

# A Dimensional Identity Connecting Gravity and Electromagnetism via the Fine-Structure Constant

Jonathan Loomis June 30, 2025

## Abstract

We present a dimensional identity that connects the gravitational constant ( $G$ ), the fine-structure constant ( $\alpha$ ), the vacuum permittivity ( $\epsilon_0$ ), and the Planck mass ( $m_p$ ). While not predictive on its own, this identity suggests a potential structural relationship between gravitational and electromagnetic constants. The formulation allows  $G$  to be expressed in terms of electromagnetic parameters and a mass scale equal to the Planck mass, offering a concise and dimensionally exact connection between domains typically treated as distinct. Though speculative, the precision and dimensional consistency of this identity motivate further scrutiny.

## 1 Introduction

The gravitational constant  $G$  and the fine-structure constant  $\alpha$  are fundamental to physics but are associated with separate interaction domains: gravity and electromagnetism, respectively. Traditionally, these constants are not considered structurally related. However, in this work we explore a dimensionally consistent identity that connects them through the Planck mass and vacuum permittivity. The result is a concise expression that, while not offering predictive power, may highlight a deeper structural symmetry among constants of nature. This formulation does not invoke unification theories or posit new physical mechanisms. Instead, it reexamines how known constants might relate through dimensional structure.

## 2 Derivation of the Identity

The standard expression for the fine-structure constant in SI units is:

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \tag{1}$$

By noting the known relation between the Planck mass and the constants  $\hbar$ ,  $c$ , and  $G$ :

$$m_p^2 = \frac{\hbar c}{G} \tag{2}$$

Substituting equation (2) into equation (1) yields:

$$\alpha = \frac{e^2}{4\pi\epsilon_0 G m_p^2} \tag{3}$$

Rearranging gives a compact identity for  $G$ :

$$G = \frac{e^2}{4\pi\varepsilon_0\alpha m_p^2} \quad (4)$$

Alternatively, solving for the mass term:

$$m = \sqrt{\frac{e^2}{4\pi\varepsilon_0\alpha G}} \quad (5)$$

Numerically, this yields a value equal to the Planck mass using empirically measured values of  $\varepsilon_0$ ,  $\alpha$ ,  $G$ , and the defined value of  $e$ . This demonstrates that the identity holds independently of the standard definition of the Planck mass, suggesting it may be treated as an auxiliary identity. This equivalence is summarized in Table 1.

Table 1: Numerical validation of the identity

Quantity	Computed	CODATA 2018
$G$	$6.67430 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$	$6.67430 \times 10^{-11}$
$m$ (derived)	$2.176434 \times 10^{-8} \text{ kg}$	$2.176434 \times 10^{-8}$
$\alpha$ (reconstructed)	$7.2973525693 \times 10^{-3}$	$7.2973525693 \times 10^{-3}$

## Electromagnetic Expression for Gravitational Coupling

The following identity provides a potential indication that the gravitational constant  $G$  may not be fundamentally independent, but instead derivable from electromagnetic and mass-scale quantities:

$$G = \frac{e^2\mu_0c^2}{4\pi\alpha m_p^2} \quad (6)$$

Here,  $G$  is expressed in terms of the elementary charge  $e$ , the vacuum permeability  $\mu_0$ , the speed of light  $c$ , the fine-structure constant  $\alpha$ , and the Planck mass  $m_p$ . While traditionally associated with gravitational physics, this construction suggests that gravitational coupling may arise from the structure of the electromagnetic vacuum.

The dimensional consistency and numerical validity of this expression imply that  $G$  could be a derived property—emerging not from an independent force law, but from the way charge, field propagation, and inertial mass interrelate through the vacuum. Although not treated here as a derivation, this relationship supports the broader possibility that gravity reflects deeper geometric or electromagnetic properties of space.

## Gravitational Coupling Hidden in Electromagnetic Structure

Several identities suggest that the gravitational constant  $G$  may be structurally encoded in quantities typically associated with electromagnetism. These relationships are not introduced as derivations, but as dimensionally consistent bridges that reveal deeper interdependence between gravity and electromagnetic vacuum behavior.

First, the gravitational constant can be expressed entirely in terms of EM constants and the Planck mass:

$$G = \frac{e^2 \mu_0 c^2}{4\pi \alpha m_p^2} \quad (7)$$

This expression shows that the permeability of free space  $\mu_0$ , the elementary charge  $e$ , the speed of light  $c$ , and the fine-structure constant  $\alpha$  together reproduce gravitational coupling strength when scaled by inertial mass.

