LAW OF UNIVERSAL GRAVITATION WITHOUT GRAVITATIONAL CONSTANT G.

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Abstract. *For over 300 years, the force of gravitational interaction was represented by a single physical law – Newton's law F = GMm/r^2. The possibility of the existence of a law of gravity other than Newton's law* $F = GMm/r^2$ *was not even considered. Excessive idealization of Newton's law* $F = GMm/r^2$ *and overestimation of its uniqueness became a brake on the development of the theory of gravity. Newton's law of gravitation does not provide a complete and accurate value for the gravitational force. It describes the local gravity of two bodies and "does not see" the additional gravitational force that actually exists as a result of the gravitational action of all bodies in the universe. Here we show that in addition to Newton's law F = GMm/r^2, there is a second law of gravitation F = mR^3/(T^2)r^2, which remained undiscovered. This law of gravity does not require the gravitational constant G. The existence of this law was first pointed out by Robert Hooke in 1679. Furthermore, we show that the additional gravitational force of the gravitational action of all bodies in the Universe is described by the third law of gravitation* $F = (mc^2)\sqrt{\Lambda}$*, which also remained undiscovered. This law of gravity also does not require a gravitational constant G. Combining the two new laws of gravitation into a single equation yields the law of universal gravitation* F_U *= mR^3/(T^2)r^2+(mc^2)*$\sqrt{\Lambda}$*. This is a complete and more accurate law of gravity than Newton's law. It replaces Newton's law of gravity. The new law of universal gravitation takes into account the accelerated expansion of the universe and Kepler's laws of planetary motion. It expresses the total force of universal gravitation, which is represented by both the local gravitational force of two bodies and the gravitational force of all N bodies in the universe. The new law of universal gravitation is a solution to the inverse N-body problem for* $N = 2$ *and for* $N \to \infty$*. A distinctive feature of the new law of universal gravitation is that it does not include the gravitational constant G. The low accuracy of the gravitational constant G ceases to be a limiting factor in gravitation.*

Keywords: *Newton's law; N-body problem; law of universal gravitation; parameters of the observable universe; dark matter; galaxy rotation curve; cosmological constant* Λ*.*

1. Introduction

Newton's law of universal gravitation $F = GMm/r^2$ is one of the greatest scientific discoveries. It is distinguished by its simplicity and mathematical perfection. Despite the fact that the law of gravitation was discovered more than 300 years ago, gravity remains the most mysterious physical phenomenon in physics and cosmology. All this time, the force of gravitational interaction was represented only by Newton's law of universal gravitation. For many practical applications, this law was sufficient. However, the predictions of Newtonian gravity fail on the scale of the universe. Newton's law shows the force of gravitational interaction of two bodies out of all N bodies in the universe. The formula for Newton's law describes the gravity of only one local source of attraction and does not take into account that bodies are simultaneously attracted to all other bodies in the universe. In reality, all bodies in the universe participate in gravitational interaction. This is not taken into account in Newton's law of universal gravitation. Newton's law does not work on the scale of the universe [1 - 9].

Newton's law of gravity does not "see" the total force that acts in the universe, but "sees" only a part of the gravitational force. Newton's law does not take into account the accelerated expansion of the universe. Newton's law does not include the parameters of the Keplerian orbit. An accurate and complete law of gravity must take into account the gravity of all N bodies in the universe. In gravity, the main fundamental law of nature remained undiscovered: the law of gravitational interaction of all N bodies in the universe. The obstacle on this path was the unsolvable N-body problem.

In order to eliminate the imperfection of Newtonian dynamics, many attempts were made to modify Newton's law [10, 11, 12]. These attempts did not lead to the discovery of a more perfect law of gravity than Newton's law. The law of gravity that takes into account the gravity of all bodies in the universe cannot be obtained by modifying and changing Newton's law of universal gravitation for the reason that Newton's law is the law of gravitational interaction of two bodies. No amount of cosmetic editing can transform the two-body law of gravitational interaction into an N-body law of gravitational interaction.

Newton's law faces an insurmountable N-body problem, requiring approximations or computational solutions for many pairs of bodies. In essence, a more accurate law of gravitation should explain the observed gravitational phenomena at all scales, including those that are currently not explained by Newton's law.

Obviously, the N-body law of gravitational interaction is not a modified two-body law of gravitation, but a new law of gravitation. It must be a new fundamental law of gravitation, complementary to the two-body law of gravitation. The additivity of gravitational forces makes it possible to represent the complete law of universal gravitation as a sum of forces: the gravitational forces of a finite number of pairs of bodies and the cosmological force of all N bodies in the Universe.

2. About theories of gravity.

Newton did not develop or use the theory of gravity in his research. This did not prevent him from discovering the fundamental law of Nature. There are now many theories of gravity, the purpose of which is to improve Newtonian dynamics, but they have not added a more perfect physical law to Newtonian dynamics than Newton's law of gravity. The abundance of theories of gravity (Metric theories, Non-metric theories, Vector theories, Scalar-tensor theories, Le Sage theory, Theories of quantum gravity, supergravity, string theory...) have not led to a better law of gravitational force than Newton's law [13]. These theories of gravity have not discovered a single new law of nature. All theories of gravity have proven powerless against the N-body problem. It is obvious that the approach "from the theory of gravity to the law of gravity" is not productive. The fact that Newton discovered the law of gravity without any theory shows that physical theories are not a tool for discovering the laws of Nature. The laws of Nature are primary, and theories have a secondary status. It is necessary to look for another approach to discover the law of gravitational interaction, taking into account the gravity of all bodies in the Universe. Here we propose a new approach to solving the N-body problem, which provides the key to discovering the laws of gravity.

3. Elimination of problems related to the accuracy of the constant G, at the cost of abandoning the constant G in gravity.

For over 300 years, the force of gravitational interaction has been represented by a single physical law—Newton's law of gravitation, F = GMm/r2. This law of gravitation includes the constant G, which is considered a fundamental constant. Newton's law of gravitation does not provide a precise value for the gravitational force. It "sees" only the gravitational force between two bodies.

Furthermore, the accuracy of the constant G is the lowest among the fundamental physical constants. This creates numerous problems in physics and cosmology. The low accuracy of G complicates the accurate modeling of gravitational interactions in space, which is critical for calculating the orbits of planets, stars, and galaxies, as well as for understanding the evolution of the Universe. The low accuracy of G becomes an obstacle to testing and refining fundamental physical theories and searching for deviations from them.

