

27 Equation (1) is, considering units, a valid physics equation. It is the consequence
 28 of the recoil kinetic energy, $E_R = h^2/(2m\lambda^2)$, which is equal to mgx_0 . Nev-
 29 ertheless, let's assume that the function representing height $x(t)$ varies within
 30 a small interval in x_0 . The interval is $\mathcal{X} = \{x \in \mathbb{R} \mid x_0 - \epsilon \leq x \leq x_0\}$. Here,
 31 the ϵ is a relatively small positive number. Equivalently, the time t is such that
 32 $x(t) \in \mathcal{X}$. As required, a low density of atoms is assumed.

33
 34 Then we may have the following expression for a t with $x(t) \in \mathcal{X}$ and $v_0 > 0$
 35 the initial velocity in this time frame, when $t = t_{max}$

$$36 \quad x(t) = v_0 t + \frac{g}{2} t^2 = \frac{h^2}{2m^2 \lambda^2 g} \quad (2)$$

37 The time set we consider for $x(t) \in \mathcal{X}$, is $\mathcal{T} = \{t \in \mathbb{R}^+ \mid 0 < t \leq t_{max}\}$. This
 38 implies $x(t) - x_0 < 0$. The t_{max} is defined in (7) below.

39
 40 The equation (2) can obviously also give,

$$41 \quad t^2 + \frac{2v_0}{g} t - \frac{h^2}{m^2 \lambda^2 g^2} = 0 \quad (3)$$

42 Which, using the abc formula, leads us to

$$43 \quad t = \frac{v_0}{g} \left(-1 \pm \sqrt{1 + \frac{h^2}{m^2 \lambda^2 v_0^2}} \right) \quad (4)$$

44 Now, we may have from the left-hand side of equation (2)

$$45 \quad \frac{dx}{dt} = \dot{x} = v_0 + gt \quad (5)$$

46 Let's assume that the

$$47 \quad \xi = \frac{h^2}{4m^2 \lambda^2 g} \in \mathcal{X} \quad (6)$$

48 is an element of \mathcal{X} and $\xi < x_0$. In our computations we have approximately
 49 $\mathcal{X} = (33, 147)$ in units of μm . The interval of fluctuations is such that $\epsilon \geq \frac{x_0}{2}$.
 50 This is an example of attaching numbers to a computation.

51
 52 To continue, $x(t)$, for t in the appropriate sub-interval of \mathcal{T} , can be equal to ξ .
 53 In our computations with ^{10}B and ^{11}B we have $\xi \approx 123\mu\text{m}$.

54 **2.1 The time interval**

55 When $x(t) \in \mathcal{X}$, we equivalently have $t \in \mathcal{T}$, and, $\mathcal{T} = \{t \in \mathbb{R}^+ \mid 0 < t \leq t_{max}\}$
 56 and for completeness from (4)

$$57 \quad t_{max} = \frac{v_0}{g} \left(-1 + \sqrt{1 + \frac{h^2}{m^2 \lambda^2 v_0^2}} \right) \quad (7)$$

58 And so, from (5) and (7) together with $x(t) \in (\xi - \epsilon', \xi + \epsilon') \subset \mathcal{X}$. It follows,
 59 via $\dot{x}(t) = v_0 + gt \leq v_0 + gt_{max}$, that

$$60 \quad \dot{x}(t) \leq v_0 \sqrt{1 + \frac{4g}{v_0^2} x(t)} \quad (8)$$

61 There is a $\alpha \geq 1$, constant in time, such that, $t \in \mathcal{T}$ gives

$$62 \quad \alpha \dot{x}(t) \approx v_0 \sqrt{1 + \frac{4g}{v_0^2} x(t)} \quad (9)$$

63 for $t \in \mathcal{T}' \subset \mathcal{T}$.

64 2.2 Differential equation

65 Let us approximate the square root in the first-order term; $\sqrt{1+y} \approx 1 + (y/2)$.
 66 Equation (9), is then approximately equal to

$$67 \quad \alpha \dot{x}(t) \approx v_0 + \frac{2g}{v_0} x(t) \quad (10)$$

68 This can be justified by $x(t)$ around x_0 in (5). Therefore, the singular solution
 69 is

$$70 \quad x_S(t) \approx f \exp\left(\frac{2gt}{\alpha v_0}\right) \quad (11)$$

71 Variation in f leads in interval $(0, t)$ to $\dot{f} \exp\left(\frac{2gt}{\alpha v_0}\right) = \frac{v_0}{\alpha}$. Then,

$$72 \quad x(t) \approx \frac{v_0^2}{2g} \left(\exp\left(\frac{2gt}{\alpha v_0}\right) - 1 \right) \approx \frac{v_0^2}{2g} \left(\frac{2gt}{\alpha v_0} \right) \quad (12)$$

73 And so, $x(t) \approx \frac{v_0 t}{\alpha}$. If we then look at $x(t)$ closely around ξ of equation (6), and
 74 τ is a time parameter in $\mathcal{T}' \subset \mathcal{T}$, then

$$75 \quad \alpha \approx \frac{4v_0 \tau m^2 \lambda^2 g}{h^2} \quad (13)$$

76 It must be noted that α in (8) and (9), is independent of time. Therefore, in an
 77 experiment the $v_0 \tau$ is selected so that $x_0(\tau) = v_0 \tau$ is in \mathcal{X} .

