LAW OF UNIVERSAL GRAVITATION WITHOUT GRAVITATIONAL CONSTANT G.

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Abstract. For over 300 years, the force of gravitational interaction was represented by a single physical law – Newton's law F = GMm/r^2. The possibility of the existence of a law of gravity other than Newton's law F = GMm/r² was not even considered. Excessive idealization of Newton's law F = GMm/r² and overestimation of its uniqueness became a brake on the development of the theory of gravity. Newton's law of gravitation does not provide a complete and accurate value for the gravitational force. It describes the local gravity of two bodies and "does not see" the additional gravitational force that actually exists as a result of the gravitational action of all bodies in the universe. Here we show that in addition to Newton's law F = GMm/r^2, there is a second law of gravitation F = mR^3/(T^2)r^2, which remained undiscovered. This law of gravity does not require the gravitational constant G. The existence of this law was first pointed out by Robert Hooke in 1679. Furthermore, we show that the additional gravitational force of the gravitational action of all bodies in the Universe is described by the third law of gravitation F = (mc²)√Λ, which also remained undiscovered. This law of gravity also does not require a gravitational constant G. Combining the two new laws of gravitation into a single equation yields the law of universal gravitation F_U = mR^3/(T^2)r^2+(mc^2)√Λ. This is a complete and more accurate law of gravity than Newton's law. It replaces Newton's law of gravity. The new law of universal gravitation takes into account the accelerated expansion of the universe and Kepler's laws of planetary motion. It expresses the total force of universal gravitation, which is represented by both the local gravitational force of two bodies and the gravitational force of all N bodies in the universe. The new law of universal gravitation is a solution to the inverse N-body problem for N = 2 and for N → ∞. The new law of universal gravitation does not include the gravitational constant G. The measured value of R³/T² based on observations of the orbits of the planets of the Solar System is: R³/T² = 132 712 440 018 (8) km³ s⁻². From the cosmological equations, the calculated value of the cosmological constant Λ = 1.36226697... •10⁻⁵² m⁻² was obtained. This is much more accurate than the constant G. The low accuracy of the gravitational constant G ceases to be a limiting factor in gravitation.

Keywords: Newton's law; N-body problem; law of universal gravitation; parameters of the observable universe; dark matter; galaxy rotation curve; cosmological constant Λ.

1. Introduction

Newton's law of universal gravitation $F = GMm/r^2$ is one of the greatest scientific discoveries. It is distinguished by its simplicity and mathematical perfection. Despite the fact that the law of gravitation was discovered more than 300 years ago, gravity remains the most mysterious physical phenomenon in physics and cosmology. All this time, the force of gravitational interaction was represented only by Newton's law of universal gravitation. For many practical applications, this law was sufficient. However, the predictions of Newtonian gravity fail on the scale of the universe. Newton's law shows the force of gravitational interaction of two bodies out of all N bodies in the

universe. The formula for Newton's law describes the gravity of only one local source of attraction and does not take into account that bodies are simultaneously attracted to all other bodies in the universe. In reality, all bodies in the universe participate in gravitational interaction. This is not taken into account in Newton's law of universal gravitation. Newton's law does not work on the scale of the universe [1 - 9].

Newton's law of gravity does not "see" the total force that acts in the universe, but "sees" only a part of the gravitational force. Newton's law does not take into account the accelerated expansion of the universe. Newton's law does not include the parameters of the Keplerian orbit. An accurate and complete law of gravity must take into account the gravity of all N bodies in the universe. In gravity, the main fundamental law of nature remained undiscovered: the law of gravitational interaction of all N bodies in the universe. The obstacle on this path was the unsolvable N-body problem.

In order to eliminate the imperfection of Newtonian dynamics, many attempts were made to modify Newton's law [10, 11, 12]. These attempts did not lead to the discovery of a more perfect law of gravity than Newton's law. The law of gravity that takes into account the gravity of all bodies in the universe cannot be obtained by modifying and changing Newton's law of universal gravitation for the reason that Newton's law is the law of gravitational interaction of two bodies. No amount of cosmetic editing can transform the two-body law of gravitational interaction into an N-body law of gravitational interaction.

Newton's law faces an insurmountable N-body problem, requiring approximations or computational solutions for many pairs of bodies. In essence, a more accurate law of gravitation should explain the observed gravitational phenomena at all scales, including those that are currently not explained by Newton's law.

Obviously, the N-body law of gravitational interaction is not a modified two-body law of gravitation, but a new law of gravitation. It must be a new fundamental law of gravitation, complementary to the two-body law of gravitation. The additivity of gravitational forces makes it possible to represent the complete law of universal gravitation as a sum of forces: the gravitational forces of a finite number of pairs of bodies and the cosmological force of all N bodies in the Universe.

2. The law of universal gravitation must take into account the gravitational force of the universe and include the parameters of the universe.

It is a mistake to consider the law of two-body gravity to be the law of universal gravitation. In reality, all bodies in the universe participate simultaneously in gravitational interactions with two bodies. The mass of the universe is enormous ($M_U \approx 10^{53}$ kg). This enormous distributed mass is not represented in the law of gravitation. Therefore, considering the law of two-body gravity, $F = GMm/r^2$, to be the law of universal gravitation is a gross exaggeration. Universal gravitation is realized with the participation of the enormous mass of all bodies in the universe, distributed throughout space. The idealization of Newton's law, $F = GMm/r^2$, and the overestimation of its predictive power have had a destructive impact on the development of gravity theory. To take into account the gravitational force of the universe, the law of universal gravitation must include the parameters of the universe. This is what is missing from Newton's law, $F = GMm/r^2$.

3. About theories of gravity.

Newton did not develop or use the theory of gravity in his research. This did not prevent him from discovering the fundamental law of Nature. There are now many theories of gravity, the purpose of which is to improve Newtonian dynamics, but they have not added a more perfect physical law to Newtonian dynamics than Newton's law of gravity. The abundance of theories of gravity (Metric theories, Non-metric theories, Vector theories, Scalar-tensor theories, Le Sage theory, Theories of quantum gravity, supergravity, string theory...) have not led to a better law of gravitational force than Newton's law [13]. These theories of gravity have not discovered a single new law of nature. All theories of gravity have proven powerless against the N-body problem. It is obvious that the approach "from the theory of gravity to the law of gravity" is not productive. The fact that Newton discovered the law of gravity without any theory shows that physical theories are not a tool for discovering the laws of Nature. The laws of Nature are primary, and theories have a secondary status. It is necessary to look for another approach to discover the law of gravitational interaction, taking into account the gravity of all bodies in the Universe. Here we propose a new approach to solving the N-body problem, which provides the key to discovering the laws of gravity.

4. Elimination of problems related to the accuracy of the constant G, at the cost of abandoning the constant G in gravity.

For over 300 years, the force of gravitational interaction has been represented by a single physical law—Newton's law of gravitation, $F = GMm/r^2$. This law of gravitation includes the constant G, which is considered a fundamental constant. Newton's law of gravitation does not provide a precise value for the gravitational force. It "sees" only the gravitational force between two bodies.

Furthermore, the accuracy of the constant G is the lowest among the fundamental physical constants. This creates numerous problems in physics and cosmology. The low accuracy of G complicates the accurate modeling of gravitational interactions in space, which is critical for calculating the orbits of planets, stars, and galaxies, as well as for understanding the evolution of the Universe. The low accuracy of G becomes an obstacle to testing and refining fundamental physical theories and searching for deviations from them.

