General Covariance as a Spontaneous Subsidiary Symmetry of Lorentz-Covariant Gravity

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Abstract Physical phenomena other than gravity are customarily assumed to be described by Lorentz-covariant theories, and the validity of the Lorentz transformation has been empirically verified to very high accuracy. But if all nongravitational phenomena really are Lorentz covariant, it would challenge physical consistency for gravity not to be Lorentz covariant as well. Here we work out the Lorentz-covariant dynamics of test bodies that interact with, respectively, any electromagnetic four-vector potential and any gravitational symmetric-second-rank-tensor potential. A subsidiary symmetry spontaneously emerges in each case: gauge invariance in electromagnetism, and general coordinate covariance in gravitation. We then work out field equations for the electromagnetic and gravitational potentials which incorporate their respective subsidiary symmetries; such field equations unavoidably have an infinite number of candidate solutions for their potentials, but candidate potentials which aren't Lorentz covariant are of course excluded, as are candidate potentials which violate causality or introduce singularities.

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1. Lorentz-covariant extension of electrostatic phenomena to dynamic electromagnetism

Electromagnetism is taken to be the result of a conserved scalar entity Q called charge. The four-vector density-flux of charge Q, i.e., the current density $j^\lambda(x)$, is the *source* of the four-vector electromagnetic potential $A_\mu(x)$. Local charge conservation implies that,

$$\partial j^\lambda(x)/\partial x^\lambda = 0. \tag{1.1}$$

In the static limit $j^\lambda(x) \to (c\rho(\mathbf{x}), \mathbf{0})$, where $\rho(\mathbf{x})$ is the static charge density, and in that limit $A_0(\mathbf{x})$ is assumed to be the electrostatic potential that follows from $\rho(\mathbf{x})$ by Coulomb's Law,

$$-\nabla^2_{\mathbf{x}} A_0(\mathbf{x}) = 4\pi\rho(\mathbf{x}) = 4\pi(j^0(\mathbf{x})/c). \tag{1.2}$$

The nonrelativistic equation of motion of a test body of mass m and charge e in the electrostatic potential $A_0(\mathbf{x})$ is of course,

$$m\, d^2\mathbf{x}/dt^2 = -e\nabla_{\mathbf{x}} A_0(\mathbf{x}), \tag{1.3a}$$

and the Eq. (1.3a) nonrelativistic equation of motion corresponds to the nonrelativistic Lagrangian,

$$L(d\mathbf{x}/dt, A_0(\mathbf{x})) = (m/2)|d\mathbf{x}/dt|^2 - eA_0(\mathbf{x}). \tag{1.3b}$$

Making use of the Lorentz invariance of differential proper time $d\tau \stackrel{\text{def}}{=} dt\sqrt{1 - |(d\mathbf{x}/dt)/c|^2}$, and assuming the Lorentz covariance of the dynamic electromagnetic four-vector potential $A_\mu(x)$, we readily extend the nonrelativistic Eq. (1.3b) Lagrangian to a Lorentz-invariant Lagrangian, i.e.,

$$L(dx^\mu/d\tau, A_\nu(x)) = -(m/2)\eta_{\mu\nu}(dx^\mu/d\tau)(dx^\nu/d\tau) - (e/c)A_\nu(x)(dx^\nu/d\tau), \tag{1.4a}$$

where $\eta_{\mu\nu}$ is the Minkowski metric,

$$\eta_{00} = 1,\ \eta_{11} = \eta_{22} = \eta_{33} = -1 \text{ and } \eta_{\mu\nu} = 0 \text{ if } \mu \neq \nu. \tag{1.4b}$$

In the nonrelativistic regime where $|d\mathbf{x}/dt| \ll c$ the Eq. (1.4a) Lorentz-invariant Lagrangian plus the dynamically-inert constant term $(m/2)c^2$ goes over into the Eq. (1.3b) nonrelativistic Lagrangian. When the electromagnetic four-vector potential $A_\mu(x)$ vanishes, the Eq. (1.4a) Lorentz-invariant Lagrangian of course becames the Lorentz-invariant Lagrangian for the free mass-m test body, namely,

$$L(dx^\mu/d\tau) = -(m/2)\eta_{\mu\nu}(dx^\mu/d\tau)(dx^\nu/d\tau). \tag{1.4c}$$

