THE LAW OF GRAVITATION THAT TAKES INTO ACCOUNT THE ACCELERATED EXPANSION OF THE UNIVERSE AND KEPLER'S LAWS OF PLANETARY MOTION

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Abstract. *The law of gravity* $F_U = mR^3/(T^2)r^2+(mc^2)\sqrt{\Lambda}$ *is investigated, which is considered as a relativistic version of the law of universal gravitation. The new law of gravitation takes into account the accelerated expansion of the Universe and Kepler's laws of planetary motion. It shows the total force of universal gravitation taking into account the gravitational interaction of all N bodies in the Universe. This law of gravitation is a solution to the inverse N-body problem for N =2 and for N → ∞. The total force of universal gravitation consists of two components: the force of gravitational interaction of two bodies* $F_{H\text{-}K} = mR^3/(T^2)r^2$ *and an additional cosmological force* $F_{Cos} = (mc^2)\sqrt{\Lambda}$*. A feature of this law of universal gravitation is that it does not include the gravitational constant G and mass M. It includes parameters available from observations. These are the parameters of the planets' orbits (R and T) and the cosmological constant Λ. The new law of gravitation shows that in addition to the force of gravitational interaction between two bodies, the bodies are acted upon by the cosmological force* $F_{Cos} = (mc^2)\sqrt{\Lambda}$ *of the universe. The presence of the speed of light in the law of gravitation together with the cosmological constant Λ represents a relativistic version of the law of universal gravitation obtained within the framework of classical gravity. The additional cosmological force causes acceleration* $A_0 = (c^2)\sqrt{\Lambda} = 10.4922 \times 10^{-10}$ m/s² *close to the acceleration value* ($a_0 = 1.2 \times 10^{-10}$ m/s²) *predicted by MOND. The new law of gravitation explains the shift of the planets' perihelion, points to the gravitational nature of the Pioneer anomaly, and provides an explanation for the rotation curve of galaxies without invoking the concept of dark matter.*

Keywords: *Newton's law; N-body problem; law of universal gravitation; parameters of the observable universe; dark matter; galaxy rotation curve; cosmological constant Λ.*

1. Introduction

Newton's law of universal gravitation $F = GMm/r^2$ is one of the greatest scientific discoveries. It is distinguished by its simplicity and mathematical perfection. Despite the fact that the law of gravitation was discovered more than 300 years ago, gravity remains the most mysterious physical phenomenon in physics and cosmology. All this time, the force of gravitational interaction was represented only by Newton's law of universal gravitation. For many practical applications, this law was sufficient. However, the predictions of Newtonian gravity fail on the scale of the universe. Newton's law shows the force of gravitational interaction of two bodies out of all N bodies in the universe. The formula for Newton's law describes the gravity of only one local source of attraction and does not take into account that bodies are simultaneously attracted to all other bodies in the universe. In reality, all bodies in the universe participate in gravitational interaction. This is not taken into account in Newton's law of universal gravitation. Newton's law does not work on the scale of the universe [1 - 9].

Newton's law of gravity does not "see" the total force that acts in the universe, but "sees" only a part of the gravitational force. Newton's law does not take into account the accelerated expansion of the universe. Newton's law does not include the parameters of the Keplerian orbit. An accurate and complete law of gravity must take into account the gravity of all N bodies in the universe. In gravity, the main fundamental law of nature remained undiscovered: the law of gravitational interaction of all N bodies in the universe. The obstacle on this path was the unsolvable N-body problem. In order to eliminate the imperfection of Newtonian dynamics, many attempts were made to modify Newton's law [10, 11, 12]. These attempts did not lead to the discovery of a more perfect law of gravity than Newton's law. The law of gravity that takes into account the gravity of all bodies in the universe cannot be obtained by modifying and changing Newton's law of universal gravitation for the reason that Newton's law is the law of gravitational interaction of two bodies. No amount of cosmetic editing can transform the two-body law of gravitational interaction into an N-body law of gravitational interaction.

