On Search For The Largest Prime Island

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Created: October 4, 2024
The last update: October 26, 2025

Abstract

There are several interesting ways to depict distribution of primes like Ulam Spiral [1], Klauber Triangle [2] or the Sacks Number Spiral [3]. In all cases, Prime Number Theorem describes the asymptotic distribution of such numbers among the positive integers. This work is devoted to illustration of primes of form $p \times q \pm C$ in a way that allows to search for clusters (so called islands of primes). The direct goal of this experimental work is to locate islands with the largest surface area and potentially discover some further patterns in distribution of primes.

1 Introduction

Work presented in this paper consists of three major parts: illustration of primes with detection of interesting clusters, formulation of patterns and examination of correctness of formulated patterns in spirit of finding new primes.

Experiments are planned against various sequences of form $p \times q \pm C$, where p and q are integer variables $(p, q > 0)$ and C is constant. Primes in the sequences are depicted as a non-zero values in X-Y graph, creating a single tile of coordinates (p, q). If two tiles are adjacent to each other on both sides, they form an island of size 2. Discovery of big islands (at least of size 4) is the first goal of this work, depicting co-located primes.

For clarity, the following definitions are in charge in this work:

Definition 1. *A prime tile is a location of coordinates (x, y) on X-Y graph with prime tile value. Primality of tile is result of a function: $isprime(F(x, y, C))$ where $F()$ is a function of form $x \times y \pm C$.*

Definition 2. *A prime island is an area composed of at least two prime tiles that have a common side.*

Lemma 3. *Formula $p \times q + C$ does not produce prime islands if C is even and positive $(p, q > 1)$.*

Proof. From lemma assumption C is even and $C > 0$. If either p or q is even, then $p \times q + C$ is also even. If $p = q = 2$ (the smallest possible values), also from lemma assumption: $C \geq 2$ (the smallest $C = 2$), then $p \times q + 2 = 6 > 2$, thus cannot be prime (because 2 is the only even prime)). If neither p nor q is even, then both $p + 1$ and $q + 1$ are even, thus $(p + 1) \times (q + 1) + C$ is even (and >2 because $(p + 1) \times (q + 1) + C > p \times q + C > 2$) and cannot be prime either. As a result p and q are building lattice with empty rows and columns around remaining cases when $p \times q + C$ is odd and prime. □

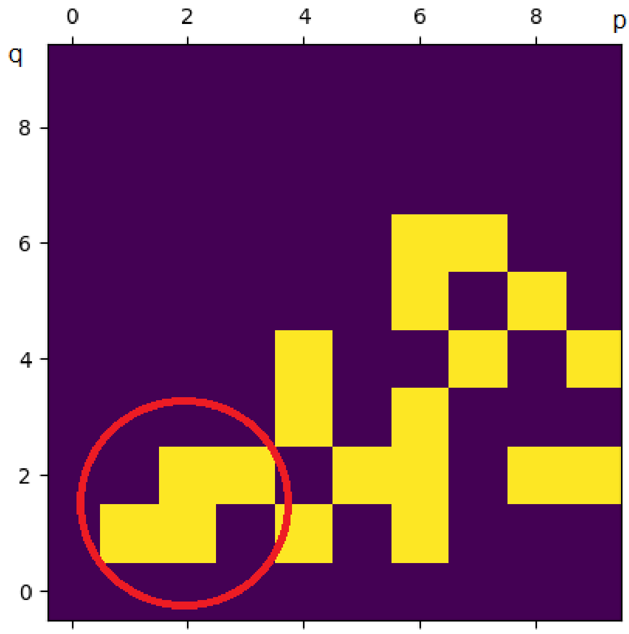

Figure 1: Illustration of prime tiles (prime islands of size 4 marked with red circle) for formula $p \times q + 1, p \geq q$.

Lemma 4. *Formula $p \times q + C$ does not produce prime islands if C is 0 $(p, q > 1)$.*

Proof. If $C = 0$ then $p \times q + 0 = p \times q$ cannot be prime by definition (because neither p nor q is 1 and $p \times q$ is always a complex number). □

In further work C is odd - this allows to focus on the most interesting findings. Framework [4] is used in all experiments. Also to simplify further calculations, notation: $p \times q \pm C$ where $p \geq q$ will be denoted as: $S_{\pm C}$.