Many theorists suggest that creating a consistent theory of gravity will require a rethinking of the very nature of gravity and, possibly, the status of G as a fundamental constant [14]. In [15, 16] it is shown that many gravitational predictions and measurements can be made without any knowledge of the gravitational constant G. In this case, is the constant G fundamental? The fundamental status of the constant G has long been questioned due to the difficulties with its precise measurement and the many problems that arise from its low accuracy. [17 - 21] Perhaps the reason for the overestimation of the role and overstated status of the constant G was the fact that Newton's law of gravitation was the only law in classical gravitation. This fueled the illusion of the uniqueness of the constant G. Problems associated with the accuracy of the constant G can be solved at the cost of abandoning G in gravitation. Rejecting the concept of the constant G leads to a new law of gravitation, more perfect than Newton's law.

If G is not a fundamental constant, this could lead to a revision of our understanding of gravity. The non-fundamental status of the constant G may indicate the non-fundamental status of Newton's law of gravity.

4. Instead of a modified Newtonian law of gravitation, a new law of gravitation without the constant G.

To go beyond Newtonian dynamics, one must abandon the gravitational mass "M" as a parameter in the law of gravitation. This is the key to discovering a new law of gravitation. Rejecting the gravitational mass "M" makes it possible to abandon the very concept of G in gravitation. The absence of G in the law of gravitation eliminates many problems caused by the low accuracy of the gravitational constant G.

Numerous attempts have been made to modify Newton's law [3, 12, 22, 23, 24, 25]. However, the modified equations remained equations of Newtonian dynamics and did not lead to a new law of gravitation. In theories where the constants are equal to unity ($G = c = \hbar = 1$), G becomes dimensionless, and the equations of gravitation are simplified. But this is merely a change in the system of units, not a rejection of the very concept of the gravitational constant. This is not a removal of the essence of the gravitational constant G, but an attempt to conceal it by making it a dimensionless unit. These are alternatives of the type: not quite "G-free". According to Espen Gaarder Haug, eliminating the gravitational constant G requires a different definition of energy and mass [22].

Thus, within the framework of Newtonian dynamics, it is impossible to completely eliminate G, since it is necessary for the quantitative description of gravitational interaction. Throughout the history of physics, there has been no deliberate attempt to derive a different physical law of gravitational force without the constant G, one more accurate than Newton's law. A physical law of gravitational force different from Newton's law must be sought beyond Newtonian dynamics, beyond Newton's law. The goal is not to hide the constant G in an artificially introduced system of units, but to completely eliminate it from the law of gravitation. I argue that this cannot be accomplished by modifying Newton's law. A new physical law of gravitational force is needed, one that does not require the constant G, and that provides a quantitative description of gravitational interaction.

5. NEW LAW OF GRAVITATION OF TWO BODIES WITHOUT CONSTANT G.

The law of two-body gravitation without the constant G was presented in [26]. Fig. 1 shows this law of gravitation.

$$F_{H-K} = \frac{mR^3}{T^2 r^2}$$

Fig. 1. Hooke-Kepler law of gravitation. Where: m is the mass of the body, R and T are orbit parameters, r is the distance.

This law is based on Kepler's law of planetary motion. The law of two-body gravitation without constant G is not burdened by the problems inherent in Newton's law of gravitation.

Robert Hooke first pointed out the existence of this law of gravitation in a letter to Newton in 1679. Scholars of Hooke's work have overlooked the fact that in his letter, Hooke provided a verbal formulation of the law of gravitation based on the elliptical orbits of the planets and Kepler's laws. It turns out that there are two laws of gravitation, both based on the inverse-square law. Newton's law is only one of them. Hooke's verbal formulation led to a more precise and perfect law of gravitation than Newton's. If Newton had accepted Hooke's proposal, we would already be celebrating the 345th anniversary of this remarkable law of two-body gravitation, which originally did not require a constant G.

If the central mass "M" is used as a parameter in the law of gravitation (as in Newton's law), then it is impossible to eliminate the constant G. Conclusion: eliminating the constant G from the law of gravitation is possible only by replacing Newton's law with a new law of gravitation. It is necessary to derive a law of gravitation that does not include the central (attracting) mass.

6. THE LAW OF THE GRAVITATIONAL FORCE OF THE UNIVERSE WITHOUT THE CONSTANT G.

The Hooke-Kepler law describes local gravity quite accurately without using the constant G. To obtain the total gravitational force, it is necessary to take into account the gravitational force of all other bodies in the universe. The gravitational force of the universe is represented by an additional cosmological force (Fig. 2).

$$F_{Cos} = mc^2\sqrt{\Lambda}$$

Fig. 2. The law of cosmological force. Where m is the mass of an object, c is the speed of light, and Λ is the cosmological constant.

Instead of the constant G, the new law of gravity includes the cosmological constant Λ. This law of gravity takes into account the accelerated expansion of the universe.

7. A new law of universal gravitation without the constant G.

The law of universal gravitation, taking into account the accelerated expansion of the Universe and Kepler's laws of planetary motion, is as follows:

$$F_U = \frac{mR^3}{T^2 r^2} + mc^2\sqrt{\Lambda}$$

Fig. 3. The law of universal gravitation, taking into account the accelerated expansion of the Universe and Kepler's laws of planetary motion. Where: m is the mass of the body, R and T are orbit parameters, r is the distance, c is the speed of light in vacuum, Λ is the cosmological constant.

The gravitational force F_U consists of two components: the local force $F_{H\text{-}K} = mR^3/T^2r^2$ and the cosmological force $F_{Cos} = mc^2\sqrt{\Lambda}$. The law of universal gravitation does not include the constant G. This law of gravitation is superior to Newton's law. The main drawback of Newton's law is that it "sees" only the local force. Moreover, the accuracy of Newton's law is limited by the low accuracy of the constant G.

The law of gravitation (Fig. 3) is the solution to the inverse N-body problem for N = 2 and for $N \to \infty$. This equation is of greatest interest as the most accurate and perfect law of gravitation. The new law of gravitation shows the total force of universal gravitation taking into account the gravitational interaction of all N bodies in the Universe. The total force of universal gravitation consists of two components: the force of gravitational interaction of two bodies $F_{H\text{-}K} = mR^3/T^2r^2$ (Hooke-Kepler law [26]) and an additional cosmological force $F_{Cos} = mc^2\sqrt{\Lambda}$. The peculiarity of the new law of universal gravitation is that it does not include the gravitational constant G and the mass M. This is a good sign. The low accuracy of the gravitational constant G ceases to be a limiting factor in gravity. The new law of gravity includes parameters available from observations. These are the parameters of the orbits of the planets (R and T) and the cosmological constant Λ.