The complete law of gravitation should ideally solve a problem that involves simultaneously calculating the gravitational interaction of all bodies in the Universe. Newton's law faces an insurmountable N-body problem, requiring approximations or computational solutions for many pairs of bodies. In essence, a more accurate law of gravitation should explain the observed gravitational phenomena at all scales, including those that are currently not explained by Newton's law.

Obviously, the N-body law of gravitational interaction is not a modified two-body law of gravitation, but a new law of gravitation. It must be a new fundamental law of gravitation, complementary to the two-body law of gravitation. The additivity of gravitational forces makes it possible to represent the complete law of universal gravitation as a sum of forces: the gravitational forces of a finite number of pairs of bodies and the cosmological force of all N bodies in the Universe.

2. About theories of gravity.

Newton did not develop or use the theory of gravity in his research. This did not prevent him from discovering the fundamental law of Nature. There are now many theories of gravity, the purpose of which is to improve Newtonian dynamics, but they have not added a more perfect physical law to Newtonian dynamics than Newton's law of gravity. The abundance of theories of gravity (Metric theories, Non-metric theories, Vector theories, Scalar-tensor theories, Le Sage theory, Theories of quantum gravity, supergravity, string theory...) have not led to a better law of gravitational force than Newton's law [13]. These theories of gravity have not discovered a single new law of nature. All theories of gravity have proven powerless against the N-body problem. It is obvious that the approach "from the theory of gravity to the law of gravity" is not productive. The fact that Newton discovered the law of gravity without any theory shows that physical theories are not a tool for discovering the laws of Nature. The laws of Nature are primary, and theories have a secondary status. It is necessary to look for another approach to discover the law of gravitational interaction, taking into account the gravity of all bodies in the Universe. Here we propose a new approach to solving the N-body problem, which provides the key to discovering the laws of gravity.

3. The law of gravity, taking into account the accelerated expansion of the Universe and Kepler's laws on the motion of the planets

The law of gravity, taking into account the accelerated expansion of the Universe and Kepler's laws on the motion of the planets, is as follows:

$$F_U = \frac{mR^3}{T^2 r^2} + mc^2\sqrt{\Lambda}$$

Fig. 1. The law of gravitation taking into account the accelerated expansion of the Universe and Kepler's laws of planetary motion. Where: m is the mass of the body, R and T are orbit parameters, r is the distance, c is the speed of light in vacuum, Λ is the cosmological constant.

This law of gravitation is a solution to the inverse problem of N bodies for N =2 and for N → ∞. This equation is of greatest interest as the most accurate and perfect law of gravitation. The new law of gravitation shows the total force of universal gravitation taking into account the gravitational interaction of all N bodies in the Universe. The total force of universal gravitation consists of two components: the force of gravitational interaction of two bodies $F_{H\text{-}K} = mR^3/T^2r^2$ (Hooke-Kepler law [14]) and an additional cosmological force $F_{Cos} = mc^2\sqrt{\Lambda}$. The peculiarity of the new law of universal gravitation is that it does not include the gravitational constant G and the mass M. This is a good sign. The low accuracy of the gravitational constant G ceases to be a limiting factor in gravity. The new law of gravity includes parameters available from observations. These are the parameters of the orbits of the planets (R and T) and the cosmological constant Λ.

The new law of gravitation shows that in addition to the force of gravitational interaction between two bodies, all bodies are affected by the cosmological force $F_{Cos} = mc^2\sqrt{\Lambda}$ of the universe. The total force of universal gravitation is represented by the vector sum of two forces: the force of gravitational interaction of two bodies FH-K and the cosmological gravitational force F_{Cos}:

$$\vec{F}_U = \vec{F}_{H-K} + \vec{F}_{Cos} \qquad (1)$$

The value of the resulting force of universal gravitation is in the range of values from $F_U = mR^3/T^2r^2 - mc^2\sqrt{\Lambda}$ to $F_U = mR^3/T^2r^2 + mc^2\sqrt{\Lambda}$. For collinear force vectors, the equation (Fig. 1) takes the form: $F_U = mR^3/T^2r^2 \pm mc^2\sqrt{\Lambda}$.