An unexpected feature of the Eq. (1.4a) Lorentz-invariant Lagrangian $L(dx^\mu/d\tau, A_\nu(x))$ is that if $A_\nu(x)$ is modified by adding a term of the form $\partial\chi(x)/\partial x^\nu$ to it, where $\chi(x)$ is an arbitrary scalar field, the dynamics of the mass-m, charge-e test body is unaffected. That is so because,

$$\begin{aligned} L(dx^\mu/d\tau, A_\nu(x) + \partial\chi(x)/\partial x^\nu) &= L(dx^\mu/d\tau, A_\nu(x)) - (e/c)(\partial\chi(x)/\partial x^\nu)(dx^\nu/d\tau) \\ &= L(dx^\mu/d\tau, A_\nu(x)) + d[-(e/c)\chi(x)]/d\tau, \end{aligned} \tag{1.4d}$$

and any term of a Lagrangian which is a derivative of an entity with respect to τ doesn't contribute to the dynamics. That adding a term of the form $\partial\chi(x)/\partial x^\mu$ to $A_\mu(x)$ doesn't alter the dynamics of Lorentz-covariant four-vector electromagnetic theory is called electromagnetic gauge transformation invariance; it is *a subsidiary effect of the strictly Lorentz-covariant four-vector nature of electromagnetic theory.*

Upon applying the Lagrangian equation of motion $d[\partial L/\partial(dx^\lambda/d\tau)]/d\tau = \partial L/\partial x^\lambda$ to the Eq. (1.4a) $L = -(m/2)\eta_{\mu\nu}(dx^\mu/d\tau)(dx^\nu/d\tau) - (e/c)A_\nu(x)(dx^\nu/d\tau)$, we obtain the Lorentz covariant dynamical equation,

$$m\,\eta_{\lambda\nu}(d^2x^\nu/d\tau^2) = (e/c)[(\partial A_\nu(x)/\partial x^\lambda) - (\partial A_\lambda(x)/\partial x^\nu)](dx^\nu/d\tau), \tag{1.5a}$$

a gauge transformation invariant result. In the normal form for Lorentzian dynamics Eq. (1.5a) reads,

$$m(d^2x^\mu/d\tau^2) = (e/c)\eta^{\mu\lambda}[(\partial A_\nu(x)/\partial x^\lambda) - (\partial A_\lambda(x)/\partial x^\nu)](dx^\nu/d\tau), \tag{1.5b}$$

where $\eta^{\mu\lambda}$ is the matrix inverse of the Minkowski metric $\eta_{\lambda\nu}$ defined by Eq. (1.4b); in fact, of course, $\eta^{\mu\lambda} = \eta_{\mu\lambda}$. Eq. (1.5b) is the well-known gauge-invariant Lorentz Force Law of dynamic electromagnetism.

We next search for a Lorentz-covariant and gauge-invariant field equation whose static limit implies the Coulomb's Law equation $-\nabla^2_{\mathbf{x}} A_0(\mathbf{x}) = 4\pi(j^0(\mathbf{x})/c)$. We note that the $\mu = 0$ component of the Lorentz-covariant four-vector entity $\eta^{\alpha\beta}(\partial^2 A_\mu(x)/\partial x^\alpha\partial x^\beta)$ reduces to $-\nabla^2_{\mathbf{x}} A_0(\mathbf{x})$ in the static limit, and that the

$\mu = 0$ component of its complementary Lorentz-covariant four-vector entity $\eta^{\alpha\beta}(\partial^2 A_\alpha(x)/\partial x^\mu \partial x^\beta)$ vanishes in the static limit. Therefore $-\nabla^2_{\mathbf{x}} A_0(\mathbf{x}) = 4\pi(j^0(\mathbf{x})/c)$ is the static limit of the $\mu = 0$ component of the following Lorentz-covariant four-vector equation,

$$\eta^{\alpha\beta}(\partial^2 A_\mu(x)/\partial x^\alpha \partial x^\beta) + K\,\eta^{\alpha\beta}(\partial^2 A_\alpha(x)/\partial x^\mu \partial x^\beta) = (4\pi/c)\,\eta_{\mu\nu}\,j^\nu(x), \tag{1.6a}$$

where K is an arbitrary constant. When K equals -1, Eq. (1.6a) is also gauge invariant. Thus a Lorentz-covariant and gauge-invariant four-vector field equation whose $\mu = 0$ component reduces in the static limit to the Coulonb's Law equation $-\nabla^2_{\mathbf{x}} A_0(\mathbf{x}) = 4\pi(j^0(\mathbf{x})/c)$ is,

$$\eta^{\alpha\beta}(\partial^2 A_\mu(x)/\partial x^\alpha \partial x^\beta) - \eta^{\alpha\beta}(\partial^2 A_\alpha(x)/\partial x^\mu \partial x^\beta) = (4\pi/c)\,\eta_{\mu\nu}\,j^\nu(x). \tag{1.6b}$$