Equation (Fig. 3) is not a modified Newtonian law formula; it is a new law of gravity. It opens up new knowledge in the field of gravity. The new law of gravity challenges the dominant paradigm in physics and cosmology.

The new law of gravitation shows that in addition to the force of gravitational interaction between two bodies, all bodies are affected by the cosmological force $F_{Cos} = mc^2\sqrt{\Lambda}$ of the universe. The total force of universal gravitation is represented by the vector sum of two forces: the force of gravitational interaction of two bodies FH-K and the cosmological gravitational force F_{Cos}:

$$\vec{F}_U = \vec{F}_{H-K} + \vec{F}_{Cos} \qquad (1)$$

The value of the resulting force of universal gravitation is in the range of values from $F_U = mR^3/T^2r^2 - mc^2\sqrt{\Lambda}$ to $F_U = mR^3/T^2r^2 + mc^2\sqrt{\Lambda}$. For collinear force vectors, the equation (Fig. 3) takes the form: $F_U = mR^3/T^2r^2 \pm mc^2\sqrt{\Lambda}$.

7.1. Two-body gravity + Universe gravity.

Newton's law $F = GMm/r^2$ "sees" the gravitational force of two bodies. The new law of gravitation $F_U = mR^3/T^2r^2+mc^2\sqrt{\Lambda}$ "sees" two forces. The first term gives the gravitational force of two bodies. The second term gives the gravitational force of the universe. The total force of gravitational interaction is the sum of the two forces. Fig. 4 conventionally shows the contribution of the cosmological force $F_{Cos} = mc^2\sqrt{\Lambda}$ to the Galaxy rotation curve.

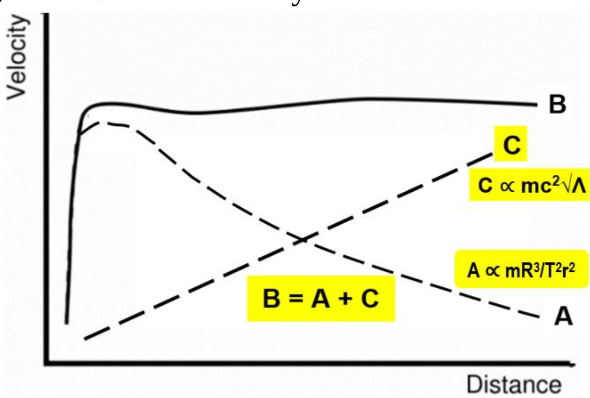

Fig. 4. Galaxy rotation curve (B) as a result of the action of two forces: B = A + C.

The law of universal gravitation $F_U = mR^3/T^2r^2+mc^2\sqrt{\Lambda}$ provides a more complete description of gravitational interactions at different scales. It combines the Hooke-Kepler law and the cosmological force law. This extension is especially relevant when considering the large-scale structure of the universe.

7.2. Accuracy and Perfection

The new law of gravitation $F_U = mR^3/T^2r^2+mc^2\sqrt{\Lambda}$ is more accurate and perfect than Newton's law F = GMm/r2, since it provides a more complete description of gravity, especially on cosmological scales. It takes into account the influence of all bodies in the Universe, not just pairs of bodies. It does not include the gravitational constant G and the mass M. The new law of gravitation is more accurate than Newton's law, since the orbital parameters (R and T) are known with greater accuracy than the constant G and the mass M. With the advent of the law of gravitation $F_U = mR^3/T^2r^2 + mc^2\sqrt{\Lambda}$, the low accuracy of the gravitational constant G ceases to be a limiting factor in gravity.

8. Cosmological force

The relativistic version of the law of universal gravitation includes the cosmological force as an integral part. This is a gravitational force that depends on the cosmological constant Λ. The formula for this force is: $F_{Cos} = mc^2\sqrt{\Lambda}$, where m is the mass of the object, c is the speed of light, Λ is the cosmological constant. The cosmological force is the gravitational force with which the mass of the entire universe acts on the body, causing acceleration $A_0 = c2\sqrt{\Lambda} = 10.4922 \times 10^{-10}$ m/s². The cosmological force shows that the gravitational interaction of an object with the Universe, represented by the cosmological constant (Λ), generates a force proportional to the mass of the body. The cosmological force does not obey the inverse square law. The cosmological gravitational force is generated by the mass of all bodies in the Universe. This is different from Newton's law, which focuses on the interaction of two bodies. In this context, the cosmological constant Λ is a coupling constant that affects the magnitude of the cosmological force. The cosmological force arises from the interaction of the body with the general mass distribution of the Universe.

8.1. The gravitational force of the Universe.

The formula for the cosmological force Fig. 2 shows that this force is directly proportional to the mass of the object and depends on the cosmological constant, which is related to the expansion of the Universe. The formula includes the speed of light, which demonstrates the relativistic version of the law of universal gravitation. The law of the cosmological force explains observed anomalies such as the Pioneer anomaly, the perihelion shifts of planets, and the rotation curves of galaxies, which are not explained by existing gravitational models. The law of the cosmological force provides a basis for understanding gravitational interactions in the Universe, especially on cosmological scales.

8.2. The Cosmological Force Law as a Supplement to the Two-Body Gravity Law

The cosmological force law is not intended to replace the two-body gravity law, but to complement it by taking into account the influence of the Universe as a whole. In essence, the cosmological force law takes into account the influence of the Universe on gravitational interactions, potentially offering a more complete understanding of gravity on both small and large scales. The additional cosmological force causes an acceleration $A_0 = (c^2)\sqrt{\Lambda}$ close to the acceleration value ($a_0 = 1.2 \times 10^{-10}$ m/s²) predicted by MOND [3, 4].

9. Hooke-Kepler Law of Gravitation

The new version of the law of universal gravitation includes the Hooke-Kepler law of gravitation $F_{H\text{-}K} = mR^3/T^2r^2$ [26]. This law includes Kepler's laws of planetary motion and the inverse square law, emphasizing the relationship between the orbital parameters (radius R and period T) and the gravitational force. In one formula, both the inverse square law and Kepler's third law, which relates the orbital radius and period, are combined. The Hooke-Kepler law (Fig. 1) provides a more accurate and complete formula for the gravitational force of two bodies than the proportionality law $F_N \propto mM/r^2$ originally proposed by Newton.