3.1. Two-body gravity + Universe gravity.

Newton's law $F = GMm/r^2$ "sees" the gravitational force of two bodies. The new law of gravitation $F_U = mR^3/T^2r^2 + mc^2\sqrt{\Lambda}$ "sees" two forces. The first term gives the gravitational force of two bodies. The second term gives the gravitational force of the universe. The total force of gravitational interaction is the sum of the two forces.

The extended law of universal gravitation $F_U = mR^3/T^2r^2+mc^2\sqrt{\Lambda}$ provides a more complete description of gravitational interactions at different scales. It combines the Hooke-Kepler law and the cosmological force law. This extension is especially relevant when considering the large-scale structure of the universe.

3.2. Accuracy and Perfection

The new law of gravitation $F_U = mR^3/T^2r^2+mc^2\sqrt{\Lambda}$ is more accurate and perfect than Newton's law F = GMm/r2, since it provides a more complete description of gravity, especially on cosmological scales. It takes into account the influence of all bodies in the Universe, not just pairs of bodies. It does not include the gravitational constant G and the mass M. The new law of gravitation is more accurate than Newton's law, since the orbital parameters (R and T) are known with greater accuracy than the constant G and the mass M. With the advent of the law of gravitation $F_U = mR^3/T^2r^2+mc^2\sqrt{\Lambda}$, the low accuracy of the gravitational constant G ceases to be a limiting factor in gravity.

4. Cosmological force

The relativistic version of the law of universal gravitation includes the cosmological force as an integral part. This is a gravitational force that depends on the cosmological constant Λ. The formula for this force is: $F_{Cos} = mc^2\sqrt{\Lambda}$, where m is the mass of the object, c is the speed of light, Λ is the cosmological constant. The cosmological force is the gravitational force with which the mass of the entire universe acts on the body, causing acceleration $A_0 = c2\sqrt{\Lambda}=10.4922 \times 10^{-10}$ m/s². The cosmological force shows that the gravitational interaction of an object with the Universe, represented by the cosmological constant (Λ), generates a force proportional to the mass of the body. The cosmological force does not obey the inverse square law. The cosmological gravitational force is generated by the mass of all bodies in the Universe. This is different from Newton's law, which focuses on the interaction of two bodies. In this context, the cosmological constant Λ is a coupling constant that affects the magnitude of the cosmological force. The cosmological force arises from the interaction of the body with the general mass distribution of the Universe.

4.1. The gravitational force of the Universe.

The formula for the cosmological force shows that this force is directly proportional to the mass of the object and depends on the cosmological constant, which is related to the expansion of the Universe. The formula includes the speed of light, which demonstrates the relativistic version of the law of universal gravitation. The law of the cosmological force explains observed anomalies such as the Pioneer anomaly, the perihelion shifts of planets, and the rotation curves of galaxies, which are not explained by existing gravitational models. The law of the cosmological force provides a basis for understanding gravitational interactions in the Universe, especially on cosmological scales.

4.2. The Cosmological Force Law as a Supplement to the Two-Body Gravity Law

The cosmological force law is not intended to replace the two-body gravity law, but to complement it by taking into account the influence of the Universe as a whole. In essence, the cosmological force law takes into account the influence of the Universe on gravitational interactions, potentially offering a more complete understanding of gravity on both small and large scales. The additional cosmological force causes an acceleration $A_0 = (c^2)\sqrt{\Lambda}$ close to the acceleration value ($a_0 = 1.2 \times 10^{-10}$ m/s^2) predicted by MOND [3, 4].

5. Hooke-Kepler Law of Gravitation

The new version of the law of universal gravitation includes the Hooke-Kepler law of gravitation $F_{H\text{-}K} = mR^3/T^2r^2$ [14]. This law includes Kepler's laws of planetary motion and the inverse square law, emphasizing the relationship between the orbital parameters (radius R and period T) and the gravitational force. In one formula, both the inverse square law and Kepler's third law, which relates the orbital radius and period, are combined. The Hooke-Kepler law provides a more accurate and complete formula for the gravitational force of two bodies than Newton's law.