We now multiply Eq. (1.6b) through by the operator $(\partial/\partial x^\lambda)\eta^{\lambda\mu}$ and sum over the index μ. Since when the expression $\eta^{\lambda\mu}\eta_{\mu\nu}$ is summed over the index μ, the result is δ^λ_ν, and since $\eta^{\lambda\mu} = \eta^{\mu\lambda}$, the upshot of so doing is to turn Eq. (1.6b) into,

$$\eta^{\alpha\beta}\eta^{\mu\lambda}(\partial^3 A_\mu(x)/\partial x^\lambda \partial x^\alpha \partial x^\beta) - \eta^{\lambda\mu}\eta^{\alpha\beta}(\partial^3 A_\alpha(x)/\partial x^\beta \partial x^\lambda \partial x^\mu) = (4\pi/c)\,\delta^\lambda_\nu\,(\partial j^\nu(x)/\partial x^\lambda), \tag{1.6c}$$

which implies that,

$$0 = \partial j^\lambda(x)/\partial x^\lambda. \tag{1.6d}$$

Thus the Eq. (1.6b) *gauge-invariant* field equation for $A_\mu(x)$ *compels the local charge conservation expressed by* Eq. (1.1). The Eq. (1.6d) result is *a particular instance* of the fact that imposing the subsidiary dynamic symmetry of gauge invariance *produces locally conserved currents;* e.g., the imposition of gauge-invariance symmetry on the structure of the coupling of the electromagnetic four-potential $A_\mu(x)$ to a charged particle's quantum wave function $\psi(x)$ *ensures* the existence of a locally conserved current constructed from $\psi(x)$.

Because Eq. (1.6b) is gauge invariant, it has an infinite number of solutions for $A_\mu(x)$: given any solution $A_\mu(x)$ *and any scalar function* $\chi(x)$ *whatsoever,* $A_\mu(x) + \partial\chi(x)/\partial x^\mu$ *is also a solution that is by no means guaranteed to be Lorentz covariant when, for example,* $\chi(x)$ *is independent of* x^0. But since gauge invariance *is a subsidiary effect of strictly Lorentz-covariant four-vector electromagnetic theory, the solutions of* Eq. (1.6b) *that aren't Lorentz covariant are excluded.* Therefore the gauge invariance of Eq. (1.6b) *must be broken in a way which ensures that* $A_\mu(x)$ *is Lorentz covariant.* The *simplest* way to ensure that $A_\mu(x)$ is Lorentz covariant is to impose *the Lorentz condition*,

$$\eta^{\alpha\beta}(\partial A_\alpha(x)/\partial x^\beta) = 0, \tag{1.7a}$$

on $A_\mu(x)$, which *reduces* Eq. (1.6b) to,

$$\eta^{\alpha\beta}(\partial^2 A_\mu(x)/\partial x^\alpha \partial x^\beta) = (4\pi/c)\,\eta_{\mu\nu}\,j^\nu(x). \tag{1.7b}$$

Although the operator $\eta^{\alpha\beta}(\partial^2/\partial x^\alpha \partial x^\beta)$ *doesn't have a unique inverse, only its retarded inverse is physically appropriate to the causality of* $A_\mu(x)$ *with respect to* $j^\nu(x)$, so Eq. (1.7b) implies that,

$$A_\mu(x) = (4\pi/c)\,[\eta^{\alpha\beta}(\partial^2/\partial x^\alpha \partial x^\beta)]^{-1}_{\mathrm{ret}}\,(\eta_{\mu\nu}\,j^\nu(x)). \tag{1.7c}$$

The Eq. (1.7c) *causal and Lorentz-covariant result for the dynamic four-vector electromagnetic potential* $A_\mu(x)$ is the consequence *of breaking the gauge invariance of* Eq. (1.6b) *by applying the retarded Lorentz gauge condition* [1] *to it.* The Eq. (1.7c) result for $A_\mu(x)$ is readily shown to be *consistent* with the Eq. (1.7a) Lorentz condition *because* $j^\lambda(x)$ *satisfies the local charge conservation condition* $\partial j^\lambda(x)/\partial x^\lambda = 0$.

