# A glimpse on Graph theory and social networks

Vicente Pedro Diogo

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**Abstract**

Mathematics is more than just numbers and equations; it is a powerful tool in which all Universe laws are based on. From the simple action of counting coins to complex engineering structure, math is very important in our daily lives. This article explores the surprising ways maths is into our everyday experiences, by diving into the relationships between Discrete Math and Network Sciences analysis : The Graph theory and its Application on Social Networks. The article gives finally a brief and accessible reflection on how Graph theory helps the information diffusion on many digital platforms like Facebook and Linkedln , this work is for a general public rather than a specialized and mathematically rigorous work.

## 1 Introduction

Social networks have become an integral part of modern life, indicating how individuals communicate, share information, and form relationships. Understanding The mains structures of these networks is essential for several applications, including marketing, community management. Graph theory, on the study of networks and their properties, offers general principles for analyzing social networks. For effect, graph theory treats social networks as mathematical structures containing nodes (representing people) and edges (representing relationships or interactions)[2].

### 1.1 Fundamental concepts :

**Graph theory**: is a field of Discrete Mathematics that studies relationships between objects represented as Graph. **Graph**G : A graph $=(V,E)$ consists of a set of objects$V = (v_1, v_2, v_3, ...)$ called vertices, and another set $E = (e_1, e_2, e_3, ...)$ whose elements are called edges, such that each edge $e_k$ is identified with an unordered pair $(v_i, v_j)$ of vertices, basically a graph is a mathematical structure used to model pairwise connections between objects[3].

**Key elements of a Graph:**

- **Vertices or nodes** : Fundamental element a graph representing an object or a point.

- **Edges or Links** : can be thought of as connection between two vertices, and can be directed (arcs) or undirected.

- **Degrees of Nodes** : This is the number of connections a node has.

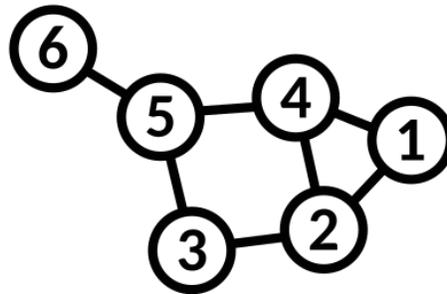- **Connected components**: Subsets of the graph where there is a path between two nodes.



Figure 1: Undirected graph with 7 edges and 6 vertices

**Real world examples** : graph theory has important implications in various real-world systems, from Sciences to Engineering , graph theory can uncover new relationships and mechanisms, such as: Transportation networks, social networks, computer networks and biology (protein structure, gene networks)[3].

# 2 Application of graph theory on social networks

Mathematically, a social network is a graph, where vertices represent people and edges represent relationships between them, for example, a friendship or working together on a project. Friends' recommendation: Social networks use graph theory to identify people with similar interests or mutual friends. Among the key features of a Graph in social networks, two gain visibility :

**Information Spread** : By analyzing how information spreads through a network and identifying popular topics.

**Detecting communities** : Thanks to graph theory, we can find groups of people on larger networks , who are closely connected.

# 3 Graph theory on social media platforms

This session gives a summary of common social medias and how Graph theory influences their functionalities.

### 3.0.1 Facebook

Facebook uses Graph theory to understand connectivity and influence , identifying influential users and communities within Facebook ,analyze how information spreads through the network.This allows Facebook to optimizes news feed algorithm and advertising by focusing on central and influential nodes[3].

### 3.0.2 Community detection on Linkedln

Linkedln's professional network identify professional communities and connections.Community detection algorithms are widely used to uncover clusters of professionals with similar skills or industry affiliations.This information is used to enhance networking opportunities and targeted job recommendations[3].

## 4 Future expectations

The future holds exciting possibilities for graph theory and social networks. We can expect advancements in personalized recommendations,development of more efficient Digital systems,more efficient network analysis, and innovative solutions to complex problems. As technology maintains its development, graph theory will play a crucial role in shaping our digital world[1].

## 5 Conclusion

The application of graph theory to social networks has profoundly advanced our understanding of relationships within these systems. This research highlights how graph theoretical concepts—such as community detection, influence analysis—can offer useful notes about network evolution, and information flow. The integration of graph theory into social network analysis not only provides a deeper comprehension of network structures but also fosters the development of more effective and resilient social networking platforms. Future research should continue to explore innovative graph-based methods to address emerging challenges and discover new opportunities in the fascinating world of social networks.

# References

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