9.1. Hooke's Contribution

Robert Hooke, in his correspondence with Newton (1679), emphasized the importance of the inverse square law and the ellipticity of planetary orbits for describing gravity. Robert Hooke relied on Kepler's laws. He argued that a complete understanding of gravity must take these factors into account. Robert Hooke had given a verbal formulation of the future law of gravity in a letter 7 years before Newton published Philosophiæ Naturalis Principia Mathematica. Newton did not accept Hooke's hint and did not include the parameters of Keplerian orbits in his law of gravity. Newton used masses. This was a different law of gravity than the one hinted at by Hooke. It is known that masses are not included in Kepler's laws at all. By rejecting Hooke's hint, Newton missed the opportunity back in 1679 to give the world a more precise law of gravity $F = mR^3/(T^2r^2)$ than the approximate law of proportionality $F_N \propto mM/r^2$, which he discovered 7 years later in 1687. It is known that in Newton's law of proportionality $F_N \propto mM/r^2$ the sign "=" ($F = GMm/r^2$) appeared only 200 years later (in 1873) with the appearance of the gravitational constant G in it [27, 28].

9.2. Relationship with Kepler's Constant

A more accurate and complete understanding of gravity can be achieved by considering elliptical orbits and the inverse square law proposed by Hooke, and by using the ratio (R^3/T^2) that follows from Kepler's law. The Hooke-Kepler law highlights the importance of Kepler's constant (R^3/T^2) in the context of the gravitational force [29].

9.3. Key Differences from Newton's Law

In essence, the Hooke-Kepler law of gravity highlights a different view of the relationship between gravity and orbital motion, highlighting the importance of Kepler's laws in understanding the force of gravity. While Newton's law focuses on the gravitational force between two masses ($F = GmM/r^2$), Hooke-Kepler's law $F_{H\text{-}K} = mR^3/(T^2r^2)$ emphasizes the relationship between the orbital parameters (radius and period) and the gravitational force. Hooke-Kepler's law does not include the mass M. It is known that Kepler's laws do not include masses at all.

9.4. Hooke-Kepler's Law - A New Law of Two-Body Gravitation

Hooke-Kepler's law of gravitation (Fig. 1) is a different formulation of the gravitational force between two bodies than Newton's law. This law of gravitation emphasizes the relationship between the orbital parameters (radius and period) and the gravitational force, offering a more accurate representation of gravity than Newton's law. It is derived by combining Hooke's idea of the inverse-square law with Kepler's laws of planetary motion. A detailed history of the Hooke-Kepler law of gravitation is given in [26].

10. Inverse N-body problem

The most important physical law of gravity $F_U = mR^3/T^2r^2 + mc^2\sqrt{\Lambda}$, which shows the force of gravitational interaction of all N bodies in the Universe, remained undiscovered in gravity. The main obstacle on this path was the N-body problem, which has no analytical solution for $N \geq 3$. To obtain the law of gravitational interaction of N bodies, the inverse N-body problem was formulated and solved in [30, 31]. The inverse N-body problem is the problem of determining the law of gravitational force based on the integral characteristics of the N-body system. Unlike the direct N-body problem, which seeks to predict the future position of the bodies, the inverse problem aims to derive the underlying law of action of forces. This problem has not been considered in physics before, possibly due to the lack of an analytical solution to the direct N-body problem.

10.1. The goal of the inverse N-body problem

The inverse N-body problem involves determining the law of gravitational force acting on a system of N bodies, given the integral properties of their motion. Essentially, instead of starting with a known gravitational force and calculating the resulting motion (as in the forward N-body problem), this problem starts with the observed motion and aims to derive the underlying law of force. Given the integral properties of the N-body system, the goal is to determine the law of action of the gravitational forces governing their interaction. The main task is to find a mathematical expression for the gravitational force that corresponds to the observed motion. Unlike the forward N-body problem, the inverse N-body problem has an analytical solution for both $N = 2$ and $N \rightarrow \infty$. For $N = 2$, the inverse problem leads to both Newton's law F = GMm/r2 and the Hooke-Kepler law ($F_{H\text{-}K} = mR^3/T^2r^2$). As N tends to infinity, it leads to the law of the cosmological force ($F_{Cos} = mc^2\sqrt{\Lambda}$). The inverse problem is solved using the integral parameters of the N-body system. Solutions to the inverse N-body problem directly lead to new laws of gravity that remained undiscovered for more than 300 years.

10.2. The Direct N-Body Problem

Knowing the masses, initial positions, and velocities of the N bodies, as well as the known law of force action (e.g., Newton's law of gravitation), the goal is to calculate their positions and velocities at all future moments of time. This problem has no analytical solution for $N \geq 3$.

10.3. The Mirror-symmetric approach to solving the inverse N-body problem.

The unsolvable direct N-body problem itself gave a hint for choosing a solution method: "If the N-body problem is unsolvable, then the mirror-symmetric problem can provide a solution." The idea of using the principles of opposition and inversion turned out to be productive. To solve the N-body gravity problem, a mirror-symmetric approach was used. Instead of the direct N-body problem, there is the inverse N-body problem. Instead of a set of individual bodies, there is one system consisting of N-bodies. Instead of the differential characteristics of individual bodies, there are integral characteristics of the N-body system. Instead of the trajectory of motion under the action of gravity, there is the law of gravity.

10.4. Instead of a set of individual bodies, there is one system consisting of N-bodies.

In the inverse N-body problem, the set of bodies is considered as a single object or one system of N bodies. The inverse N-body problem involves reducing the N-body problem to an equivalent two-body problem, where one of the bodies is a system of N bodies.

10.5. Integral characteristics of the N-body system instead of differential characteristics of bodies.

In the direct N-body problem, differential characteristics of bodies are used as parameters. The inverse N-body problem requires a transition from the differential approach (describing the motion of individual bodies) to the integral approach (describing the system as a whole). This means considering the general characteristics of the system, rather than the characteristics of individual pairs of bodies. This means using the integral characteristics of the N-body system instead of the characteristics of individual bodies.

The additivity of gravitational forces in the differential approach leads to the equation of the resultant force, represented by an infinite series. The additivity of gravitational forces in the integral approach leads to the equation of the resultant force, represented by a finite series. In the simplest version, two terms of the series.