78 2.3 Acceleration $\ddot{x}(t)$

79 The previous subsections imply, via (9),

$$80 \quad \alpha \ddot{x}(t) \approx \frac{v_0}{2\sqrt{1 + \frac{4g}{v_0^2} x(t)}} \frac{4g}{v_0^2} \dot{x}(t) = \frac{1}{\alpha} 2g \quad (14)$$

81 Hence, at a certain point in time where $x(t)$ closely approximates ξ as given by
82 (6), the g acceleration is

$$83 \quad \ddot{x}(t) \approx \frac{2g}{\alpha^2} \quad (15)$$

84 The $x(t)$ is close to ξ

85 2.3.1 What is the physics

86 The question we raise here is; "Is it possible to have $\sqrt{2} > \alpha > 1$?" Because, if
87 so, $\frac{\sqrt{2}}{\alpha} > 1$, and so, $\ddot{x} > g$. Therefore, a question can be raised about where the
88 increase in acceleration of certain atoms in the gas came from.

89
90 The criterium is to compute $x_0(\tau) = v_0\tau$ and see if $x_0(\tau) \in \mathcal{X}$. The inequality
91 we inspect is

$$92 \quad \frac{h}{\sqrt{g}} \sqrt{(2^{1/2})/4} > \sqrt{v_0\tau} m\lambda > 2^{-1} \frac{h}{\sqrt{g}} \quad (16)$$

93
94
95 The estimate is $\sqrt{v_0\tau} m\lambda \approx (h/\sqrt{g})(2^{-1} + \sqrt{(2^{1/2})/4})/2$. We take Boron atoms
96 $m \approx 1.7934 \times 10^{-26}$ (kg) and $\lambda = 5.32 \times 10^{-7}$ (m). The evaporation energy
97 for 1 mole of Boron is 508 kJ. The constants we work with are $g = 9.8$ (ms^{-2})
98 together with Planck's constant $h \approx 6.6261 \times 10^{-34}$ (kgm^2s^{-1}). According to
99 the computations in the section below, from (1), is $x_0(0) \approx 2.461 \times 10^{-4}$. The
100 interval \mathcal{X} is more exactly defined by $(2.461 \times 10^{-4} - \epsilon, 2.461 \times 10^{-4})$. The
101 subset $\mathcal{X}' \subset \mathcal{X}$ for $\tau \in \mathcal{T}'$ contains $x_0(\tau)$. The code, section-2.3.2, below gives
102 $x_0(\tau) \approx 1.474 \times 10^{-4}$. The minimal eps defining $x_0(\tau) \in \mathcal{X}$ is $\epsilon_{mi} \approx 9.866 \times 10^{-5}$.
103 Hence. an epsilon such as e.g. $\epsilon \approx 9.766 \times 10^{-5}$, gives indeed, $x_0(\tau) \in \mathcal{X}$.

104 2.3.2 Code

105 In order to have easy computations, the following R code describes the equation
106 numerically.

```
107 h<-6.6261e-34
108 m<-1.7934e-26
109 lam<-5.32e-7
110 g<-9.8
111 fc<-(sqrt((2^(0.5))/4)+(1/2))/2
112 x0<-h^2/(2*g*m^2*lam^2)
113 fchg<-fc*h/(sqrt(g))
114 v0t<-(fchg/(m*lam))^2
115 epsmi<-x0-v0t
116 alf<-(4*v0t*m^2*lam^2*g)/(h^2)
117 print(x0)
```

```

118 print(v0t)
119 print(epsmi)
120 print(alf)
121 dg<-1.393
122 declpr<-dg*g/2
123 declpr<-declpr*((3.317e-3)^2)
124 x10b<-v0t-declpr
125 decl<-declpr/dg
126 x11b<-1.40e-4
127 x11b<-x11b-decl
128 print(x10b)
129 print(x11b)
130 stop("x")

```

131 3 Conclusion and discussion

132 Because $x_0(\tau) \in \mathcal{X}$ we found that in $x(t) \approx \xi$,

$$133 \quad \ddot{x}(t) \approx \left(\frac{\sqrt{2}}{\alpha} \right)^2 g > g \quad (17)$$