Many theorists suggest that creating a consistent theory of gravity will require a rethinking of the very nature of gravity and, possibly, the status of G as a fundamental constant [14]. In [15, 16] it is shown that many gravitational predictions and measurements can be made without any knowledge of the gravitational constant G. In this case, is the constant G fundamental? The fundamental status of the constant G has long been questioned due to the difficulties with its precise measurement and the many problems that arise from its low accuracy [17 - 21]. Perhaps the reason for the overestimation of the role and overstated status of the constant G was the fact that Newton's law of gravitation was the only law in classical gravitation. This fueled the illusion of the uniqueness of the constant G. Problems associated with the accuracy of the constant G can be solved at the cost of abandoning G in gravitation. Rejecting the concept of the constant G leads to a new law of gravitation, more perfect than Newton's law.

If G is not a fundamental constant, this could lead to a revision of our understanding of gravity. The non-fundamental status of the constant G may indicate the non-fundamental status of Newton's law of gravity.

5. Emphasis on eliminating the constant G from the law of gravitation.

We emphasize eliminating the constant G for the following reasons:

- The constant G is one of the least precise of all known fundamental constants. The difficulty of measuring it does not inspire hope for a quick solution to this problem.

- The constant G was not included in Newton's original works. Its inclusion in the law of gravitation did not add completeness to Newton's law. Newton's law of gravitation applies to the gravity of two bodies. It does not take into account the gravitational attraction of all bodies in the universe, nor the cosmological expansion of the universe.

- Doubts about the fundamental status of the constant G. The fundamental status of the constant G has long been questioned by many scientists.

- A complete description of both local and cosmological gravitational effects requires a new law of gravitation without the constant G, one that is more precise than Newton's law.

6. Instead of a modified Newtonian law of gravitation, a new law of gravitation without the constant G.

To go beyond Newtonian dynamics, one must abandon the gravitational mass "M" as a parameter in the law of gravitation. This is the key to discovering a new law of gravitation. Rejecting the gravitational mass "M" makes it possible to abandon the very concept of G in gravitation. The absence of G in the law of gravitation eliminates many problems caused by the low accuracy of the gravitational constant G.

Numerous attempts have been made to modify Newton's law [3, 12, 22, 23, 24, 25]. However, the modified equations remained equations of Newtonian dynamics and did not lead to a new law of gravitation. In theories where the constants are equal to unity ($G = c = \hbar = 1$), G becomes dimensionless, and the equations of gravitation are simplified. But this is merely a change in the system of units, not a rejection of the very concept of the gravitational constant. This is not a removal of the essence of the gravitational constant G, but an attempt to conceal it by making it a dimensionless unit. These are alternatives of the type: not quite "G-free".

In [22], it is noted that to eliminate the gravitational constant G, it is necessary to shift to a different definition of energy and mass. Here, we propose a different approach. Instead of a different definition of energy and mass, it is necessary to completely abandon the use of the mass "M" as a parameter in the law of gravity. This leads to a new law of gravity without the gravitational constant G. But this will not be Newtonian dynamics, nor will it be modified Newtonian dynamics.

Thus, within the framework of Newtonian dynamics, it is impossible to completely eliminate G, since it is necessary for the quantitative description of gravitational interaction. Throughout the history of physics, there has been no deliberate attempt to derive a different physical law of gravitational force without the constant G, one more accurate than Newton's law. A physical law of gravitational force different from Newton's law must be sought beyond Newtonian dynamics, beyond Newton's law. The goal is not to hide the constant G in an artificially introduced system of units, but to completely eliminate it from the law of gravitation. I argue that this cannot be accomplished by modifying Newton's law. A new physical law of gravitational force is needed, one that does not require the constant G, and that provides a quantitative description of gravitational interaction.

7. NEW LAW OF GRAVITATION OF TWO BODIES WITHOUT CONSTANT G.

The law of two-body gravitation without the constant G was presented in [26]. Fig. 1 shows this law of gravitation.

$$F_{H-K} = \frac{mR^3}{T^2 r^2}$$

Fig. 1. Hooke-Kepler law of gravitation. Where: m is the mass of the body, R and T are orbit parameters, r is the distance.

This law is based on Kepler's law of planetary motion. The law of two-body gravitation without constant G is not burdened by the problems inherent in Newton's law of gravitation.

Robert Hooke first pointed out the existence of this law of gravitation in a letter to Newton in 1679. Scholars of Hooke's work have overlooked the fact that in his letter, Hooke provided a verbal formulation of the law of gravitation based on the elliptical orbits of the planets and Kepler's laws. It turns out that there are two laws of gravitation, both based on the inverse-square law. Newton's law is only one of them. Hooke's verbal formulation led to a more precise and perfect law of gravitation than Newton's. If Newton had accepted Hooke's proposal, we would already be celebrating the 345th anniversary of this remarkable law of two-body gravitation, which originally did not require a constant G.

If the central mass "M" is used as a parameter in the law of gravitation (as in Newton's law), then it is impossible to eliminate the constant G. Conclusion: eliminating the constant G from the law of gravitation is possible only by replacing Newton's law with a new law of gravitation. It is necessary to derive a law of gravitation that does not include the central (attracting) mass.

8. THE LAW OF THE GRAVITATIONAL FORCE OF THE UNIVERSE WITHOUT THE CONSTANT G.

The Hooke-Kepler law describes local gravity quite accurately without using the constant G. To obtain the total gravitational force, it is necessary to take into account the gravitational force of all other bodies in the universe. The gravitational force of the universe is represented by an additional cosmological force (Fig. 2).

$$F_{Cos} = mc^2 \sqrt{\Lambda}$$

Fig. 2. The law of cosmological force. Where m is the mass of an object, c is the speed of light, and Λ is the cosmological constant.

Instead of the constant G, the new law of gravity includes the cosmological constant Λ. This law of gravity takes into account the accelerated expansion of the universe.

9. A new law of universal gravitation without the constant G.

The law of universal gravitation, taking into account the accelerated expansion of the Universe and Kepler's laws of planetary motion, is as follows:

$$F_U = \frac{mR^3}{T^2 r^2} + mc^2 \sqrt{\Lambda}$$

Fig. 3. The law of universal gravitation, taking into account the accelerated expansion of the Universe and Kepler's laws of planetary motion. Where: m is the mass of the body, R and T are orbit parameters, r is the distance, c is the speed of light in vacuum, Λ is the cosmological constant.

The gravitational force F_U consists of two components: the local force $F_{H-K} = mR^3/T^2 r^2$ and the cosmological force $F_{Cos} = mc^2\sqrt{\Lambda}$. The law of universal gravitation does not include the constant G. This law of gravitation is superior to Newton's law. The main drawback of Newton's law is that it "sees" only the local force. Moreover, the accuracy of Newton's law is limited by the low accuracy of the constant G.

The law of gravitation (Fig. 3) is the solution to the inverse N-body problem for $N = 2$ and for $N \to \infty$. This equation is of greatest interest as the most accurate and perfect law of gravitation. The new law of gravitation shows the total force of universal gravitation taking into account the gravitational interaction of all N bodies in the Universe. The total force of universal gravitation consists of two components: the force of gravitational interaction of two bodies $F_{H-K} = mR^3/T^2 r^2$ (Hooke-Kepler law [26]) and an additional cosmological force $F_{Cos} = mc^2\sqrt{\Lambda}$. The peculiarity of the new law of universal gravitation is that it does not include the gravitational constant G and the mass M. This is a good sign. The low accuracy of the gravitational constant G ceases to be a limiting factor in gravity. The new law of gravity includes parameters available from observations. These are the parameters of the orbits of the planets (R and T) and the cosmological constant Λ.