In contrast to the direct N-body problem, the inverse N-body problem has solutions both for $N = 2$. and for $N \to \infty$. The use of integral characteristics of the N-body system leads to the fact that the inverse N-body problem has a solution for an arbitrary value of N. The inverse N-body problem is applicable both to the infinite universe model and to the finite universe model. In the infinite universe model, $N \to \infty$. In the finite universe model, N takes a finite value. The main requirement for both the finite universe model and the infinite universe model is the known value of at least one integral parameter of the N-body system. The known value of at least one integral parameter of the N-body system allows one to obtain a solution to the inverse N-body problem for an arbitrary value of N.

10.6. Results of the N-body inverse problem solution

The solution of the inverse problem leads to the discovery of new force laws that describe gravitational interactions better than existing models, potentially explaining phenomena not explained by current theories. All solutions of the N-body inverse problem are given in [30, 31]. Among the solutions of the N-body inverse problem, the solution in the form: $F_U = mR^3/T^2r^2+mc^2\sqrt{\Lambda}$ is of the greatest interest. It gives the most accurate and perfect law of universal gravitation. It gives a more complete and more accurate description of gravity, especially on a cosmological scale, taking into account the influence of all bodies in the Universe, and not just pairs of bodies. The law of gravity does not include the gravitational constant G and the mass M. For this reason, this solution compares favorably with solutions containing the constant G. It is known that the constant G has the lowest accuracy among all fundamental physical constants.

Solving the inverse N-body problem has implications for understanding dark matter and dark energy, as well as improving models of galaxy dynamics. The inverse N-body problem has the potential to reveal new insights into the nature of gravity and the fundamental laws that govern the Universe. It can help us improve our understanding of gravitational interactions at various scales, from planetary systems to the large-scale structure of the Universe. In fact, the inverse N-body problem is a fascinating area of research that can lead to a deeper understanding of the fundamental forces that govern the Universe.

11. Relativistic version of the law of universal gravitation without the constant G.

Newtonian gravity assumes instantaneous transfer of gravitational force. This contradicts the theory of relativity, which states that nothing can travel faster than the speed of light. Researchers have explored various approaches to resolving these inconsistencies [32, 33]. These approaches often involve modifying Newtonian gravity by ad hoc introducing the speed of light into the equation. Here we present the equation $F_U = mR^3/T^2r^2 + mc^2\sqrt{\Lambda}$, which is not a modification of Newton's law. The equation (Fig. 3) is completely different from Newton's law. At the same time, it is an equation of classical gravity. In the equation (Fig. 3), the speed of light enters naturally together with the cosmological constant Λ. The presence of the speed of light in the classical law of gravitation together with the cosmological constant Λ represents the relativistic version of the law of universal gravitation. The equation (Fig. 3) is a relativistic version of the law of universal gravitation, obtained within the framework of classical gravity.

12. Evolution of the law of universal gravitation from Hooke and Newton to the present day.

For over 300 years, the force of gravity was considered to be Newton's single law, which describes the gravitational attraction of only one local source of gravity and ignores the fact that bodies are simultaneously attracted by all other bodies in the universe. The path from the two-body law of gravitation to the true law of universal gravitation (Fig. 3) was long. The stages of this long journey are shown in Fig. 5.

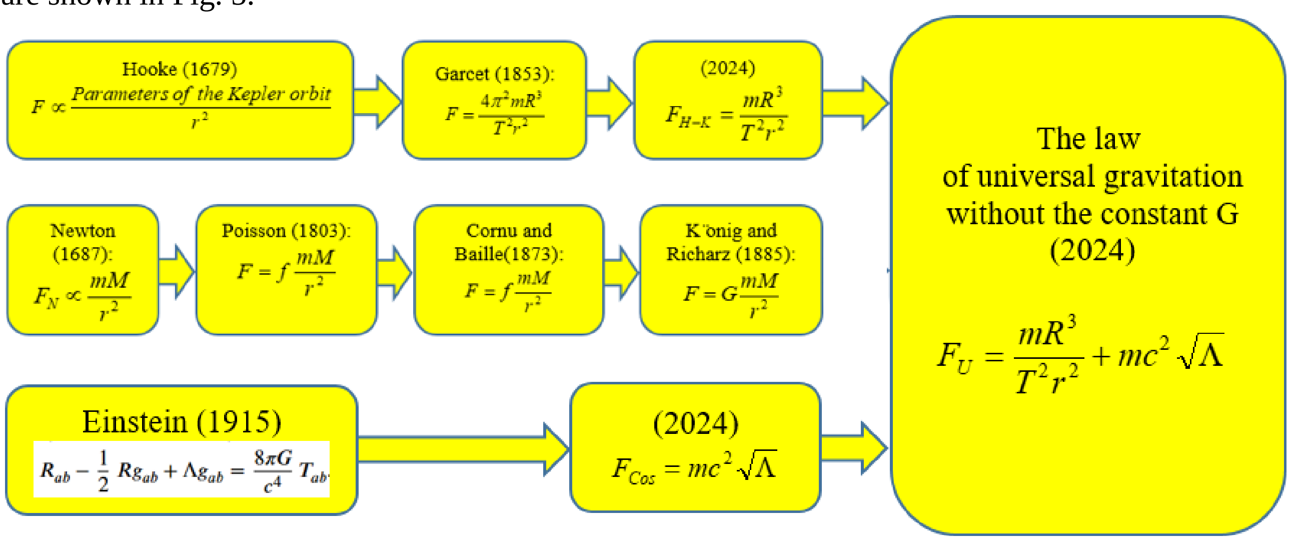

Fig. 5. The evolution of the law of universal gravitation from Hooke and Newton.

In 1687, Newton formulated the proportional relationship $F_N \propto mM/r^2$. This was not an exact law of gravitation. The proportional relationship $F_N \propto mM/r^2$ evolved into the law of gravitation F =

GMm/r^2 only after the introduction of the constant G. This occurred almost 200 years later, between 1803 and 1885. With the advent of the law of universal gravitation $F = GMm/r^2$, the world assumed that the law of universal gravitation had been discovered and there was nothing more to discover. However, space exploration revealed discrepancies with the law of gravitation $F = GMm/r^2$. It turned out that on large scales, gravity behaves differently than predicted by Newton's law. Clearly, a physical law other than Newton's law is needed to describe gravity.