By extension, this implies gravitational structure is reflected in other canonical EM quantities. For example, Coulomb's law can be rewritten as:

$$F = \frac{e^2}{4\pi \epsilon_0 r^2} = \frac{\alpha G m_p^2}{r^2} \quad (8)$$

This expression interprets electrostatic force as a curvature-scaled gravitational interaction between charged masses. Similarly, vacuum impedance and field energy expressions inherit gravitational scaling:

$$Z_0 = \mu_0 c = \frac{4\pi \alpha G m_p^2}{e^2 c}, \quad U = \frac{1}{2} \epsilon_0 E^2 V = \frac{e^2 E^2 V}{8\pi \alpha G m_p^2}$$

Finally, the vacuum response coefficient in Maxwell's equations,

$$\mu_0 \epsilon_0 = \frac{1}{c^2},$$

remains dimensionally consistent when both constants are expressed via gravitational-coupled forms.

Together, these identities support the view that gravitational coupling is embedded within the electromagnetic structure of the vacuum, and that  $G$  may be a relational product of mass, charge, and vacuum geometry.

### 3 On Circularity and Definitions

A potential objection is that this identity is circular, since  $m_p$  is traditionally defined in terms of  $G$ ,  $\hbar$ , and  $c$ . However, when the identity is used to derive  $m$  numerically from  $\epsilon_0$ ,  $\alpha$ ,  $G$ , and  $e$ , the resulting mass matches the Planck mass without referencing its definition. This renders the identity self-consistent and empirically grounded, rather than tautological. Similar apparent circularities exist in the Planck unit system (e.g.,  $c = \ell_p/t_p$ ), but these reflect structural coherence rather than logical inconsistency.

### 4 Interpretation

The identity highlights a dimensionally exact overlap between gravitational and electromagnetic constants. If  $G$  can be fully expressed in terms of  $\epsilon_0$ ,  $\alpha$ , and  $m_p$ , it raises the possibility of a structural (if not causal) link between gravity and electromagnetism. This may suggest

that gravitational interactions reflect a suppressed or neutralized aspect of electromagnetic structure, or that both arise from a shared dimensional foundation. While these interpretations remain speculative, the precision and dimensional symmetry warrant further theoretical and empirical investigation.

## 5 Implications and Future Work

The identity allows for  $G$  to be estimated using constants with smaller relative uncertainties. As of CODATA 2018:

- $e = 1.602176634 \times 10^{-19}$  C (exact)
- $\epsilon_0 = 8.8541878128 \times 10^{-12}$  F/m
- $\alpha = 7.2973525693 \times 10^{-3}$  (measured)
- $m_p = 2.176434 \times 10^{-8}$  kg (measured)

With these values, the identity can yield  $G$  with a relative uncertainty dominated by  $m_p$ , potentially offering a more precise estimation than direct gravitational measurement. Further work will focus on exploring whether this identity arises naturally within broader theoretical frameworks or remains a coincidental dimensional resonance.

These findings, while dimensionally grounded, remain interpretively open. Whether the identity reflects a deeper coupling between physical constants or is merely a formal coincidence, it offers a compact, exact link between traditionally disconnected domains. Such identities may provide structural guidance for future theoretical developments that unify or structurally constrain fundamental constants.

It is important to note that this identity does not alter any known predictions or observations of gravity or electromagnetism. Rather, it suggests the possibility of a deeper foundational link between the two, as revealed through dimensional structure. The identity itself is non-predictive, but it may represent a useful constraint or point of convergence in future unification attempts.

## 6 Conclusion

We have presented a dimensionally exact identity connecting gravitational and electromagnetic constants via the fine-structure constant and the Planck mass. While the identity does not currently yield new predictions, its precision and structural symmetry warrant attention. Whether this relation reflects a deeper physical constraint or remains a numerical artifact is a question for future theoretical and empirical exploration.

## 7 References

P.J. Mohr, D.B. Newell, and B.N. Taylor, “CODATA Recommended Values of the Fundamental Physical Constants: 2018,” *Rev. Mod. Phys.* **93**, 025010 (2021). <https://doi.org/10.1103/RevModPhys.93.025010>