2. Lorentz-covariant extension of weak static Newtonian gravity to dynamic metric gravity

Gravity is taken to be the result of conserved energy-momentum, a four-vector P^μ. The symmetric-second-rank-tensor density-flux of energy-momentum P^μ, i.e., the energy-momentum tensor $T^{\mu\nu}(x)$, is the *source* of the symmetric-second-rank-tensor gravitational potential $\phi_{\mu\nu}(x)$. Gravity *itself*, however, *also contributes to the total conserved energy-momentum, a nonlinear effect that can be neglected in the weak-gravity static limit*, where it is assumed that the static energy density $T^{00}(\mathbf{x})$ divided by c^2 *is an effective mass density which acts as the source of the weak-gravity static gravitational potential component* $\phi_{00}(\mathbf{x})$, where $|\phi_{00}(\mathbf{x})| \ll c^2$, *in accord with the weak-gravity static differential Newtonian Law of Gravity*,

$$\nabla^2_{\mathbf{x}}\phi_{00}(\mathbf{x}) = 4\pi\,G\,(T^{00}(\mathbf{x})/c^2). \tag{2.1}$$

The nonrelativistic equation of motion of a test body of mass m in the weak-gravity static Newtonian gravitational potential $\phi_{00}(\mathbf{x})$, where $|\phi_{00}(\mathbf{x})| \ll c^2$, is of course,

$$m\, d^2\mathbf{x}/dt^2 = -m\nabla_{\mathbf{x}}\, \phi_{00}(\mathbf{x}). \tag{2.2a}$$

The Eq. (2.2a) nonrelativistic equation of motion corresponds to the nonrelativistic Lagrangian,

$$L(d\mathbf{x}/dt, \phi_{00}(\mathbf{x})) = (m/2)|d\mathbf{x}/dt|^2 - m\, \phi_{00}(\mathbf{x}), \tag{2.2b}$$

which, exactly as the Eq. (1.3b) nonrelativistic Lagrangian leads to its corresponding Eq. (1.4a) Lorentz-invariant Lagrangian, leads to the following corresponding Lorentz-invariant Lagrangian,

$$L(dx^\mu/d\tau, \phi_{\mu\nu}(x)) = -(m/2)\eta_{\mu\nu}(dx^\mu/d\tau)(dx^\nu/d\tau) - (m/c^2)\phi_{\mu\nu}(x)(dx^\mu/d\tau)(dx^\nu/d\tau), \tag{2.3a}$$

that can be reexpressed *in the astonishingly simple dynamic gravitational metric form,*

$$L(dx^\mu/d\tau, g_{\mu\nu}(x)) = -(m/2)g_{\mu\nu}(x)(dx^\mu/d\tau)(dx^\nu/d\tau), \tag{2.3b}$$

where,

$$g_{\mu\nu}(x) \stackrel{\text{def}}{=} \eta_{\mu\nu} + (2/c^2)\, \phi_{\mu\nu}(x). \tag{2.3c}$$

The Eq. (2.3b) Lorentz-invariant Lagrangian $-(m/2)g_{\mu\nu}(x)(dx^\mu/d\tau)(dx^\nu/d\tau)$ for a test body in the Lorentz-covariant dynamic gravitational metric $g_{\mu\nu}(x)$ *merely swaps the Minkowski metric* $\eta_{\mu\nu}$ in the Eq. (1.4c) Lorentz-invariant Lagrangian $-(m/2)\eta_{\mu\nu}(dx^\mu/d\tau)(dx^\nu/d\tau)$ of a free test body *for the Lorentz-covariant dynamic gravitational metric* $g_{\mu\nu}(x)$. Thus gravity is very simply and precisely characterized as *a Lorentz-covariant distortion of the Minkowski metric of space-time.*

The Eq. (2.3b) Lagrangian $-(m/2)g_{\mu\nu}(x)(dx^\mu/d\tau)(dx^\nu/d\tau)$, *in addition to being Lorentz-invariant has the form of an invariant under general coordinate transformations when it is assumed that* $g_{\mu\nu}(x)$ *transforms as a covariant second-rank symmetric tensor under general coordinate transformations.* Thus, just as gauge invariance *is a spontaneous subsidiary effect of Lorentz-covariant four-vector electromagnetic theory*, general coordinate transformation covariance *is a spontaneous subsidiary effect of Lorentx-covariant symmetric-second-rank-tensor gravity theory.*