Figure 1 depicts sample prime islands found for sequence S_{+1}, found for the first values of p and q. For the first island we have the following set of contiguous primes/tiles/formula:

- $2/(1, 1)/1 \times 1 + 1$
- $3/(1, 2)/1 \times 2 + 1$
- $5/(2, 2), 2 \times 2 + 1$
- $7/(2, 3)/2 \times 3 + 1$

The island is marked with a red circle and its size is 4.

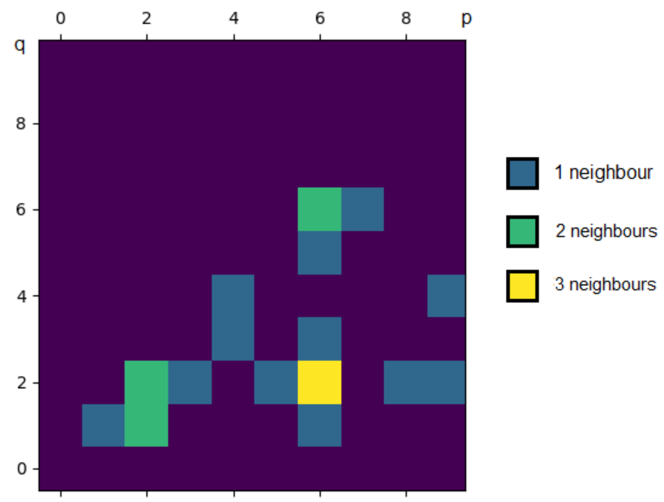

Figure 2: Illustration of prime tiles (skipping single tiles to reinforce prime islands) for S_{+1}. Additional coloring added to the picture, reflecting number of prime neighbours.

2 Search for prime islands (methodology)

Two methods to create X-Y matrices were considered and tested out.

Method A. Method faster. Each consecutive iteration equals study to one consecutive prime and its possible representation as a form of $S_{\pm C}$. For instance, if examined formula $F()$ is $p \times q + 1$ and we check 19 (the 8th prime), then 19 has two possible representations ($19 = 2 \times 9 + 1 = 3 \times 6 + 1$), meaning that 19 brings two prime tiles of coordinates $(2, 9)$ and $(3, 6)$ to the global picture.

Method B. Method slower but after acceptance tests was found as more accurate/reliable (mainly because we do not add new points to already examined area). In this approach we check every combination of p and q, where $p \geq q$, one-by-one.

3 Results of experiments and conclusions

Table 1 presents sample results for few formulas - a list of prime islands found with Method A. Generally, there are cases where there is just one example of largest island (ex. check of first $10k$ iterations for S_{-983} results in 1 islands of size 11) in the given set but there are also cases that largest prime island's surface is low and such list is long (ex. check of first $10k$ iterations for S_{-999} results in 13456 islands of size 3). Table 2 depicts how many islands of the given size were found for few selected values of C.

Executed experiments allowed to gather interesting observations. The largest prime island found was of size 15 and was located with Method B for three formulas: the first island is for: S_{-293} $[(2,148), (3,148), (2,149), (2,150), (3,150), (4,150), (5,150), (6,150), (7,150), (8,150), (6,149), (6,151), (6,152), (5,152), (4,151)]$ (Figure 7), the second one: S_{-641} - $[(4,210), (5,210), (6,210), (7,210), (8,210), (9,210), (10,210), (11,210), (12,210), (13,210), (12,209), (8,209), (6,209), (6,208), (6,207)]$ (Figure 8) and the third one: S_{+851} - $[(838,1092), (839,1092), (840,1092), (841,1092), (840,1091), (840,1090), (840,1089), (840,1093), (840,1094), (840,1095),$

Table 1: Number of prime islands of given size for each formula $p \times q + C$, results after $100k$ iterations, method A.