Equation (Fig. 3) is not a modified Newtonian law formula; it is a new law of gravity. It opens up new knowledge in the field of gravity. The new law of gravity challenges the dominant paradigm in physics and cosmology.

The new law of gravitation shows that in addition to the force of gravitational interaction between two bodies, all bodies are affected by the cosmological force $F_{Cos} = mc^2\sqrt{\Lambda}$ of the universe. The total force of universal gravitation is represented by the vector sum of two forces: the force of gravitational interaction of two bodies FH-K and the cosmological gravitational force F_{Cos}:

$$\vec{F}_U = \vec{F}_{H-K} + \vec{F}_{Cos} \qquad (1)$$

The value of the resulting force of universal gravitation is in the range of values from $F_U = mR^3/T^2 r^2 - mc^2\sqrt{\Lambda}$ to $F_U = mR^3/T^2 r^2 + mc^2\sqrt{\Lambda}$. For collinear force vectors, the equation (Fig. 3) takes the form: $F_U = mR^3/T^2 r^2 \pm mc^2\sqrt{\Lambda}$.

9.1. Two-body gravity + Universe gravity.

Newton's law $F = GMm/r^2$ "sees" the gravitational force of two bodies. The new law of gravitation $F_U = mR^3/T^2r^2 + mc^2\sqrt{\Lambda}$ "sees" two forces. The first term gives the gravitational force of two bodies. The second term gives the gravitational force of the universe. The total force of gravitational interaction is the sum of the two forces. Fig. 4 conventionally shows the contribution of the cosmological force $F_{Cos} = mc^2\sqrt{\Lambda}$ to the Galaxy rotation curve.
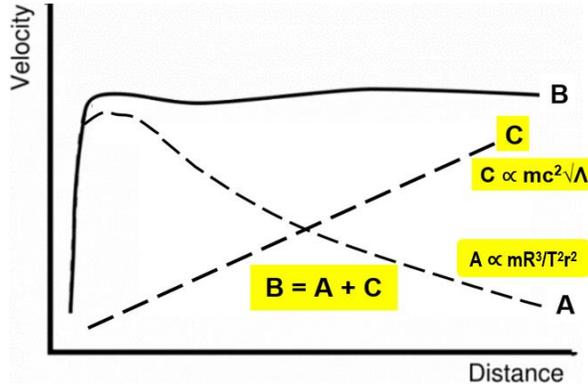
Fig. 4. Galaxy rotation curve (B) as a result of the action of two forces: B = A + C.

The law of universal gravitation $F_U = mR^3/T^2r^2 + mc^2\sqrt{\Lambda}$ provides a more complete description of gravitational interactions at different scales. It combines the Hooke-Kepler law and the cosmological force law. This extension is especially relevant when considering the large-scale structure of the universe.

9.2. Accuracy and Perfection

The new law of gravitation $F_U = mR^3/T^2r^2 + mc^2\sqrt{\Lambda}$ is more accurate and perfect than Newton's law $F = GMm/r2$, since it provides a more complete description of gravity, especially on cosmological scales. It takes into account the influence of all bodies in the Universe, not just pairs of bodies. It does not include the gravitational constant G and the mass M. The new law of gravitation is more accurate than Newton's law, since the orbital parameters (R and T) are known with greater accuracy than the constant G and the mass M. With the advent of the law of gravitation $F_U = mR^3/T^2r^2 + mc^2\sqrt{\Lambda}$, the low accuracy of the gravitational constant G ceases to be a limiting factor in gravity.

10. Cosmological force

The relativistic version of the law of universal gravitation includes the cosmological force as an integral part. This is a gravitational force that depends on the cosmological constant Λ. The formula for this force is: $F_{Cos} = mc^2\sqrt{\Lambda}$, where m is the mass of the object, c is the speed of light, Λ is the cosmological constant. The cosmological force is the gravitational force with which the mass of the entire universe acts on the body, causing acceleration $A_0 = c2\sqrt{\Lambda} = 10.4922 \times 10^{-10}$ m/s². The cosmological force shows that the gravitational interaction of an object with the Universe, represented by the cosmological constant (Λ), generates a force proportional to the mass of the body. The cosmological force does not obey the inverse square law. The cosmological gravitational force is generated by the mass of all bodies in the Universe. This is different from Newton's law, which focuses on the interaction of two bodies. In this context, the cosmological constant Λ is a coupling constant

that affects the magnitude of the cosmological force. The cosmological force arises from the interaction of the body with the general mass distribution of the Universe.

10.1. The gravitational force of the Universe.

The formula for the cosmological force Fig. 2 shows that this force is directly proportional to the mass of the object and depends on the cosmological constant, which is related to the expansion of the Universe. The formula includes the speed of light, which demonstrates the relativistic version of the law of universal gravitation. The law of the cosmological force explains observed anomalies such as the Pioneer anomaly, the perihelion shifts of planets, and the rotation curves of galaxies, which are not explained by existing gravitational models. The law of the cosmological force provides a basis for understanding gravitational interactions in the Universe, especially on cosmological scales.

10.2. The Cosmological Force Law as a Supplement to the Two-Body Gravity Law

The cosmological force law is not intended to replace the two-body gravity law, but to complement it by taking into account the influence of the Universe as a whole. In essence, the cosmological force law takes into account the influence of the Universe on gravitational interactions, potentially offering a more complete understanding of gravity on both small and large scales. The additional cosmological force causes an acceleration $A_0 = (c^2)\sqrt{\Lambda}$ close to the acceleration value ($a_0 = 1.2 \times 10^{-10}$ m/s^2) predicted by MOND [3, 4].

11. Hooke-Kepler Law of Gravitation

The new version of the law of universal gravitation includes the Hooke-Kepler law of gravitation $F_{H-K} = mR^3/T^2r^2$ [26]. This law includes Kepler's laws of planetary motion and the inverse square law, emphasizing the relationship between the orbital parameters (radius R and period T) and the gravitational force. In one formula, both the inverse square law and Kepler's third law, which relates the orbital radius and period, are combined. The Hooke-Kepler law (Fig. 1) provides a more accurate and complete formula for the gravitational force of two bodies than the proportionality law $F_N \propto mM/r^2$ originally proposed by Newton.

11.1. Hooke's Contribution

Robert Hooke, in his correspondence with Newton (1679), emphasized the importance of the inverse square law and the ellipticity of planetary orbits for describing gravity. Robert Hooke relied on Kepler's laws. He argued that a complete understanding of gravity must take these factors into account. Robert Hooke had given a verbal formulation of the future law of gravity in a letter 7 years before Newton published Philosophiæ Naturalis Principia Mathematica. Newton did not accept Hooke's hint and did not include the parameters of Keplerian orbits in his law of gravity. Newton used masses. This was a different law of gravity than the one hinted at by Hooke. It is known that masses are not included in Kepler's laws at all. By rejecting Hooke's hint, Newton missed the opportunity back in 1679 to give the world a more precise law of gravity $F = mR^3/(T^2r^2)$ than the approximate law of proportionality $F_N \propto mM/r^2$, which he discovered 7 years later in 1687. It is known that in Newton's

law of proportionality $F_N \propto mM/r^2$ the sign "=" ($F = GMm/r^2$) appeared only 200 years later (in 1873) with the appearance of the gravitational constant G in it [27, 28].

11.2. Relationship with Kepler's Constant

A more accurate and complete understanding of gravity can be achieved by considering elliptical orbits and the inverse square law proposed by Hooke, and by using the ratio (R^3/T^2) that follows from Kepler's law. The Hooke-Kepler law highlights the importance of Kepler's constant (R^3/T^2) in the context of the gravitational force [29].