134 This is so because it turned out that, $\sqrt{2} > \alpha > 1$ is possible with $x_0(\tau) \approx$
135 $1.47 \times 10^{-4} \in \mathcal{X}$. In fact $\alpha \approx 1.1982$. Hence, $\frac{\sqrt{2}}{\alpha} \approx 1.1803$ which gives
136 $\ddot{x} \approx 1.3931g$, and so an increase of $\Delta g \approx 0.3931g$ is found. Let's take e.g.
137 $\tau \approx 33.17\mu\text{s}$. And so $v_0 \approx 4.432$ meters per second. This is a valid velocity
138 in a classical treatment. The initial velocity $v_0 > 0$ is possible. Therefore, an
139 approach with $x(t) = v_0t + (g/2)t^2$ is justified. Furthermore, the attention is
140 directed to the part of the interval \mathcal{X} , where $\xi \approx 123\mu\text{m}$ can be found. The size
141 of \mathcal{X} is perhaps, however, not physical. That's nevertheless unimportant for the
142 principle of a $\Delta g > 0$. Of course, to increase the approximation the ξ can be
143 chosen closer to x_0 . A $\Delta g > 0$ can be derived. The example of $\xi = \frac{x_0}{2} \approx 123\mu\text{m}$
144 serves the purpose of setting up a calculation in a realistic empirical setting.

145
146 Despite considerations of the degree of approximation, the following question
147 can be raised. Where does such an increase Δg of \ddot{x} come from? It cannot be
148 something in the surroundings of the Boron Bloch interferometer. In addition
149 and quite importantly, is an experiment possible?

150 3.1 Experiment setup

151 Suppose, we take a 1:1 mixture of ^{10}B and ^{11}B . Starting from the position below
152 ξ , e.g. at $33\mu\text{m}$. Then for ^{11}B it will be a little bit more difficult to reach ξ
153 than for ^{10}B . This means that a setup can be created where ^{10}B can obtain the
154 alleged increased acceleration $g + \Delta g$, while the ^{11}B can not.

155

156 First, we turn on the laser lattice during approximately $4.1\mu\text{s}$ and let the mix-
157 ture of ^{10}B and ^{11}B have a $v_0 \approx 4.432\text{m/s}$. Then, secondly, turn off the lattice
158 during $\delta t = 3.317\text{ms}$. When the lattice is turned off, the initial velocity is zero
159 for both ^{10}B and ^{11}B in free fall.

160
161 The computations, see the section-2.3.2, labeled Code, show that the ^{10}B , start-
162 ing from $123\mu\text{m}$ will have arrived at $x(^{10}\text{B}) \approx 4.793 \times 10^{-5}\text{m}$, i.e. approx $48\mu\text{m}$.
163 The ^{11}B , starting from $120\mu\text{m}$, will have arrived at $x(^{11}\text{B}) \approx 6.609 \times 10^{-5}\text{m}$,
164 i.e. $66\mu\text{m}$. In $\delta t = 3.317\text{ms}$, the ^{10}B arrive at $48\mu\text{m}$. The ^{11}B arrive at $66\mu\text{m}$
165 in that same time. If after $\delta t = 3.317\text{ms}$ at $48\mu\text{m}$ the ^{10}B and ^{11}B are counted,
166 there will be more ^{10}B than ^{11}B .

167
168 Obviously, the absorption/emission of photons by Boron atoms in the laser
169 lattice has been left out of consideration in this classical approach. The energy
170 in eV at 532nm is approximately $E = \frac{6.582 \times 10^{-16}}{2\pi} \frac{2.998 \times 10^8}{5.32 \times 10^{-7}} \approx 0.0589$ eV. The
171 absorption of a photon ¹ is expected not to be very high. The classical physics
172 approach of the atom as a little inert ball can be maintained in approximation.
173 However, it can be questioned if it is a realistic way to set up a laser lattice for
174 Boron.

175
176 Finally, the example with ^{10}B and ^{11}B is provided as a computational clue with
177 a connection to physical empirical reality. Perhaps the ^{10}B and ^{11}B example
178 is unrealistic in an actual experiment. Nevertheless, it enables an experimental
179 setup to study this classical physics "out of nowhere" $\Delta g > 0$ in a low-density
180 gas of atoms.

181 3.2 Time invariance

182 One thing is clear about the question of time invariance. Equation (2) is not
183 time invariant. We have $t > 0$. A "-t" gives, via $x(t) = x_{\text{start}} + v_0 t + (g/2)t^2$, a
184 different x than a "+t". Therefore, if, $x(t) = x_{\text{start}} + v_0 t + (g/2)t^2$, can be seen
185 as a limit of quantum mechanics, time non-invariance appears to be related to
186 the "out of nowhere" Δg .

187 References

- 188 [1] P. Clade, Bloch oscillations in atom interferometry, 2014. fhal-00989685ff.
189 [2] V. Kuz'menko, On the time reversal noninvariance in quantum physics, DOI:
190 10.13140/RG.2.2.20696.19201, 2018.

¹Looking at <https://qedfusion.org/LIB/PROPS/PHOTON/borondata.html>