We note that Eq. (2.3c) implies that $\phi_{\mu\nu}(x) = (c^2/2)(g_{\mu\nu}(x) - \eta_{\mu\nu})$, which permits the Eq. (2.1) weak-gravity static differential Newtonian Law of Gravity to be reexpressed entirely in terms of $g_{00}(\mathbf{x})$ and $T^{00}(\mathbf{x})$,

$$\nabla^2_{\mathbf{x}} g_{00}(\mathbf{x}) = (8\pi/c^4)\, G\, T^{00}(\mathbf{x}). \tag{2.4}$$

Applying the Lagrangian equation of motion $d[\partial L/\partial(dx^\lambda/d\tau)]/d\tau = \partial L/\partial x^\lambda$ to the Eq. (2.3b) gravitational metric Lagrangian $L = -(m/2)g_{\mu\nu}(x)(dx^\mu/d\tau)(dx^\nu/d\tau)$ yields the following Lorentz covariant result,

$$m\, g_{\lambda\nu}(x)\left(d^2x^\nu/d\tau^2\right) = -(m/2)\left[\partial g_{\lambda\mu}(x)/\partial x^\nu + \partial g_{\lambda\nu}(x)/\partial x^\mu - \partial g_{\mu\nu}(x)/\partial x^\lambda\right](dx^\mu/d\tau)\,(dx^\nu/d\tau). \tag{2.5a}$$

Because of the matrix metric factor $g_{\lambda\nu}(x)$ *on its left side*, Eq. (2.5a) *isn't in the normal form for Lorentzian dynamics.* However, *if for all* x *the metric* $g_{\lambda\nu}(x)$ *has the matrix inverse* $g^{\kappa\lambda}(x)$ such that, when summed over the index λ, the product $g^{\kappa\lambda}(x)g_{\lambda\nu}(x)$ yields δ^κ_ν, *then* Eq. (2.5a) *can be put into the following normal form for Lorentzian dynamics*,

$$m\left(d^2x^\kappa/d\tau^2\right) = -(m/2)\, g^{\kappa\lambda}(x)\left[\partial g_{\lambda\mu}(x)/\partial x^\nu + \partial g_{\lambda\nu}(x)/\partial x^\mu - \partial g_{\mu\nu}(x)/\partial x^\lambda\right](dx^\mu/d\tau)\,(dx^\nu/d\tau), \tag{2.5b}$$

which is *the gravitational geodesic equation* for the motion of a test body in the metric $g_{\mu\nu}(x)$. The Eq. (2.5b) gravitational geodesic equation's *usual presentation* [2] is,

$$d^2x^\kappa/d\tau^2 + \Gamma^\kappa_{\mu\nu}(x)\,(dx^\mu/d\tau)\,(dx^\nu/d\tau) = 0, \tag{2.5c}$$

where the *affine connection* $\Gamma^\kappa_{\mu\nu}(x)$ [3] is defined as,

$$\Gamma^\kappa_{\mu\nu}(x) \stackrel{\text{def}}{=} (1/2)\, g^{\kappa\lambda}(x)\left[\partial g_{\lambda\mu}(x)/\partial x^\nu + \partial g_{\lambda\nu}(x)/\partial x^\mu - \partial g_{\mu\nu}(x)/\partial x^\lambda\right]. \tag{2.5d}$$

We've noted that when the metric $g_{\mu\nu}(x)$ transforms as a second-rank symmetric covariant tensor under general coordinate transformations, the Eq. (2.3b) Lagrangian which yields the gravitational geodesic equation *is a general invariant.* Therefore it isn't at all surprising that under those circumstances the gravitational geodesic equation *itself* of Eq. (2.5b) (or Eq. (2.5c)) transforms *as a generally contravariant vector* [4].