Size of island	-1	$+1$	-7	-9
3	1987	1920	1693	1541
4	207	179	212	1
5	42	29	43	0
6	9	4	10	0
7	7	0	3	0
8	0	1	2	0
9	0	0	1	0

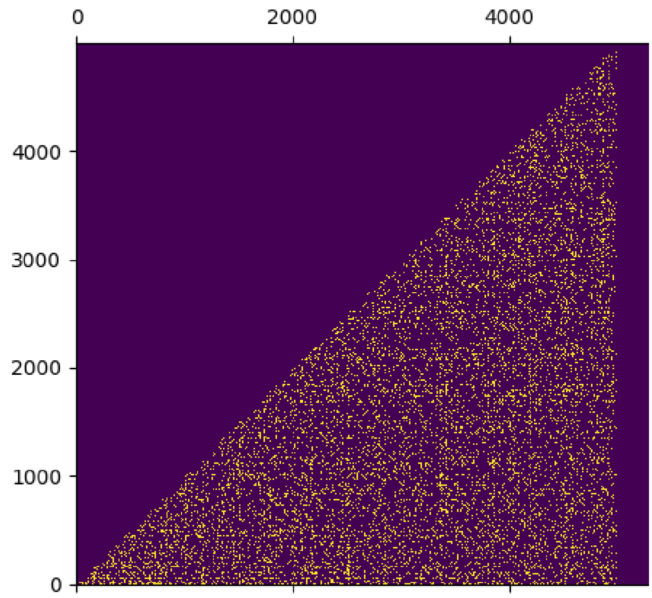

Figure 3: Prime tiles for S_{-293}, $10k$ iterations, method B in use.

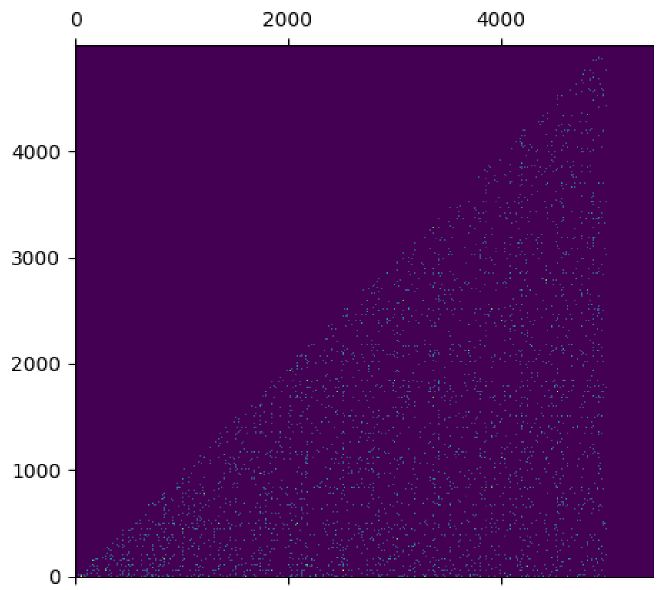

Figure 4: Prime islands for S_{-293}, $10k$ iterations, method B in use; this figure is result of post-processing of Figure 3 (single prime tiles were removed, all other prime tiles were colored, depending on number of prime neighbours).

(840, 1096), (840, 1097), (840, 1098), (839, 1098), (841, 1098)] (Figure 9).

Appendix A contains results for $-1001 \leq C \leq 1001$ ($10k$ iterations, C is either negative or positive odd number), all gathered with the use of Method B. Figure 5 + Figure 6 visualize these results and provide foundation to the following hypothesis - there is periodic relationship (period = 6) between odd positive parameter C and the size and number of the largest first islands found - the colors in the figures are arranged in clear, regular stripes. This observation led to formulation of Lemma 5.

Lemma 5. *If $C = 3+6a$ (a is integer ≥ 0) then the largest size of prime island in S_{+C} is 3.*

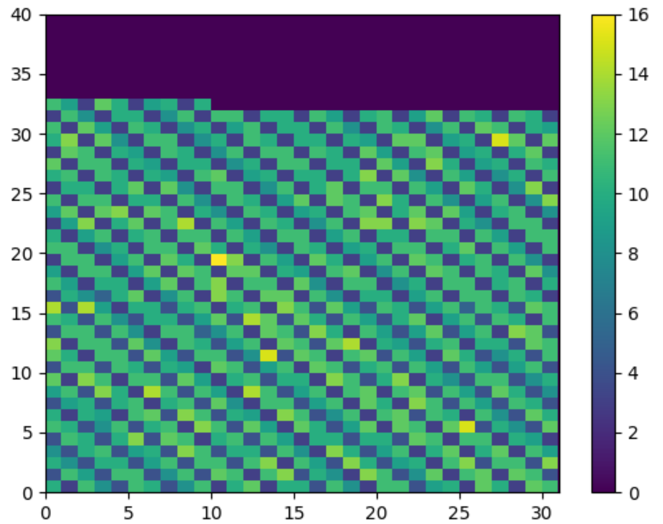

Figure 5: Size of largest island found for experiments depicted in appendix A: a set of results from S_{-1001} to S_{+1001} (step +2), method B, the first $10k$ iterations.