11.3. Key Differences from Newton's Law

In essence, the Hooke-Kepler law of gravity highlights a different view of the relationship between gravity and orbital motion, highlighting the importance of Kepler's laws in understanding the force of gravity. While Newton's law focuses on the gravitational force between two masses ($F = GmM/r^2$), Hooke-Kepler's law $F_{H-K} = mR^3/(T^2r^2)$ emphasizes the relationship between the orbital parameters (radius and period) and the gravitational force. Hooke-Kepler's law does not include the mass M. It is known that Kepler's laws do not include masses at all.

11.4. Hooke-Kepler's Law - A New Law of Two-Body Gravitation

Hooke-Kepler's law of gravitation (Fig. 1) is a different formulation of the gravitational force between two bodies than Newton's law. This law of gravitation emphasizes the relationship between the orbital parameters (radius and period) and the gravitational force, offering a more accurate representation of gravity than Newton's law. It is derived by combining Hooke's idea of the inverse-square law with Kepler's laws of planetary motion. A detailed history of the Hooke-Kepler law of gravitation is given in [26].

12. Inverse N-body problem

The most important physical law of gravity $F_U = mR^3/T^2r^2 + mc^2\sqrt{\Lambda}$, which shows the force of gravitational interaction of all N bodies in the Universe, remained undiscovered in gravity. The main obstacle on this path was the N-body problem, which has no analytical solution for $N \geq 3$. To obtain the law of gravitational interaction of N bodies, the inverse N-body problem was formulated and solved in [30, 31]. The inverse N-body problem is the problem of determining the law of gravitational force based on the integral characteristics of the N-body system. Unlike the direct N-body problem, which seeks to predict the future position of the bodies, the inverse problem aims to derive the underlying law of action of forces. This problem has not been considered in physics before, possibly due to the lack of an analytical solution to the direct N-body problem.

12.1. The goal of the inverse N-body problem

The inverse N-body problem involves determining the law of gravitational force acting on a system of N bodies, given the integral properties of their motion. Essentially, instead of starting with a known gravitational force and calculating the resulting motion (as in the forward N-body problem), this problem starts with the observed motion and aims to derive the underlying law of force. Given

the integral properties of the N-body system, the goal is to determine the law of action of the gravitational forces governing their interaction. The main task is to find a mathematical expression for the gravitational force that corresponds to the observed motion. Unlike the forward N-body problem, the inverse N-body problem has an analytical solution for both N = 2 and N → ∞. For N = 2, the inverse problem leads to both Newton's law $F = GMm/r2$ and the Hooke-Kepler law ($F_{H-K} = mR^3/T^2r^2$). As N tends to infinity, it leads to the law of the cosmological force ($F_{Cos} = mc^2\sqrt{\Lambda}$). The inverse problem is solved using the integral parameters of the N-body system. Solutions to the inverse N-body problem directly lead to new laws of gravity that remained undiscovered for more than 300 years.

12.2. The Direct N-Body Problem

Knowing the masses, initial positions, and velocities of the N bodies, as well as the known law of force action (e.g., Newton's law of gravitation), the goal is to calculate their positions and velocities at all future moments of time. This problem has no analytical solution for N ≥ 3.

12.3. The Mirror-symmetric approach to solving the inverse N-body problem.

The unsolvable direct N-body problem itself gave a hint for choosing a solution method: "If the N-body problem is unsolvable, then the mirror-symmetric problem can provide a solution." The idea of using the principles of opposition and inversion turned out to be productive. To solve the N-body gravity problem, a mirror-symmetric approach was used. Instead of the direct N-body problem, there is the inverse N-body problem. Instead of a set of individual bodies, there is one system consisting of N-bodies. Instead of the differential characteristics of individual bodies, there are integral characteristics of the N-body system. Instead of the trajectory of motion under the action of gravity, there is the law of gravity.

12.4. Instead of a set of individual bodies, there is one system consisting of N-bodies.

In the inverse N-body problem, the set of bodies is considered as a single object or one system of N bodies. The inverse N-body problem involves reducing the N-body problem to an equivalent two-body problem, where one of the bodies is a system of N bodies.

12.5. Integral characteristics of the N-body system instead of differential characteristics of bodies.

In the direct N-body problem, differential characteristics of bodies are used as parameters. The inverse N-body problem requires a transition from the differential approach (describing the motion of individual bodies) to the integral approach (describing the system as a whole). This means considering the general characteristics of the system, rather than the characteristics of individual pairs of bodies. This means using the integral characteristics of the N-body system instead of the characteristics of individual bodies.

The additivity of gravitational forces in the differential approach leads to the equation of the resultant force, represented by an infinite series. The additivity of gravitational forces in the integral

approach leads to the equation of the resultant force, represented by a finite series. In the simplest version, two terms of the series.

In contrast to the direct N-body problem, the inverse N-body problem has solutions both for N = 2. and for N → ∞. The use of integral characteristics of the N-body system leads to the fact that the inverse N-body problem has a solution for an arbitrary value of N. The inverse N-body problem is applicable both to the infinite universe model and to the finite universe model. In the infinite universe model, N → ∞. In the finite universe model, N takes a finite value. The main requirement for both the finite universe model and the infinite universe model is the known value of at least one integral parameter of the N-body system. The known value of at least one integral parameter of the N-body system allows one to obtain a solution to the inverse N-body problem for an arbitrary value of N.

12.6. Results of the N-body inverse problem solution

The solution of the inverse problem leads to the discovery of new force laws that describe gravitational interactions better than existing models, potentially explaining phenomena not explained by current theories. All solutions of the N-body inverse problem are given in [30, 31]. Among the solutions of the N-body inverse problem, the solution in the form: $F_U = mR^3/T^2r^2 + mc^2\sqrt{\Lambda}$ is of the greatest interest. It gives the most accurate and perfect law of universal gravitation. It gives a more complete and more accurate description of gravity, especially on a cosmological scale, taking into account the influence of all bodies in the Universe, and not just pairs of bodies. The law of gravity does not include the gravitational constant G and the mass M. For this reason, this solution compares favorably with solutions containing the constant G. It is known that the constant G has the lowest accuracy among all fundamental physical constants.

Solving the inverse N-body problem has implications for understanding dark matter and dark energy, as well as improving models of galaxy dynamics. The inverse N-body problem has the potential to reveal new insights into the nature of gravity and the fundamental laws that govern the Universe. It can help us improve our understanding of gravitational interactions at various scales, from planetary systems to the large-scale structure of the Universe. In fact, the inverse N-body problem is a fascinating area of research that can lead to a deeper understanding of the fundamental forces that govern the Universe.

13. Relativistic version of the law of universal gravitation without the constant G.

Newtonian gravity assumes instantaneous transfer of gravitational force. This contradicts the theory of relativity, which states that nothing can travel faster than the speed of light. Researchers have explored various approaches to resolving these inconsistencies [32, 33]. These approaches often involve modifying Newtonian gravity by ad hoc introducing the speed of light into the equation. Here we present the equation $F_U = mR^3/T^2r^2 + mc^2\sqrt{\Lambda}$, which is not a modification of Newton's law. The equation (Fig. 3) is completely different from Newton's law. At the same time, it is an equation of classical gravity. In the equation (Fig. 3), the speed of light enters naturally together with the cosmological constant Λ. The presence of the speed of light in the classical law of gravitation together with the cosmological constant Λ represents the relativistic version of the law of universal gravitation.

The equation (Fig. 3) is a relativistic version of the law of universal gravitation, obtained within the framework of classical gravity.