5.1. Hooke's Contribution

Robert Hooke, in his correspondence with Newton (1679), emphasized the importance of the inverse square law and the ellipticity of planetary orbits for describing gravity (). Robert Hooke relied on Kepler's laws. He argued that a complete understanding of gravity must take these factors into account. Robert Hooke had given a verbal formulation of the future law of gravity in a letter 7 years before Newton published Philosophiæ Naturalis Principia Mathematica. Newton did not accept Hooke's hint and did not include the parameters of Keplerian orbits in his law of gravity. Newton used masses. This was a different law of gravity than the one hinted at by Hooke. It is known that masses are not included in Kepler's laws at all. By rejecting Hooke's hint, Newton missed the opportunity back in 1679 to give the world a more precise law of gravity $F = mR^3/(T^2r^2)$ than the approximate law of proportionality $F_N \propto mM/r^2$, which he discovered 7 years later in 1687. It is known that Newton's law of proportionality $F_N \propto mM/r^2$ was transformed into the precise formula $F = GMm/r^2$ only 200 years later (in 1873) with the appearance of the gravitational constant G in it [15, 16].

5.2. Relationship with Kepler's Constant

A more accurate and complete understanding of gravity can be achieved by considering elliptical orbits and the inverse square law proposed by Hooke, and by incorporating the relationship (R^3/T^2) that follows from Kepler's law. The Hooke-Kepler law highlights the importance of Kepler's constant (R^3/T^2) in the context of the gravitational force [17].

5.3. Key Differences from Newton's Law

In essence, the Hooke-Kepler law of gravity highlights a different view of the relationship between gravity and orbital motion, highlighting the importance of Kepler's laws in understanding the force of gravity. While Newton's law focuses on the gravitational force between two masses ($F = GmM/r^2$), Hooke-Kepler's law $F_{H\text{-}K} = mR^3/(T^2r^2)$ emphasizes the relationship between the orbital parameters (radius and period) and the gravitational force. Hooke-Kepler's law does not include the mass M. It is known that Kepler's laws do not include masses at all.

5.4. Hooke-Kepler's Law - A New Law of Two-Body Gravitation

Hooke-Kepler's law of gravitation is a different formulation of the gravitational force between two bodies than Newton's law. This law of gravitation emphasizes the relationship between the orbital parameters (radius and period) and the gravitational force, offering a more accurate representation of gravity than Newton's law. It is derived by combining Hooke's idea of the inverse-square law with Kepler's laws of planetary motion.

6. Inverse N-body problem

The most important physical law of gravity $F_U = mR^3/T^2r^2 + mc^2\sqrt{\Lambda}$, which shows the force of gravitational interaction of all N bodies in the Universe, remained undiscovered in gravity. The main obstacle on this path was the N-body problem, which has no analytical solution for $N \geq 3$. To obtain the law of gravitational interaction of N bodies, the inverse N-body problem was formulated and solved in [18, 19]. The inverse N-body problem is the problem of determining the law of gravitational force based on the integral characteristics of the N-body system. Unlike the direct N-body problem, which seeks to predict the future position of the bodies, the inverse problem aims to derive the underlying law of action of forces. This problem has not been considered in physics before, possibly due to the lack of an analytical solution to the direct N-body problem.

6.1. The goal of the inverse N-body problem

The inverse N-body problem involves determining the law of gravitational force acting on a system of N bodies, given the integral properties of their motion. Essentially, instead of starting with a known gravitational force and calculating the resulting motion (as in the forward N-body problem), this problem starts with the observed motion and aims to derive the underlying law of force. Given the integral properties of the N-body system, the goal is to determine the law of action of the gravitational forces governing their interaction. The main task is to find a mathematical expression for the gravitational force that corresponds to the observed motion. Unlike the forward N-body problem, the inverse N-body problem has an analytical solution for both $N = 2$ and $N \to \infty$ (). For $N = 2$, the inverse problem leads to both Newton's law $F = GMm/r^2$ and the Hooke-Kepler law ($F = mR^3/T^2r^2$). As N tends to infinity, it leads to the law of the cosmological force ($F = (mc^2)\sqrt{\Lambda}$). The inverse problem is solved using the integral parameters of the N-body system. Solutions to the inverse N-body problem directly lead to new laws of gravity that remained undiscovered for more than 300 years.