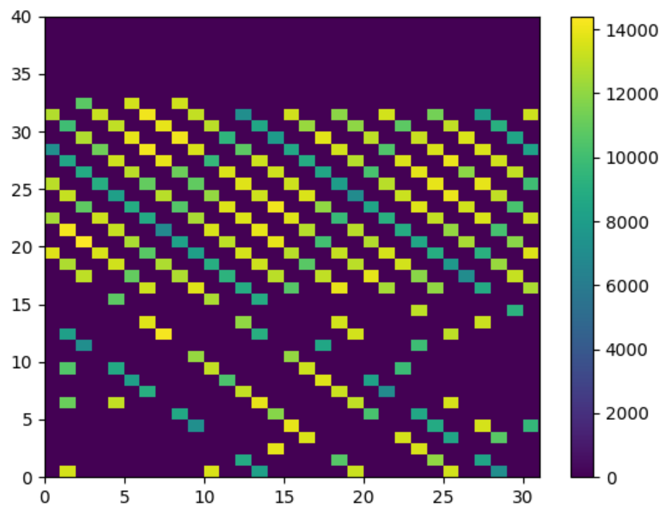

Figure 6: Number of islands of largest size found for experiments depicted in appendix A: a set from S_{-1001} to S_{+1001} (step +2), method B, the first $10k$ iterations.

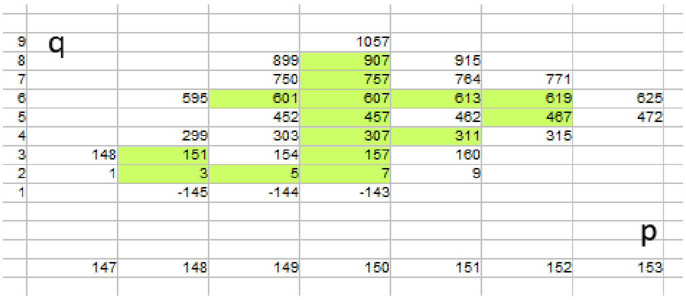

Figure 7: The largest prime island found (of size 15, S_{-641}).

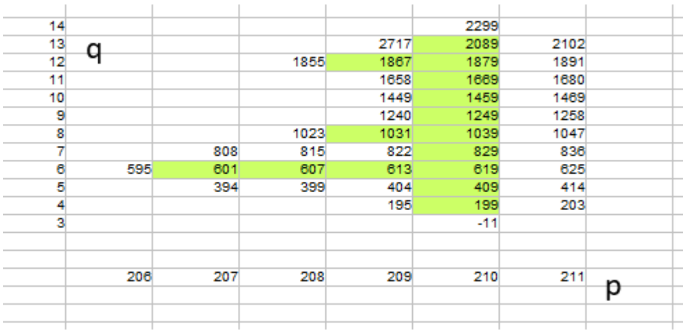

Figure 8: The largest prime island found (of size 15, S_{-293}).

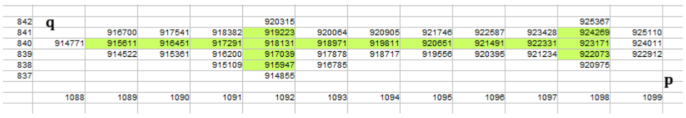

Figure 9: The largest prime island found (of size 15, S_{+851}).

4 Further experiments for Lemma 5

Experiments to verify the Lemma 5 are planned as follows:

1. Let C be a very big positive odd number, greater than these used in appendix A.
2. Verify if periodic observation related to largest prime islands (its size and count) is still present for: C, $C+2$, $C+4$, and $C+6$.

Results of these experiments are presented in Table 3. They all confirm that if C is of form $3+6a$, the size of largest prime island found is 3 (and the number of such islands is big).