14. Evolution of the law of universal gravitation from Hooke and Newton to the present day.

For over 300 years, the force of gravity was considered to be Newton's single law, which describes the gravitational attraction of only one local source of gravity and ignores the fact that bodies are simultaneously attracted by all other bodies in the universe. The path from the two-body law of gravitation to the true law of universal gravitation (Fig. 3) was long. The stages of this long journey are shown in Fig. 5.
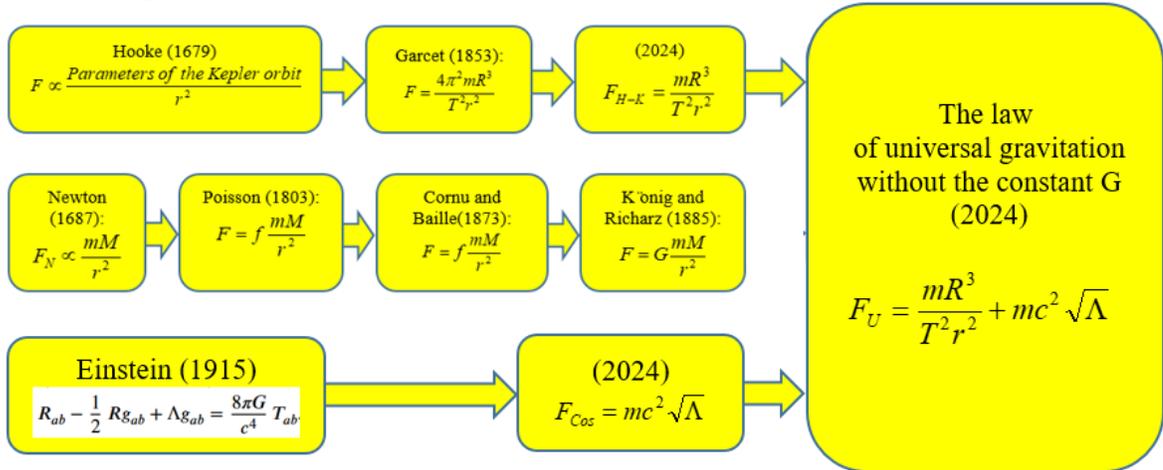
Fig. 5. The evolution of the law of universal gravitation from Hooke and Newton.

In 1687, Newton formulated the proportional relationship $F_N \propto mM/r^2$. This was not an exact law of gravitation. The proportional relationship $F_N \propto mM/r^2$ evolved into the law of gravitation $F = GMm/r^2$ only after the introduction of the constant G. This occurred almost 200 years later, between 1803 and 1885. With the advent of the law of universal gravitation $F = GMm/r^2$, the world assumed that the law of universal gravitation had been discovered and there was nothing more to discover. However, space exploration revealed discrepancies with the law of gravitation $F = GMm/r^2$. It turned out that on large scales, gravity behaves differently than predicted by Newton's law. Clearly, a physical law other than Newton's law is needed to describe gravity.

A more accurate law of gravitation without the constant G could have been developed as early as 1679, seven years before Newton's law. Hooke formulated a more accurate law of gravitation, emphasizing elliptical orbits and the inverse square law [26]. Combining the parameters of Keplerian orbits with the inverse-square law paved the way for a complete and more accurate law of two-body gravitation: $F = mR^3/T^2r^2$. However, this opportunity was missed in 1679. Newton used mass instead of the parameters of Keplerian orbits and, instead of a complete law of gravitation, obtained a proportional relationship: $F_N \propto mM/r^2$.

A second law of gravitation without the constant G could have appeared in 1853. This could have been done by Garcet, who derived the formula $F = 4\pi^2 mR^3/T^2r^2$. However, he considered this formula a special case for a circular orbit. He did not see in this formula a new law of gravitation, different and more perfect than Newton's law. Thus, the second opportunity to discover the law of gravitation without the constant G was missed.

In 1915, Einstein published the equations of general relativity, which included the cosmological constant Λ and the constant G. With the introduction of the cosmological constant Λ into the equations of general relativity, it became possible to take into account the influence of all bodies in the universe on the force of gravity and derive the law of gravitation for the universe: $F_{Cos} = mc^2\sqrt{\Lambda}$. However, Einstein abandoned the cosmological constant Λ. As a result, the opportunity to derive the law of gravitation for the universe was missed. At that time, this opportunity was provided by the constant Λ and the formula $E = mc^2$. The evolution of gravity theory shows that science is progressing at a snail's pace...

Combining the Hooke-Kepler law $F_{H-K} = mR^3/T^2r^2$ and the law of cosmological forces $F_{Cos} = mc^2\sqrt{\Lambda}$, we obtain the law of universal gravitation $F_U = mR^3/T^2r^2 + mc^2\sqrt{\Lambda}$, which takes into account both local gravity and the gravity of the universe. It takes into account both Kepler's law and the accelerated expansion of the universe.

15. Important consequences of the new law of universal gravitation.

The emergence of a new law of universal gravitation, which takes into account the effect of the accelerated expansion of the Universe and describes local systems more accurately than Newton's law, has important implications for the theory of gravity and our understanding of the Universe:

1. Rejection of dark matter and dark energy. The main consequence is that the hypotheses of dark matter and dark energy are no longer necessary [34]. The observed "anomalies" in the rotation of galaxies are a direct consequence of the new law of universal gravitation. The accelerated expansion of the Universe itself is explained by an inherent property of gravitational interaction on large scales, without invoking mysterious "dark energy."

2. Changes in the foundations of physics and cosmology and a revision of the cosmological model. A deeper understanding of the nature of gravity itself, possibly considering it an emergent phenomenon.

3. Impact on local calculations and the accuracy of orbital calculations: Even in the Solar System, where the expansion effect is minimal, the new law predicts small but measurable deviations in the planetary orbits that can be detected by modern high-precision instruments. For example, for Earth, the law gives the magnitude of the additional acceleration, which is $A_0 = c^2\sqrt{\Lambda} = 10.4922 ... \cdot 10^{-10}$ ms^{-2}.

16. The gravitational constant G is a composite constant.

Since the law of universal gravitation can be represented without the constant G, this calls into question its fundamental status. The fundamental status of the constant G has long been questioned by many scientists [17-21]. However, I was unable to find rigorous justification for such assertions in published sources. Duff and Veneziano, in their dispute with Okun, also limited themselves to assertions and did not provide evidence that G is not a fundamental constant [17].

The main justification for the non-fundamental nature of the constant G is the new law of universal gravitation (Fig. 3), which does not require the constant G. The absence of the constant G in the law of universal gravitation does not mean that the constant G becomes "completely unnecessary." It is very necessary in cosmology for calculating the parameters of the universe. Even the currently known value of $G = 6.67430...\cdot 10^{-11}$ kg^{-1}m^{3}s^{-2} allows us to calculate the parameters of the universe with "unprecedented" accuracy (5-6 digits!) [38].

Further improvement in the accuracy of the constant G will provide even greater accuracy for the parameters of the universe. Therefore, solving the problem of the accuracy of the constant G remains relevant in cosmology. Despite the fact that the law of universal gravitation can be imagined without the constant G, we have not lost interest in the constant G in cosmology.

Here we present a justification for the fact that the gravitational constant G is not independent. We show that G is a composite constant. To prove this, we use the well-known cosmological equations of Milne and Blakeslee, which include the parameters of the Universe and the constant G [35, 36].

Milne proposed the following equation in 1936:

$$Mu = c^3Tu/G \qquad (2)$$

where: Mu is the mass of the universe; c is the speed of light; Tu is the time of the universe; G is the gravitational constant.