A more accurate law of gravitation without the constant G could have been developed as early as 1679, seven years before Newton's law. Hooke formulated a more accurate law of gravitation, emphasizing elliptical orbits and the inverse square law [26]. Combining the parameters of Keplerian orbits with the inverse-square law paved the way for a complete and more accurate law of two-body gravitation: $F = mR^3/T^2r^2$. However, this opportunity was missed in 1679. Newton used mass instead of the parameters of Keplerian orbits and, instead of a complete law of gravitation, obtained a proportional relationship: $F_N \propto mM/r^2$.

A second law of gravitation without the constant G could have appeared in 1853. This could have been done by Garcet, who derived the formula $F = 4\pi^2 mR^3/T^2r^2$. However, he considered this formula a special case for a circular orbit. He did not see in this formula a new law of gravitation, different and more perfect than Newton's law. Thus, the second opportunity to discover the law of gravitation without the constant G was missed.

In 1915, Einstein published the equations of general relativity, which included the cosmological constant Λ and the constant G. With the introduction of the cosmological constant Λ into the equations of general relativity, it became possible to take into account the influence of all bodies in the universe on the force of gravity and derive the law of gravitation for the universe: $F_{Cos} = mc^2\sqrt{\Lambda}$. However, Einstein abandoned the cosmological constant Λ. As a result, the opportunity to derive the law of gravitation for the universe was missed. At that time, this opportunity was provided by the constant Λ and the formula $E = mc^2$. The evolution of gravity theory shows that science is progressing at a snail's pace...

Combining the Hooke-Kepler law $F_{H\text{-}K} = mR^3/T^2r^2$ and the law of cosmological forces $F_{Cos} = mc^2\sqrt{\Lambda}$, we obtain the law of universal gravitation $F_U = mR^3/T^2r^2+mc^2\sqrt{\Lambda}$, which takes into account both local gravity and the gravity of the universe. It takes into account both Kepler's law and the accelerated expansion of the universe.

13. Important consequences of the new law of universal gravitation.

The emergence of a new law of universal gravitation, which takes into account the effect of the accelerated expansion of the Universe and describes local systems more accurately than Newton's law, has important implications for the theory of gravity and our understanding of the Universe:

1. Rejection of dark matter and dark energy. The main consequence is that the hypotheses of dark matter and dark energy are no longer necessary [34]. The observed "anomalies" in the rotation of galaxies are a direct consequence of the new law of universal gravitation. The accelerated expansion of the Universe itself is explained by an inherent property of gravitational interaction on large scales, without invoking mysterious "dark energy."

2. Changes in the foundations of physics and cosmology and a revision of the cosmological model. A deeper understanding of the nature of gravity itself, possibly considering it an emergent phenomenon.

3. Impact on local calculations and the accuracy of orbital calculations: Even in the Solar System, where the expansion effect is minimal, the new law predicts small but measurable deviations in the planetary orbits that can be detected by modern high-precision instruments. For example, for

Earth, the law gives the magnitude of the additional acceleration, which is $A_0 = c^2\sqrt{\Lambda} = 10.4922 ... \cdot 10^{-10}\ ms^{-2}$.

14. The gravitational constant G is a composite constant.

Since the law of universal gravitation can be represented without the constant G, this calls into question its fundamental status. The fundamental status of the constant G has long been questioned by many scientists [17-21]. However, I was unable to find rigorous justification for such assertions in published sources. Duff and Veneziano, in their dispute with Okun, also limited themselves to assertions and did not provide evidence that G is not a fundamental constant [17].

Here we present a justification for the fact that the gravitational constant G is not independent. We show that G is a composite constant. To prove this, we use the well-known cosmological equations of Milne and Blakeslee, which include the parameters of the Universe and the constant G [35, 36].

Milne proposed the following equation in 1936:

$$\mathbf{Mu = c^3Tu/G} \qquad (2)$$

where: Mu is the mass of the universe; c is the speed of light; Tu is the time of the universe; G is the gravitational constant.

Bleksley proposed the following equation in 1951:

$$\mathbf{Mu = c^2Ru/G} \qquad (3)$$

where: Mu is the mass of the universe; c is the speed of light; Ru is the radius of the universe; G is the gravitational constant.

Equations (2) and (3) show the relationship between the parameters of the universe and the gravitational constant G. H. Weyl was the first to point out in 1918 that the parameters of the universe and the parameters of the electron are related [37]. According to H. Weyl, the coupling coefficients are large numbers. He proposed the following approximate formula for the radius of the universe:

$$\mathbf{Ru \approx 10^{40}r_e} \qquad (4)$$

where: Ru is the radius of the universe; r_e is the classical radius of the electron.

In [38], we presented precise formulas for the relationship between the parameters of the universe and the parameters of the electron. In [38], we showed that the parameters of the universe are related to the parameters of the electron as follows:

$$\mathbf{Mu = m_e\alpha D_0^2} \qquad (5)$$

$$\mathbf{Ru = r_e\alpha D_0} \qquad (6)$$

$$\mathbf{Tu = t_e\alpha D_0} \qquad (7)$$

where: Mu is the mass of the universe; Ru is the radius of the universe; Tu is the time of the universe; r_e is the classical radius of the electron; m_e is the electron mass; $t_e = r_e/c$; α is the fine structure constant; D_0 is the large Weyl number (D_0 = 4.16561... x 10^42).

Substituting Mu, Ru, and Tu from formulas (5) – (7) into the Milne and Blackley formulas, we obtain an expression for the gravitational constant G [39]:

$$\mathbf{G = c^2Ru/Mu = c^2r_e/m_eD_0 = r_e^3/t_e^2m_eD_0} \qquad (8)$$

$$\mathbf{G = c^3Tu/Mu = c^3t_e/m_eD_0 = r_e^3/t_e^2m_eD_0} \qquad (9)$$

Thus, the gravitational constant G is a combination of the electron constants (Fig. 6) [39].

$$G = r_e^3/t_e^2 m_e D_o$$

Fig. 6. The gravitational constant G is a composite constant. Where r_e is the electron radius; $t_e = r_e /c$; m_e is the electron mass; D_0 is the large Weyl number ($D_0 = 4.16561... \times 10^{42}$).

Thus, the gravitational constant G is not primary and is not an independent constant. It is a composite constant. In Newton's formula, it is a dimensionality adjustment coefficient.