6.2. The Direct N-Body Problem

Knowing the masses, initial positions, and velocities of the N bodies, as well as the known law of force action (e.g., Newton's law of gravitation), the goal is to calculate their positions and velocities at all future moments of time. This problem has no analytical solution for $N \geq 3$.

6.3. The Mirror-symmetric approach to solving the inverse N-body problem.

The unsolvable direct N-body problem itself gave a hint for choosing a solution method: "If the N-body problem is unsolvable, then the mirror-symmetric problem can provide a solution." The idea of using the principles of opposition and inversion turned out to be productive. To solve the N-body gravity problem, a mirror-symmetric approach was used. Instead of the direct N-body problem, there is the inverse N-body problem. Instead of a set of individual bodies, there is one system consisting of N-bodies. Instead of the differential characteristics of individual bodies, there are integral characteristics of the N-body system. Instead of the trajectory of motion under the action of gravity, there is the law of gravity.

6.4. Instead of a set of individual bodies, there is one system consisting of N-bodies.

In the inverse N-body problem, a set of bodies is considered as a single object or system. The inverse N-body problem involves reducing the N-body problem to an equivalent two-body problem, where one of the bodies is a system of N bodies.

6.5. Integral characteristics of the N-body system instead of differential characteristics of bodies.

In the direct N-body problem, differential characteristics of bodies are used as parameters. The inverse N-body problem requires a transition from the differential approach (describing the motion of individual bodies) to the integral approach (describing the system as a whole). This means considering the general characteristics of the system, rather than the characteristics of individual pairs of bodies. This means using the integral characteristics of the N-body system instead of the characteristics of individual bodies.

The additivity of gravitational forces in the differential approach leads to the equation of the resultant force, represented by an infinite series. The additivity of gravitational forces in the integral approach leads to the equation of the resultant force, represented by a finite series. In the simplest version, two terms of the series.

In contrast to the direct N-body problem, the inverse N-body problem has solutions both for $N = 2$. and for $N \to \infty$. The use of integral characteristics of the N-body system leads to the fact that the inverse N-body problem has a solution for an arbitrary value of N. The inverse N-body problem is applicable both to the infinite universe model and to the finite universe model. In the infinite universe model, $N \to \infty$. In the finite universe model, N takes a finite value. The main requirement for both the finite universe model and the infinite universe model is the known value of at least one integral parameter of the N-body system. The known value of at least one integral parameter of the N-body system allows one to obtain a solution to the inverse N-body problem for an arbitrary value of N.

6.6. Results of the N-body inverse problem solution

The solution of the inverse problem leads to the discovery of new force laws that describe gravitational interactions better than existing models, potentially explaining phenomena not explained by current theories. All solutions of the N-body inverse problem are given in [18, 19]. Among the solutions of the N-body inverse problem, the solution in the form: $F_U = mR^3/T^2r^2+mc^2\sqrt{\Lambda}$ is of the greatest interest. It gives the most accurate and perfect law of universal gravitation. It gives a more complete and more accurate description of gravity, especially on a cosmological scale, taking into account the influence of all bodies in the Universe, and not just pairs of bodies. The law of gravity does not include the gravitational constant G and the mass M. For this reason, this solution compares favorably with solutions containing the constant G. It is known that the constant G has the lowest accuracy among all fundamental physical constants.

Solving the inverse N-body problem has implications for understanding dark matter and dark energy, as well as improving models of galaxy dynamics. The inverse N-body problem has the potential to reveal new insights into the nature of gravity and the fundamental laws that govern the Universe. It can help us improve our understanding of gravitational interactions at various scales, from planetary systems to the large-scale structure of the Universe. In fact, the inverse N-body problem is a fascinating area of research that can lead to a deeper understanding of the fundamental forces that govern the Universe.