Bleksley proposed the following equation in 1951:

$$Mu = c^2Ru/G \qquad (3)$$

where: Mu is the mass of the universe; c is the speed of light; Ru is the radius of the universe; G is the gravitational constant.

Equations (2) and (3) show the relationship between the parameters of the universe and the gravitational constant G. H. Weyl was the first to point out in 1918 that the parameters of the universe and the parameters of the electron are related [37]. According to H. Weyl, the coupling coefficients are large numbers. He proposed the following approximate formula for the radius of the universe:

$$Ru \approx 10^{40}r_e \qquad (4)$$

where: Ru is the radius of the universe; r_e is the classical radius of the electron.

In [38], we presented precise formulas for the relationship between the parameters of the universe and the parameters of the electron. In [38], we showed that the parameters of the universe are related to the parameters of the electron as follows:

$$Mu = m_e\alpha D_0^2 \qquad (5)$$
$$Ru = r_e\alpha D_0 \qquad (6)$$
$$Tu = t_e\alpha D_0 \qquad (7)$$
$$\Lambda = 1/r_e^2\alpha^2D_0^2 \qquad (8)$$

where: Mu is the mass of the universe; Ru is the radius of the universe; Tu is the time of the universe; Λ is the cosmological constant; r_e is the classical radius of the electron; m_e is the electron mass; $t_e = r_e/c$; α is the fine structure constant; D_0 is the large Weyl number ($D_0 = 4.16561... \times 10^{42}$).

Substituting Mu, Ru, and Tu from formulas (5) – (7) into the Milne and Blackley formulas, we obtain an expression for the gravitational constant G [39, 40]:

$$G = c^2Ru/Mu = c^2r_e/m_eD_0 = r_e^3/t_e^2m_eD_0 \qquad (9)$$
$$G = c^3Tu/Mu = c^3t_e/m_eD_0 = r_e^3/t_e^2m_eD_0 \qquad (10)$$

Thus, the gravitational constant G is a combination of the electron constants (Fig. 6) [39, 40]:

$$G = r_e^3/t_e^2m_eD_o$$

Fig. 6. The gravitational constant G is a composite constant. Where r_e is the electron radius; $t_e = r_e/c$; m_e is the electron mass; D_0 is the large Weyl number ($D_0 = 4.16561... \times 10^{42}$).

Thus, the gravitational constant G is not primary and is not an independent constant. It is a composite constant. In Newton's formula, it is a dimensionality adjustment coefficient.

17. How can the constant G be expressed in terms of the fundamental constants $\hbar$, α, e, m_e, and r_e, c?

Problems associated with measuring the gravitational constant G [41, 42] compel the search for methods for analytically calculating it. Attempts to theoretically derive the gravitational constant G from the fundamental constants have been undertaken for decades. The main goal of such work is to move beyond the problems of empirically measuring G and find a method for its analytical calculation.

Above, we showed that the constant G can be decomposed into simpler components. This means that the constant G is not a fundamental "building block" of physical laws, but is derived from other known quantities. The formula for the gravitational constant G shown in Fig. 6 is not the only one. Below, I present three more formulas using the fundamental constants $\hbar$, α, e, m_e, and r_e, c.

The gravitational constant G can be expressed in terms of the Planck constant $\hbar$ and the electron parameters:

$$G = r_e^5/t_e^3\hbar\alpha D_o \qquad (11)$$

The gravitational constant G can be expressed in terms of the Planck constant $\hbar$, the speed of light c, and the electron mass:

$$G = \hbar\alpha c/m_e^2 D_o \qquad (12)$$

The gravitational constant G can be expressed in terms of the electron charge e:

$$G = 4\pi\varepsilon_0 r_e^6/t_e^4 e^2 D_o \qquad (13)$$

18. Four equivalent formulas for the gravitational constant G.

Formulas (9) - (13) are equivalent (Fig. 7). Despite the fact that the constant G is represented by different fundamental constants, all formulas yield the same value for G.

$$G = r_e^3/t_e^2 m_e D_o$$

$$G = r_e^5/t_e^3\hbar\alpha D_o$$

$$G = \hbar\alpha c/m_e^2 D_o$$

$$G = 4\pi\varepsilon_0 r_e^6/t_e^4 e^2 D_o$$

Fig. 7. Four equivalent formulas for the gravitational constant G represented by the fundamental constants $\hbar$, α, e, m_e, r_e, and c. Where re is the classical radius of the electron; $t_e = r_e/c$; m_e is the electron mass; c is the speed of light; $\hbar$ is Planck's constant; α is the fine structure constant; e is the electron charge; and D_0 is the large Weyl number.

The composite nature of the constant G (Fig. 7) opens the possibility of its analytical calculation, since G is a derivative of other constants and can be calculated from other, more basic quantities.

19. Solving the circular problem associated with calculating the gravitational constant G.

All equivalent formulas for calculating the gravitational constant G (Fig. 7) contain the dimensionless constant D_0—a large Weyl number. A large Weyl number D_0 allows one to resolve a long-standing circular problem associated with calculating the gravitational constant G. In this circular problem, the gravitational constant G as an object of study is confirmed by the fact that it itself requires confirmation by this object [43, 44].

To avoid this circularity, it is necessary to calculate the value of D_0 without using the gravitational constant G. A large Weyl number D_0 can be obtained not only in gravity (as the ratio of the electric force to the gravitational force). There is another method for obtaining the large Weyl number D_0. The value of D_0 can be obtained from constants of the microcosm. The origins of the dimensionless constant D_0 should be sought not in the dimensional constant G, but in dimensionless fundamental constants such as the fine structure constant "alpha" and the ratio of elementary particle masses [40, 45, 46, 47, 48]. First and foremost, this is the dimensionless constant m_p/m_e: the proton-electron mass ratio. This constant is known with high precision: m_p/m_e = 1836.152 673 426...

My early research [40, 45, 46, 47, 48] demonstrated how to obtain the large Weyl number D0 from the constant m_p/m_e = 1836.152 673 426... and other elementary particle mass ratios. This eliminates the circular problem associated with calculating the gravitational constant G.

20. The refined value of the Large Weyl number D_0 and the calculated value of the gravitational constant G.

The method of obtaining the large Weyl number D_0 from the ratios of the masses of elementary particles allows us to approximate the accuracy of the number D_0 to the accuracy of the fine structure constant "alpha" and obtain 8-9 digits for D_0: D_0 = 4.16650364... x 10^{42} [40, 45, 46, 47, 48] (Fig.8).

$$D_0 = 4{,}16650364...\bullet 10^{42}$$

Fig.8. Refined value of the Large Weyl number D_0.

The value D_0 = 4.16650364... x 10^{42}, when substituted into formulas (9) - (13), yields the following calculated value for the composite constant G: G = 6.6728674...•10^{-11}kg^{-1}m^{3}s^{-2} (Fig.9).

$$G =$$
$$= r_e^3/t_e^2 m_e D_0 = r_e^5/t_e^3 \hbar \alpha D_0 = \hbar \alpha c/m_e^2 D_0 = 4\pi\varepsilon_0 r_e^6/t_e^4 e^2 D_0 =$$
$$= 6.6728674...\bullet 10^{-11}kg^{-1}m^3s^{-2}$$

Fig.9. Calculated value of the gravitational constant G.

The large Weyl number $D_0 = 4.16650364... \times 10^{42}$ becomes a theoretical "standard" and allows us to break the vicious circle in the circular problem associated with calculating the gravitational constant G.