References

[1] Weisstein, Eric W., *Prime Spiral.* From MathWorld – A Wolfram Resource. https://mathworld.wolfram.com/PrimeSpiral.html

[2] *The Klauber Triangle, accessed at 07/19/2025.* https://scipython.com/blog/the-klauber-triangle/

[3] *The Sacks Number Spiral, accessed at 07/19/2025.* https://www.naturalnumbers.org/sparticle.html

[4] *Library for various operations on primes.* https://github.com/mbarylsk/primes

Table 2: Formulas S subjected to experiments, results after $100k$ iterations - example results, method A.

i	S	List of largest prime islands found.
1	S_{+1}	[(5, 6872), (6, 6872), (6, 6871), (6, 6870), (5, 6870), (4, 6870), (3, 6870), (7, 6870)]
2	S_{-1}	[(1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (4, 5), (2, 7)], [(2, 120), (3, 120), (4, 120), (5, 120), (6, 120), (7, 120), (2, 121)], [(3, 18), (4, 18), (5, 18), (6, 18), (6, 17), (6, 19), (4, 17)], [(3, 798), (4, 798), (5, 798), (6, 798), (6, 799), (6, 800), (4, 797)], [(4, 587), (4, 588), (5, 588), (6, 588), (6, 589), (6, 590), (7, 590)], [(4, 4452), (5, 4452), (6, 4452), (6, 4453), (6, 4454), (7, 4454), (6, 4455)], [(4, 9300), (5, 9300), (6, 9300), (7, 9300), (6, 9299), (6, 9298), (5, 9298)]
3	S_{-3}	[(1, 5), (2, 5), (2, 4), (2, 3), (1, 6)]
4	S_{+3}	A lot (one of examples: [(1093, 1120), (1094, 1120), (1094, 1121)])
5	S_{-5}	[(2, 4), (3, 4), (4, 4), (2, 5), (2, 6), (3, 6), (4, 6), (4, 7)]
6	S_{-7}	[(1, 60), (2, 60), (3, 60), (4, 60), (5, 60), (6, 60), (6, 59), (6, 61), (4, 59)]
7	S_{-9}	[(1, 11), (2, 11), (2, 10), (1, 12)]

Table 3: Experiments with various C to check presence of periodic prime islands of size 3, method B, $10k$ iterations

Experiments	List of largest prime islands found.
$C = +2001 \cdots + 2009$	**2001: (3, 12178)**, 2003: (11, 3), 2005: (9, 1), **2007: (3, 14027)**, 2009: (11, 1)
$C = +9999 \cdots + 10011$	**9999: (3, 11276)**, 10001: (11, 1), 10003: (10, 1), **10005: (3, 8158)**, 10007: (11, 1), 10009: (10, 3), **10011: (3, 12781)**
$C = +49999 \cdots + 50007$	49999: (11, 1), **50001: (3, 10232)**, 50003: (10, 2), 50005: (8, 7), **50007: (3, 13242)**
$C = +99999 \cdots + 100011$	**99999: (3, 12272)**, 100001: (12, 1), 100003: (12, 1), **100005: (3, 7988)**, 100007: (11, 1), 100009: (9, 3), **100011: (3, 11369)**

A

Set of results from $C = -1001$ to $C = 1001$ (step 2), method B

Set is of form of: { C : (greatest size of prime island found, number of such islands found), ...}. Calculations performed for the first $10k$ iterations of $p \times q + C$ ($p \geq q$) with method B. Bolded the greatest ever islands found.

