 y}\left(\frac{1-u}{\Delta}e^{i\sigma^3 y} + \frac{u}{\Delta}\frac{\sin y}{y}\right)e^{-is\varphi}e^{isum^2 - isu^2m^2} \quad =$$
$$\Delta^{-1/2} e^{-is\varphi} e^{isum^2 - isu^2m^2}\left((1-u)\left(\frac{1-u}{\Delta} + \frac{u}{\Delta}\frac{\sin y}{y}e^{-i\sigma^3 y}\right)\right). \tag{45}$$

Using the C_i we then have:

$$M(p) = \frac{\alpha m}{2\pi}\int_0^\infty \frac{ds}{s}\int_0^1 du e^{-isu^2m^2}\left\{\Delta^{-1/2}e^{-is\varphi} \quad \times\right.$$
$$\left(1 + e^{-2i\sigma^3 y} + (1-u)e^{-2i\sigma^3 y}\frac{\gamma\Pi}{m} \quad +\right.$$
$$\left.\left.(1-u)\left(\frac{1-u}{\Delta} + \frac{u}{\Delta}\frac{\sin y}{y}e^{-i\sigma^3 y} - e^{-2i\sigma^3 y}\frac{\gamma\Pi_\perp}{m}\right) + C.T.\right\} \tag{46}$$

with

$$\varphi = u(1-u)\left(m^2 - (\gamma\Pi)^2\right) + \frac{u}{y}(\beta + (1-u)y)\Pi_\perp^2 - u^2\frac{e}{2}\sigma_{\mu\nu}F^{\mu\nu} \tag{47}$$

and

$$\Delta = (1-u)^2 + 2u(1-u)\frac{\sin y \cos y}{y} + u^2\left(\frac{\sin y}{y}\right)^2. \tag{48}$$

At this stage we are prepared to compute the contact terms. We have said they are determined by equations

$$M(\gamma\Pi = -m; B = 0) = 0 \tag{49}$$

and)

$$\frac{\partial M}{\partial(\gamma\Pi)}(\gamma\Pi = -m; B = 0) = 0 \tag{50}$$

To satisfy the first condition, we have

$$\beta = \arctan\left(\frac{(1-u)\sin y}{(1-u)\cos y + u\sin y/y}\right)_{y\to 0} =$$

$$\arctan[(1-u)y + O(y^2)] = (1-u)y + O(y^2). \tag{51}$$

On the other hand, using eq. (51) we have:

$$\varphi(B=0) = u(1-u)(m^2 - (\not{\Pi})^2) + u\left(\frac{\beta}{y}(1-u)\right)\Pi_\perp^2 \quad -$$

$$u^2\frac{e^2}{2}\sigma_{\mu\nu}F^{\mu\nu} = u(1-u)(m^2 - (\gamma\Pi)^2) \tag{52}$$

and

$$\Delta(B=0) = (1-u^2) + 2u(1-u)\frac{\sin y}{y}\cos y + u^2\left(\frac{\sin y}{y}\right)^2 = 1. \tag{53}$$

The field free case mass operator is therefore

$$M_0(p) = \frac{\alpha m}{2\pi}\int_0^\infty \frac{ds}{s}\int_0^1 du e^{-isu^2m^2} \quad \times$$

$$\left\{e^{-isu(1-u)}(m^2 - (\not{p})^2)(2 + (1-u)\frac{\not{p}}{m}) + C.T.\right\}. \tag{54}$$

This mas term on the mass shell is then

$$M_0(\gamma\Pi = -m) = \frac{\alpha m}{2\pi}\int_0^\infty \frac{ds}{s}\int_0^1 du e^{-isu^2m^2}\left(2 + (1-u)\frac{(-m)}{m}\right) \quad =$$

$$\frac{\alpha m}{2\pi}\int_0^\infty \frac{ds}{s}\int_0^1 due^{-isu^2m^2}(1+u)+C.T.. \tag{55}$$

As $M_0(\gamma\Pi=-m)=0$, we get

$$C.T.^{(1)}=-(1-u). \tag{56}$$

Now, let us determine $\partial M_0/\partial(\gamma p)$:

$$\frac{\partial M_0}{\partial(\gamma p)}=\frac{\alpha m}{2\pi}\int_0^\infty \frac{ds}{s}\int_0^1 due^{-isu^2m^2}\quad\times$$

$$\Big\{(-is)u(1-u)(-2\not p)e^{-isu(1-u)(m^2-p^2)}(2+(1-u)\frac{\not p}{m})\quad+$$

$$e^{-isu(1-u)(m^2-(\not p)^2)}\frac{1-u}{m}\Big\}. \tag{57}$$

The evaluation of the last equation on the mass shell gives

$$\frac{\partial M_0}{\partial(\not p)}(\not p=-m)\quad=$$

$$\frac{\alpha m}{2\pi}\int_0^\infty \frac{ds}{s}\int_0^1 due^{-isu^2m^2}\left\{-2ismu(1-u^2)+\frac{1-u}{m}\right\}, \tag{58}$$

which determines the contact term in the form

$$C.T.^{(2)}=-(\not\Pi+m)\left\{\frac{1-u}{m}-2ismu(1-u^2)\right\} \tag{59}$$

Together with these contact terms we have

$$M(\Pi)=\frac{\alpha m}{2\pi}\int_0^\infty \frac{ds}{s}\int_0^1 due^{-isu^2m^2}\quad\times$$

$$\Big\{\Delta^{-1/2}e^{-is\varphi}\Big(1+e^{-2i\sigma^3 y}+(1-u)e^{-2i\sigma^3 y}\frac{\gamma\Pi}{m}\quad+$$

$$(1-u)\left(\frac{1-u}{\Delta}+\frac{u}{\Delta}\frac{\sin y}{y}e^{-i\sigma^3 y}-e^{-2is\sigma^3 y}\right)\frac{\gamma\Pi_\perp}{m}-(1-u)\quad-$$

$$(m+\not\Pi)\left(\frac{1-u}{m}-2imu(1-u^2)s\right)\Big\} \tag{60}$$

The free limit of $M(\Pi)$ is as expected:

$$M_0(p)=-(\gamma p+m)^2\frac{\alpha}{4\pi}\int_m^\infty \frac{dM}{M}\left(1-\frac{m^2}{M^2}\right)\quad\times$$

$$\left(\frac{1-\frac{2mM}{(M-m)^2}}{\gamma p+M-i\varepsilon}+\frac{1+\frac{2mM}{(M+m)^2}}{\gamma p-M+i\varepsilon}\right). \tag{61}$$

For $M \to m$ the well-known infrared divergences appear because of the zero mass of the photon.

3 Discussion

We have reconsidered the calculation of the one-loop mass correction of the electron in a homogeneous magnetic field. There are two kind of representations known in the literature. One is in terms of eigenfunctions, and the other one uses the proper time representation and the operator method. We focused on the Schwinger source method which is mathematically simple and pedagogically clear.

We exhibited the space-time form of couplings that involve only the electromagnetic field, and we also used these forms directly for calculations, in the special circumstance of slowly varying fields. With more general situations, however, it is usually preferable to consider an appropriate causal arrangement and then perform the space-time extrapolation. We are recognizing now that source theory is flexible; it is not committed to any special calculation method and is free to choose the most convenient one. Indeed, it is the interplay and synthesis of various calculation devices, each adapted to specific circumstances, that constitutes the general source theory computational method.

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