Measurements of the constant G made by different teams give values from 6.6757×10^{-11} $kg^{-1}m^3s^{-2}$ to $6.6719 \times 10^{-11} kg^{-1}m^3s^{-2}$ [49, 50]. Comparison of the calculated value of $G = 6.6728674... \cdot 10^{-11} kg^{-1}m^3s^{-2}$ with the measured values shows that the calculated value of G falls within the range of measured values and is at the lower end of the range of measured values.

21. The cosmological constant Λ in the new law of universal gravitation as a quantitative measure of the gravitational force of the Universe.

For the first time, the constant Λ was introduced into the law of gravitation as a fundamental constant, replacing the constant G. The complete law of gravitational force (Fig. 3) takes into account both local gravity and the accelerated expansion of the Universe. The new law of universal gravitation (Fig. 3) is represented by observable parameters: the orbital radius R, the period T, and the cosmological constant Λ. Using the cosmological constant Λ in the law of universal gravitation allows us to integrate the "cosmological factor"—the influence of the entire mass of the Universe on the motion of individual bodies—into classical mechanics. The cosmological force is represented by the second term in Fig. 3. This force describes the gravitational effect of the total mass of the Universe on a specific body. Unlike the Newtonian force, it does not depend on the distance between two objects and does not obey the inverse-square law.

The new law of universal gravitation combines local (Keplerian) motion and the global expansion of the Universe into a single formula, where the role of the fundamental constant shifts from G to observable parameters and the cosmological constant Λ.

The parameter Λ is associated with the cosmological acceleration $A_0 = c^2 \sqrt{\Lambda}$, whose value is approximately 10.49×10^{-10} m/s². This acceleration, which is caused by the cosmological force, was not taken into account in classical Newtonian dynamics.

22. A method for obtaining the value of the cosmological constant Λ without using the vacuum energy density.

The law of universal gravitation (Fig. 3) includes the cosmological constant Λ. Its value must be known with high accuracy. The observed value: $\Lambda \approx 1.1 \times 10^{-52} m^{-2}$ ($\Lambda = 1.089(29) \times 10^{-52}$ m^{-2} [51]) has very low accuracy, even worse than the constant G. It is known that attempts to derive Λ in quantum theory have yielded no results. It is known that the method of calculating the cosmological constant Λ using the vacuum energy density leads to a discrepancy between theory and experiment of 120 orders of magnitude (10^{120}). This discrepancy is known as the cosmological constant problem or the "vacuum catastrophe". Therefore, both a more precise observed value and an accurate calculated value are relevant.

Here we demonstrate how to obtain the value of the cosmological constant Λ with high accuracy without using the vacuum energy density. We will use the cosmological equations that we previously obtained in [52, 53]. The cosmological equations include the parameters of the Universe. Cosmological equations, combined into systems of cosmological equations, open up a new method for calculating the value of the cosmological constant Λ without using the vacuum energy density.

We will demonstrate a new method for calculating the value of the cosmological constant Λ using several systems of cosmological equations. To calculate the value of the cosmological constant Λ, we will use the four previously obtained systems of cosmological equations [52, 53] (Fig. 10). We will also use the calculated value of the gravitational constant $G = 6.6728674...\cdot10^{-11}kg^{-1}m^3s^{-2}$ (Fig. 9).
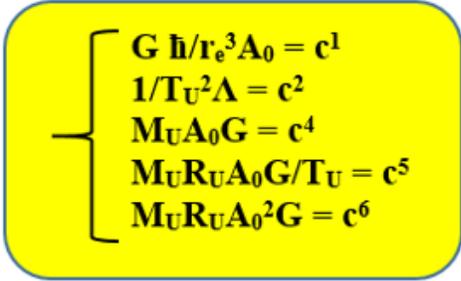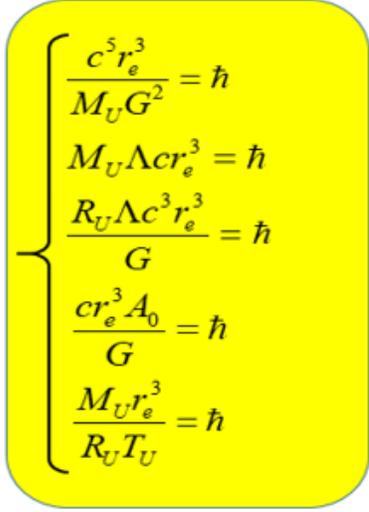
a)
$$\begin{cases} G\,\hbar/r_e^3A_0 = c^1 \\ 1/T_U^2\Lambda = c^2 \\ M_U A_0 G = c^4 \\ M_U R_U A_0 G/T_U = c^5 \\ M_U R_U A_0^2 G = c^6 \end{cases}$$
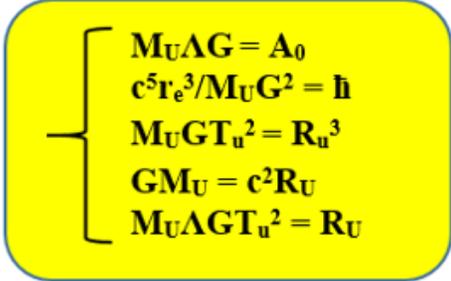
b)
$$\begin{cases} M_U\Lambda G = A_0 \\ c^5 r_e^3/M_U G^2 = \hbar \\ M_U G T_u^2 = R_u^3 \\ G M_U = c^2 R_U \\ M_U \Lambda G T_u^2 = R_U \end{cases}$$

c)
$$\begin{cases} \dfrac{c^5 r_e^3}{M_U G^2} = \hbar \\ M_U\Lambda c r_e^3 = \hbar \\ \dfrac{R_U\Lambda c^3 r_e^3}{G} = \hbar \\ \dfrac{c r_e^3 A_0}{G} = \hbar \\ \dfrac{M_U r_e^3}{R_U T_U} = \hbar \end{cases}$$
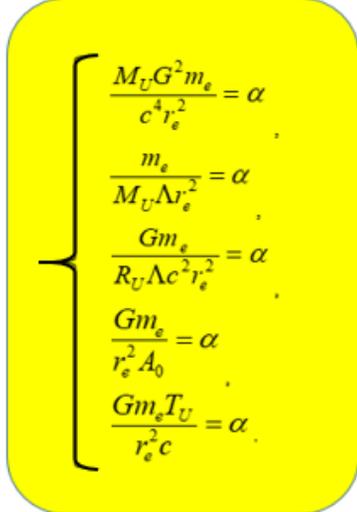
d)
$$\begin{cases} \dfrac{M_U G^2 m_e}{c^4 r_e^2} = \alpha \\ \dfrac{m_e}{M_U\Lambda r_e^2} = \alpha \\ \dfrac{G m_e}{R_U\Lambda c^2 r_e^2} = \alpha \\ \dfrac{G m_e}{r_e^2 A_0} = \alpha \\ \dfrac{G m_e T_U}{r_e^2 c} = \alpha \end{cases}$$

Fig. 10. Systems of cosmological equations for calculating the parameters of the observable Universe. Where: α is the fine structure constant, $\hbar$ is Planck's constant, M_U is the mass of the observable Universe, G is Newton's gravitational constant, Λ is the cosmological constant, R_U is the radius of the observable Universe, T_U is time, A_0 is the cosmological acceleration, r_e is the classical radius of the electron; c is the speed of light in a vacuum; m_e is the mass of the electron.