{−1001 : (10, 3), −999 : (3, 13456), −997 : (11, 2), −995 : (8, 6), −993 : (4, 1), −991 : (11, 3), −989 : (10, 2), −987 : (4, 2), −985 : (8, 11), −983 : (11, 1), −981 : (3, 13586), −979 : (10, 4), −977 : (12, 1), −975 : (3, 8124), −973 : (10, 2), −971 : (10, 3), −969 : (4, 1), −967 : (11, 1), −965 : (8, 9), −963 : (3, 13271), −961 : (11, 1), −959 : (11, 1), −957 : (4, 3), −955 : (9, 1), −953 : (10, 3), −951 : (3, 13430), −949 : (12, 1), −947 : (10, 2), −945 : (3, 7073), −943 : (12, 1), −941 : (11, 1), −939 : (4, 1), −937 : (11, 2), −935 : (9, 3), −933 : (4, 1), −931 : (11, 1), −929 : (11, 2), −927 : (4, 1), −925 : (8, 8), −923 : (11, 2), −921 : (4, 2), −919 : (12, 1), −917 : (11, 1), −915 : (3, 8776), −913 : (11, 1), −911 : (13, 1), −909 : (4, 2), −907 : (10, 2), −905 : (8, 11), −903 : (3, 10613), −901 : (13, 1), −899 : (11, 2), −897 : (4, 2), −895 : (8, 7), −893 : (12, 1), −891 : (3, 12019), −889 : (11, 1), −887 : (12, 2), −885 : (3, 8439), −883 : (12, 1), −881 : (11, 1), −879 : (4, 1), −877 : (10, 5), −875 : (8, 3), −873 : (4, 1), −871 : (10, 4), −869 : (10, 5), −867 : (4, 1), −865 : (9, 1), −863 : (11, 1), −861 : (4, 1), −859 : (11, 1), −857 : (11, 1), −855 : (4, 1), −853 : (11, 1), −851 : (13, 1), −849 : (3, 13824), −847 : (11, 1), −845 : (9, 1), −843 : (4, 1), −841 : (13, 1), −839 : (10, 9), −837 : (4, 2), −835 : (9, 2), −833 : (10, 1), −831 : (3, 13527), −829 : (11, 3), −827 : (13, 1), −825 : (4, 1), −823 : (11, 1), −821 : (11, 2), −819 : (4, 1), −817 : (13, 1), −815 : (8, 14), −813 : (4, 1), −811 : (10, 4), −809 : (11, 2), −807 : (4, 1), −805 : (9, 2), −803 : (10, 1), −801 : (4, 1), −799 : (13, 1), −797 : (11, 1), −795 : (4, 1), −793 : (10, 6), −791 : (10, 1), −789 : (4, 1), −787 : (11, 1), −785 : (8, 15), −783 : (3, 13581), −781 : (13, 1), −779 : (10, 4), −777 : (4, 2), −775 : (9, 2), −773 : (11, 1), −771 : (3, 13511), −769 : (10, 6), −767 : (12, 1), −765 : (3, 8460), −763 : (10, 1), −761 : (12, 1), −759 : (3, 10696), −757 : (11, 1), −755 : (9, 1), −753 : (4, 1), −751 : (11, 1), −749 : (10, 3), −747 : (4, 2), −745 : (10, 1), −743 : (13, 1), −741 : (4, 1), −739 : (12, 1), −737 : (11, 2), −735 : (3, 7130), −733 : (10, 1), −731 : (11, 2), −729 : (4, 1), −727 : (11, 1), −725 : (9, 2), −723 : (3, 13877), −721 : (10, 3), −719 : (10, 3), −717 : (4, 5), −715 : (10, 1), −713 : (10, 4), −711 : (4, 1), −709 : (12, 1), −707 : (11, 1), −705 : (3, 8764), −703 : (11, 3), −701 : (11, 2), −699 : (3, 13488), −697 : (10, 2), −695 : (9, 1), −693 : (3, 9562), −691 : (11, 4), −689 : (10, 3), −687 : (4, 2), −685 : (8, 10), −683 : (11, 2), −681 : (4, 2), −679 : (10, 5), −677 : (11, 4), −675 : (3, 8651), −673 : (13, 1), −671 : (11, 1), −669 : (4, 1), −667 : (12, 1), −665 : (9, 1), −663 : (3, 11769), −661 : (12, 1), −659 : (10, 3), −657 : (4, 3), −655 : (9, 2), −653 : (12, 1), −651 : (3, 10210), −649 : (12, 1), −647 : (11, 4), −645 : (3, 8603), −643 : (12, 1), **-641: (15, 1)**, −639 : (4, 1), −637 : (10, 2), −635 : (8, 12), −633 : (4, 1), −631 : (12, 1), −629 : (10, 2), −627 : (3, 11096), −625 : (10, 1), −623 : (9, 8), −621 : (3, 13070), −619 : (11, 1), −617 : (12, 1), −615 : (4, 1), −613 : (13, 1), −611 : (11, 1), −609 : (4, 1), −607 : (10, 9), −605 : (9, 1), −603 : (3, 13893), −601 : (13, 1), −599 : (11, 2), −597 : (4, 5), −595 : (8, 2), −593 : (11, 2), −591 : (3, 13023), −589 : (11, 2), −587 : (12, 1), −585 : (4, 2), −583 :

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