15. Conclusion

Newton's law is insufficient to describe gravity. For over 300 years, a law of gravity that simultaneously accounts for the gravity of two bodies and all other bodies in the universe remained undiscovered. To obtain a new law of gravity, the inverse N-body problem was formulated and solved. The solution to the inverse N-body problem for N = 2 and for $N \rightarrow \infty$ yields a new law of gravitation $F_U = mR^3/T^2r^2 + mc^2\sqrt{\Lambda}$, which takes into account the accelerated expansion of the Universe. The total force of universal gravitation consists of two components: the force of gravitational interaction of two bodies $F_{H\text{-}K} = mR^3/(T^2)r^2$ and an additional cosmological force $F_{Cos} = (mc^2)\sqrt{\Lambda}$. A special feature of the new law of universal gravitation is that it does not include the gravitational constant G and mass M. The new law of gravitation includes parameters available from observations. These are the parameters of the planets' orbits (R and T) and the cosmological constant Λ. The new law of gravitation shows that in addition to the force of gravitational interaction between two bodies, the bodies are acted upon by the cosmological force $F_{Cos} = mc^2\sqrt{\Lambda}$ of the universe. The presence of the speed of light in the new law of gravitation together with the cosmological constant Λ represents a relativistic version of the law of universal gravitation.

16. Conclusions

1. For over 300 years, the force of gravitational interaction was described by a single physical law – Newton's law $F = GMm/r^2$. Here we show that Newton's law $F = GMm/r^2$ is not the only law of universal gravitation.

2. In addition to Newton's law $F = GMm/r^2$, there is a second law of gravitation $F = mR^3/T^2r^2$, which remained undiscovered. It does not include the gravitational constant G. The existence of this law was first pointed out by Robert Hooke in 1679.

3. With the discovery of the law of two-body gravitation $F_{H\text{-}K} = mR^3/T^2r^2$, which does not include the gravitational constant G, Newton's law of universal gravitation $F = GMm/r^2$ loses its unique status.

4. The third law of gravity, $F_{Cos} = mc^2\sqrt{\Lambda}$, describes the additional gravitational force exerted by all bodies in the universe. This is a new law of gravity. It complements Newtonian dynamics and completes the classical theory of gravity. It reveals an additional gravitational force that always exists, but Newton's law "doesn't see" it.

5. The appearance of the cosmological constant Λ in the third law of gravity shows that, in addition to the gravitational force of two bodies, these bodies are acted upon by the cosmological force $F_{Cos} = mc^2\sqrt{\Lambda}$ of the universe.

6. Combining the two new laws of gravity into a single equation yields the law of universal gravitation $F_U = mR^3/T^2r^2+mc^2\sqrt{\Lambda}$.

7. The new law of universal gravitation $F_U = mR^3/T^2r^2+mc^2\sqrt{\Lambda}$ has two components: the two-body law of gravity $F_{H\text{-}K} = mR^3/T^2r^2$ and the law of cosmological force $F_{Cos} = mc^2\sqrt{\Lambda}$.

8. The inverse N-body problem was formulated and solved. The inverse N-body problem is the problem of determining the law of gravitational force based on the integral characteristics of a system of N bodies. Unlike the direct N-body problem, which seeks to predict the future positions of the bodies, the inverse problem aims to derive the underlying law of force action.

9. The new law of gravity $F_U = mR^3/T^2r^2+mc^2\sqrt{\Lambda}$ was obtained as a solution to the inverse N-body problem for N = 2 and for $N \to \infty$.

10. All solutions to the N-body inverse problem are given in [30, 31]. Among the solutions to the N-body inverse problem, the most interesting is the solution in the form: $F_U = mR^3/T^2r^2 + mc^2\sqrt{\Lambda}$.

11. The new law of gravitation explains the shift in the perihelion of planets, indicates the gravitational nature of the Pioneer anomaly, and explains the rotation curve of galaxies without using the concept of dark matter.

12. The new law of universal gravitation $F_U = mR^3/T^2r^2 + mc^2\sqrt{\Lambda}$ is more accurate and perfect than Newton's law $F = GMm/r^2$, since it provides a more complete description of gravity. It does not include the gravitational constant G and the mass M. The orbital parameters (R and T) are known with greater accuracy than the constant G and the mass M. The low accuracy of the gravitational constant G ceases to be a limiting factor in gravitation.

References.

1. “Galaxy rotation curve” Wikipedia, Wikimedia Foundation, 14 April 2025, https://en.wikipedia.org/wiki/Galaxy_rotation_curve

2. Rubin, Vera; Thonnard, N.; Ford, Jr., W. K. (1980). "Rotational Properties of 21 SC Galaxies With a Large Range of Luminosities and Radii, From NGC 4605 (R=4kpc) to UGC 2885 (R=122kpc)". The Astrophysical Journal. 238: 471ff. doi:10.1086/158003.).

3. Milgrom, M. A modification of the Newtonian dynamics - Implications for galaxies.Astrophysical Journal, Vol. 270, p. 371-383 (1983). DOI: 10.1086/161131 Page 9

4. Milgrom M. 2022, Phys. Rev. D, 106, 064060 10.1103/PhysRevD.106.064060

5. U. Le Verrier (1859), (in French), "Lettre de M. Le Verrier à M. Faye sur la théorie de Mercure et sur le mouvement du périhélie de cette planète", Comptes rendus hebdomadaires des séances de l'Académie des sciences (Paris), vol. 49 (1859), pp. 379–383. https://en.wikipedia.org/wiki/Urbain_Le_Verrier