7. Relativistic version of the law of universal gravitation

Newtonian gravity assumes instantaneous transfer of gravitational force. This contradicts the theory of relativity, which states that nothing can travel faster than the speed of light. Researchers have explored various approaches to resolving these inconsistencies [20, 21]. These approaches often involve modifying Newtonian gravity by ad hoc introducing the speed of light into the equation. Here we present the equation $F_U = mR^3/T^2r^2+mc^2\sqrt{\Lambda}$, which is not a modification of Newton's law. The equation (Fig. 1) is completely different from Newton's law. At the same time, it is an equation of classical gravity. In the equation (Fig. 1), the speed of light enters naturally together with the cosmological constant Λ. The presence of the speed of light in the classical law of gravitation together with the cosmological constant Λ represents the relativistic version of the law of universal gravitation. The equation (Fig. 1) is a relativistic version of the law of universal gravitation, obtained within the framework of classical gravity.

8. Conclusion

In gravity, the most important physical law of gravity, which takes into account both the gravity of two bodies and all bodies in the Universe, remained undiscovered. To obtain this law of gravity, the inverse N-body problem was formulated and solved. The solution to the inverse N-body problem for N = 2 and for $N \to \infty$ yields a new law of gravitation $F_U = mR^3/T^2r^2+mc^2\sqrt{\Lambda}$, which takes into account the accelerated expansion of the Universe. The total force of universal gravitation consists of two components: the force of gravitational interaction of two bodies $F_{H\text{-}K}$ = mR^3/(T^2)r^2 and an additional cosmological force F_{Cos} = (mc^2)$\sqrt{\Lambda}$. A special feature of the new law of universal gravitation is that it does not include the gravitational constant G and mass M. The new law of gravitation includes parameters available from observations. These are the parameters of the planets' orbits (R and T) and the cosmological constant Λ. The new law of gravitation shows that in addition to the force of gravitational interaction between two bodies, the bodies are acted upon by the cosmological force $F_{Cos} = mc^2\sqrt{\Lambda}$ of the universe. The presence of the speed of light in the new law of gravitation together with the cosmological constant Λ represents a relativistic version of the law of universal gravitation.

9. Conclusions

1. The law of gravitation $F_U = mR^3/T^2r^2+mc^2\sqrt{\Lambda}$ was discovered, which takes into account the accelerated expansion of the Universe and Kepler's laws of planetary motion. It shows the total force of universal gravitation taking into account the gravitational interaction of all N bodies in the Universe and is a relativistic version of the law of universal gravitation.

2. The inverse N-body problem was formulated and solved. The inverse N-body problem is the problem of determining the law of gravitational force based on the integral characteristics of an N-body system. Unlike the direct N-body problem, which seeks to predict the future position of the bodies, the inverse problem aims to derive the underlying law of action of forces.

3. The new law of gravitation $F_U = mR^3/T^2r^2+mc^2\sqrt{\Lambda}$ was obtained as a solution to the inverse N-body problem for N =2 and for $N \to \infty$.

4. All solutions of the inverse N-body problem are given in [18, 19]. Among the solutions of the inverse N-body problem, the solution in the form: $F_U = mR^3/T^2r^2+mc^2\sqrt{\Lambda}$ is of the greatest interest. It provides the most accurate and perfect law of universal gravitation.

5. The new law of gravitation shows that in addition to the force of gravitational interaction of two bodies, the bodies are acted upon by the cosmological force $F_{Cos} = mc^2\sqrt{\Lambda}$ of the universe.

6. The new law of gravitation explains the shift of the perihelion of the planets, indicates the gravitational nature of the Pioneer Anomaly, and explains the rotation curve of galaxies without involving the concept of dark matter.

7. The new law of universal gravitation $F_U = mR^3/T^2r^2+mc^2\sqrt{\Lambda}$ is more accurate and perfect than Newton's law $F = GMm/r^2$, since it provides a more complete description of gravity. It does not include the gravitational constant G and the mass M. The orbital parameters (R and T) are known with greater accuracy than the constant G and the mass M. The low accuracy of the gravitational constant G ceases to be a limiting factor in gravity.

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