The systems of cosmological equations (Fig. 10) are systems of algebraic equations. The solutions are the parameters of the Universe, including the cosmological constant Λ. All four systems of cosmological equations (Fig. 10) yield the same value of the cosmological constant $\Lambda = 1.36226697...\cdot10^{-52}$ m^{-2}. This is not the only method for calculating the cosmological constant Λ without using the vacuum energy density. The same value of the cosmological constant $\Lambda = 1.36226697...\cdot10^{-52}$ m^{-2} is given by formula (8) (Fig. 11).

$$\Lambda = 1/r_e^2\alpha^2D_0^2 = 1.36226697... \cdot 10^{-52} \text{ m}^{-2}$$

Fig. 11. Calculated value of the cosmological constant Λ without using the vacuum energy density.

The calculated value of the cosmological constant $\Lambda = 1.36226697... \cdot 10^{-52}$ m^{-2} is very close to the observed value: $\Lambda \approx 1.1 \times 10^{-52}$ m^{-2} ($\Lambda = 1.089(29) \times 10^{-52}$ m^{-2} [51]). As we can see, the calculated value of the cosmological constant Λ is not burdened by the problem of discrepancies between theory and experiment. In quantum field theory, the difference between theory and observations is 120 orders of magnitude (10^{120}).

While there is complete agreement in order of magnitude (10^{-52}), there is a discrepancy in the second digit of 2 units. Due to the low accuracy of the observed value, this discrepancy is difficult to explain. The discrepancy may be due to the fact that the experimental value of the cosmological constant $\Lambda \approx 1.1 \times 10^{-52}$ m^{-2} ($\Lambda = 1.089(29) \times 10^{-52}$ m^{-2}) was obtained using the averaged value of the constant G, which is included in the vacuum energy density formula.

Therefore, in the law of universal gravitation without the constant G (Fig. 3), the exact calculated value of the cosmological constant $\Lambda = 1.36226697... \cdot 10^{-52}$ m^{-2} can be used. This allows the cosmological force ($F_{Cos} = mc^2\sqrt{\Lambda}$) to be determined with high accuracy.

23. Gravitational interpretation of the rest energy E=mc2 given by Erwin Schrödinger.

The law of universal gravitation without the constant G ($F_U = mR^3/T^2r^2 + mc^2\sqrt{\Lambda}$) has a two-component structure. The second component of the gravitational force is represented by the cosmological force $F_{Cos} = mc^2\sqrt{\Lambda}$. The cosmological force F_{Cos} is represented by the rest energy mc^2 and the cosmological constant Λ.

To explain this unification of the rest energy mc^2 and the cosmological constant Λ in a single equation, I refer to Schrödinger's paper [54], which is one of the key works that anticipated modern cosmological ideas. In this paper, Schrödinger emphasizes the fundamental connection between the local and the global in the universe. In this paper, Schrödinger showed that the combined influence of all the masses in the universe manifests itself at the local level. Schrödinger showed that the rest energy $E = mc^2$ can be interpreted as the potential energy of the interaction of a particle with all other masses in the universe.

Schrödinger also presented the gravitational constant G as related to the parameters of the Universe: $G \approx c^2 Ru/Mu$. In his article [54], Schrödinger considered the rest energy of a particle as the potential energy in the gravitational field of all other masses in the Universe.

24. Conclusion

Newton's law is insufficient to describe gravity. For over 300 years, a law of gravity that simultaneously accounts for the gravity of two bodies and all other bodies in the universe remained undiscovered. To obtain a new law of gravity, the inverse N-body problem was formulated and solved.

The solution to the inverse N-body problem for N = 2 and for N → ∞ yields a new law of gravitation $F_U = mR^3/T^2r^2 + mc^2\sqrt{\Lambda}$, which takes into account the accelerated expansion of the Universe. The total force of universal gravitation consists of two components: the force of gravitational interaction of two bodies $F_{H-K} = mR^3/(T^2)r^2$ and an additional cosmological force $F_{Cos} = (mc^2)\sqrt{\Lambda}$. A special feature of the new law of universal gravitation is that it does not include the gravitational constant G and mass M. The new law of gravitation includes parameters available from observations. These are the parameters of the planets' orbits (R and T) and the cosmological constant Λ. The new law of gravitation shows that in addition to the force of gravitational interaction between two bodies, the bodies are acted upon by the cosmological force $F_{Cos} = mc^2\sqrt{\Lambda}$ of the universe. The presence of the speed of light in the new law of gravitation together with the cosmological constant Λ represents a relativistic version of the law of universal gravitation.

25. Conclusions

1. For over 300 years, the force of gravitational interaction was described by a single physical law – Newton's law $F = GMm/r^2$. Here we show that Newton's law $F = GMm/r^2$ is not the only law of universal gravitation.

2. In addition to Newton's law $F = GMm/r^2$, there is a second law of gravitation $F = mR^3/T^2r^2$, which remained undiscovered. It does not include the gravitational constant G. The existence of this law was first pointed out by Robert Hooke in 1679.

3. With the discovery of the law of two-body gravitation $F_{H-K} = mR^3/T^2r^2$, which does not include the gravitational constant G, Newton's law of universal gravitation $F = GMm/r^2$ loses its unique status.

4. The third law of gravity, $F_{Cos} = mc^2\sqrt{\Lambda}$, describes the additional gravitational force exerted by all bodies in the universe. This is a new law of gravity. It complements Newtonian dynamics and completes the classical theory of gravity. It reveals an additional gravitational force that always exists, but Newton's law "doesn't see" it.

5. The appearance of the cosmological constant Λ in the third law of gravity shows that, in addition to the gravitational force of two bodies, these bodies are acted upon by the cosmological force $F_{Cos} = mc^2\sqrt{\Lambda}$ of the universe.

6. Combining the two new laws of gravity into a single equation yields the law of universal gravitation $F_U = mR^3/T^2r^2 + mc^2\sqrt{\Lambda}$.

7. The new law of universal gravitation $F_U = mR^3/T^2r^2 + mc^2\sqrt{\Lambda}$ has two components: the two-body law of gravity $F_{H-K} = mR^3/T^2r^2$ and the law of cosmological force $F_{Cos} = mc^2\sqrt{\Lambda}$.

8. The inverse N-body problem was formulated and solved. The inverse N-body problem is the problem of determining the law of gravitational force based on the integral characteristics of a system of N bodies. Unlike the direct N-body problem, which seeks to predict the future positions of the bodies, the inverse problem aims to derive the underlying law of force action.

9. The new law of gravity $F_U = mR^3/T^2r^2 + mc^2\sqrt{\Lambda}$ was obtained as a solution to the inverse N-body problem for N = 2 and for N → ∞.

10. All solutions to the N-body inverse problem are given in [30, 31]. Among the solutions to the N-body inverse problem, the most interesting is the solution in the form: $F_U = mR^3/T^2r^2 + mc^2\sqrt{\Lambda}$.

11. The new law of gravitation explains the shift in the perihelion of planets, indicates the gravitational nature of the Pioneer anomaly, and explains the rotation curve of galaxies without using the concept of dark matter.

12. The new law of universal gravitation $F_U = mR^3/T^2r^2 + mc^2\sqrt{\Lambda}$ is more accurate and perfect than Newton's law $F = GMm/r^2$, since it provides a more complete description of gravity. It does not include the gravitational constant G and the mass M. The orbital parameters (R and T) are known with greater accuracy than the constant G and the mass M. The measured value of R^3/T^2 based on observations of the orbits of the planets of the Solar System is: $R^3/T^2 = 132\ 712\ 440\ 018\ (8)\ km^3\ s^{-2}$. This is much more accurate than the constant G. The low accuracy of the gravitational constant G ceases to be a limiting factor in gravitation.

13. A new method for obtaining the value of the cosmological constant Λ without using the vacuum energy density is proposed. The calculated value of the cosmological constant $\Lambda = 1.36226697... \cdot 10^{-52}\ m^{-2}$.

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