6. Pavel Kroupa, Tereza Jerabkova, Ingo Thies, Jan Pflamm-Altenburg, Benoit Famaey, Henri M J Boffin, Jörg Dabringhausen, Giacomo Beccari, Timo Prusti, Christian Boily, et al. Asymmetrical tidal tails of open star clusters: stars crossing their cluster's práh challenge Newtonian gravitation. Monthly Notices of the Royal Astronomical Society, Volume 517, Issue 3, December 2022, Pages 3613–3639, https://doi.org/10.1093/mnras/stac2563
7. Кю-Хён Чхэ 2023 ApJ 952 128 DOI 10.3847/1538-4357/ace101
8. Kroupa, P., et al. (2023) The Many Tensions with Dark-Matter Based Models and Implications on the Nature of the Universe. https://arxiv.org/abs/2309.11552 https://doi.org/10.22323/1.436.0231
9. X. Hernandez, Pavel Kroupa. A recent confirmation of the wide binary gravitational anomaly. arXiv:2410.17178 [astro-ph.GA].
10. "Alexis_Clairaut" Wikipedia, Wikimedia Foundation, 14 April 2025, https://en.wikipedia.org/wiki/Alexis_Clairaut
11. "Inverse-square_law" Wikipedia, Wikimedia Foundation, 14 April 2025, https://en.wikipedia.org/wiki/Inverse-square_law.
12. A suggestion in the theory of Mercury // Astr. J. — 1894. — Vol. 14. — P. 49—51.
13. Alternatives to general relativity. Wikipedia, Wikimedia Foundation, 14 April 2025, https://en.wikipedia.org/wiki/Alternatives_to_general_relativity
14. A Schlatter and R E Kastner 2023 J. Phys. Commun. 7 065009DOI 10.1088/2399-6528/acd6d7.
15. Espen Gaarder Haug. Gravity Without Newton's Gravitational Constant and No Knowledge of the Mass Size. (2022). European Journal of Applied Physics, 4(6), 4-10. https://doi.org/10.24018/ejphysics.2022.4.6.223)
16. Haug, E. (2023) Not Relying on the Newton Gravitational Constant Gives More Accurate Gravitational Predictions. Journal of Applied Mathematics and Physics, 11, 3124-3158. doi: 10.4236/jamp.2023.1110205.
17. M. J. Duff, L. B. Okun and G. Veneziano. Trialogue on the number of fundamental constants. JHEP 03. 2002; 023. physics/0110060.
18. Duff, M. J. (2014). How fundamental are fundamental constants? Contemporary Physics, 56(1), 35–47. https://doi.org/10.1080/00107514.2014.980093
George T Gillies 1997 Rep. Prog. Phys. 60 151DOI 10.1088/0034-4885/60/2/001
19. Quinn Terry and Speake Clive. 2014. The Newtonian constant of gravitation — a constant too difficult to measure? An introduction. Phil. Trans. R. Soc. A.372 2014 0253 http://doi.org/10.1098/rsta.2014.0253
20. zhou, . changzheng ., & ZHOU, . ziqing . (2025). Gravity is Not a Fundamental Force A; Spacetime Thermodynamics and the Emergence of Gravity. Zenodo. https://doi.org/10.5281/zenodo.17111933
21. Daniel J. Cleary. January 31, 2025. The Fundamental Nature of Gravity:
Why G is Not a Constant but a Consequence of Spacetime Structure
22. Haug, E.G. Different Mass Definitions and Their Pluses and Minuses Related to Gravity. Foundations 2023, 3, 199-219. https://doi.org/10.3390/foundations3020017
23. Indranil Banik, Srikanth T Nagesh, Hosein Haghi, Pavel Kroupa, Hongsheng Zhao,Overestimated inclinations of Milgromian disc galaxies: the case of the ultradiffuse galaxy AGC114905, Monthly Notices of the Royal Astronomical Society, Volume 513, Issue 3, July 2022, Pages 3541–3548, https://doi.org/10.1093/mnras/stac1073.
24. J. W. Moffat; V. T. Toth (2007). "Modified Gravity: Cosmology without dark matter or Einstein'scosmological constant". arXiv:0710.0364

25. T.C. Scott. From Modified Newtonian Dynamics to Superfluid Vacuum Theory // Entropy. — 2023. — T. 25 (December). — C. 12. — doi:10.3390/e25010012
26. Kosinov, M. (2025). HOOKE-KEPLER LAW OF GRAVITATION: the rejected law of gravitation, which turned out to be more accurate and perfect than Newton's law. *Cambridge Open Engage*. doi:10.33774/coe-2025-53vlv-v5.
27. Cornu, Marie-Alfred, and Jean-Baptistin Baille. 1873. Détermination nouvelle de la constante de l'attraction et de la densité moyenne de la terre. Comptes rendus hebdomadaires des séances de l'Académie des Sciences 76 (n° 13, Semestre 1): 954-958.
28. A. Cornu and J. B. Baille. D´etermination nouvelle de la constante de l'attraction et de la densit´e moyennede la terre. C. R. Acad. Sci. Paris, 76, 1873.
29. Kosinov, M. (2024). Kepler's constant in celestial mechanics, in electromagnetism and in cosmology. Cambridge Open Engage. doi:10.33774/coe-2024-qx0d6
30. Kosinov, M. (2025). INVERSE N-BODY PROBLEM. *Cambridge Open Engage*. doi:10.33774/coe-2025-6pxkg-v2
31. Kosinov, M. (2025). INVERSE N-BODY PROBLEM - THE KEY TO SOLVING GRAVITY PROBLEMS. *Cambridge Open Engage*. doi:10.33774/coe-2025-7w4k7-v4
32. Biswas, T. (1988) Minimally Relativistic Newtonian Gravity. American Journal of Physics, 56, 1032. https://doi.org/10.1119/1.15385
33. Haug, E. (2020) Relativistic Newtonian Gravitation That Gives the Correct Prediction of Mercury Precession. Journal of High Energy Physics, Gravitation and Cosmology, 6, 238-243. doi: 10.4236/jhepgc.2020.62017
34. Moni Bidin C. et al. Kinematical and chemical vertical structure of the Galactic thick disk. II. A lack of dark matter in the solar neighborhood // The Astrophysical Journal. — IOP Publishing, 2012
35. Milne, E. A. On the foundations of dynamics. Proc. R. Soc. A 1936, 154, 22–52.
36. A. E. H. Bleksley. A new approach to cosmology: Iii. South African Journal of Science, 48 (1), 1951.
37. H.Weyl, Raum-Zeit-Materie (Berlin,1918).
38. Mykola Kosinov. SYSTEMS OF COSMOLOGICAL EQUATIONS OF THE UNIVERSE. THE FINE STRUCTURE CONSTANT "ALPHA" IN A NEW CAPACITY AS THE MAIN CONSTANT OF COSMOLOGY. March 2024. DOI: 10.13140/RG.2.2.13152.29445
39. Kosinov, M. (2024). FIVE PRIMARY FUNDAMENTAL CONSTANTS FOR EXPRESSING THE ENTIRE SET OF PHYSICAL LAWS, SECONDARY CONSTANTS AND PARAMETERS OF THE UNIVERSE. *Cambridge Open Engage*. doi:10.33774/coe-2024-fxz88