Just as in electromagnetic theory the Eq. (1.2) static Coulomb's Law is parlayed into the Eq. (1.6b) gauge-invariant equation for the four-vector potential $A_\mu(x)$, so in gravity theory the Eq. (2.4) weak-gravity static differential Newtonian Law of Gravity is parlayed into the general coordinate transformation covariant Einstein equation for the metric $g_{\mu\nu}(x)$. The Einstein equation's left side must be a symmetric second-rank tensor in order to match that property of its energy-momentum source on its right side. This, together with the requirements that the Einstein equation must be general coordinate transformation covariant and must imply the Eq. (2.4) differential Newtonian Law of Gravity in the weak-gravity static limit, pretty much pins down the left side of the Einstein equation as a linear combination of the Ricci tensor and the product of the curvature scalar with the metric, both of which *are nonlinear in the metric.* The final details of the Einstein equation arise from the vanishing of the generally covariant divergence of its left side in order to match that property of its energy-momentum source on its right side, and of course also from the requirement that the Einstein equation must imply Eq. (2.4) in the weak-gravity static limit.

Because the Einstein equation is generally covariant, every general coordinate transformation of any of its metric solutions is also a metric solution, so it has an infinite number of solutions. But since general covariance *is a subsidiary effect of strictly Lorentz-covariant symmetric-second-rank-tensor gravity theory, the metric solutions of the Einstein equation that aren't Lorentz covariant are excluded.* Furthermore, the Ricci tensor and the curvature scalar are constructed from the affine connection and its first derivatives with respect to space-time, but the affine connection *isn't well defined unless the metric has a matrix inverse for all values of* x (see Eq. (2.5d)). Thus *we must require that* $\det(g_{\mu\nu}(x)) \neq 0$ *for all values of* x. This requirement *can be combined with an affirmation of the Lorentz covariance of the metric* $g_{\mu\nu}(x)$ *in an astonishingly simple way by stipulating that* $\det(g_{\mu\nu}(x)) = C$ *for all values of* x, *where* C *is a fixed nonzero constant.* But because $\det(\eta_{\mu\nu}) = -1$, the *only* value that the fixed nonzero constant C can have is -1.

In 1915 Einstein *also* arrived at this coordinate condition $\det(g_{\mu\nu}(x)) = -1$ for all x, but via *a temporary foray* into the idea that physics is covariant under linear coordinate transformations of unit determinant [5] *in place of* his 1913 "general relativity" idée fixe that all of physics is covariant under general coordinate transformations. *Because of his very different mode of arrival at the coordinate condition* $\det(g_{\mu\nu}(x)) = -1$ *for all* x, Einstein apparently was *oblivious* to the twin facts *that it crucially guarantees the existence of the matrix inverse of the metric* $g_{\mu\nu}(x)$ *and is consistent with the Lorentz covariance of* $g_{\mu\nu}(x)$. Regardless, Einstein's application of the coordinate condition $\det(g_{\mu\nu}(x)) = -1$ for all x in his landmark November 18, 1915 paper *produced the correct values for both Mercury's remnant perihelion shift and the deflection of starlight by the sun's gravity* [5], but *before* he adopted the coordinate condition $\det(g_{\mu\nu}(x)) = -1$ for all x, Einstein had spent approximately two years struggling with results for Mercury's remnant perihelion shift *which were substantially too small* [5]. Furthermore, the *correct* result for the deflection of starlight by the sun's gravity which results from application of the coordinate condition $\det(g_{\mu\nu}(x)) = -1$ for all x *is twice a previous result firmly predicted by Einstein's highly-touted Principle of Equivalence!*

In spite of these hard facts, Einstein in a closely-subsequent November 1915 paper *renewed his commitment to his* 1913 *"general relativity" idée fixe that all of physics is generally covariant, which caused him to deny the physical importance of the coordinate condition* $\det(g_{\mu\nu}(x)) = -1$ *for all* x. Consequently the coordinate condition $\det(g_{\mu\nu}(x)) = -1$ for all x, which is *central* to the tremendous achievements of Einstein's landmark November 18, 1915 paper, *goes unmentioned in virtually all gravity textbooks!*

Einstein's *denial* of the physical importance of his highly successful November 18, 1915 coordinate condition $\det(g_{\mu\nu}(x)) = -1$ for all x *paved the way for* Alexandre Friedmann's 1922 *promotion of his Galilean-relativity-consistent coordinate condition* $g_{00}(x) = 1$ *for all* x, *which cannot accommodate the empirically well-established phenomenon of gravitational time dilation because that effect is given by* [6],

$$[(\text{the tick rate of a clock at } x_2)/(\text{the tick rate of a clock at } x_1)] = \sqrt{g_{00}(x_2)/g_{00}(x_1)}\,. \tag{2.6}$$

In spite of so unphysical a consequence, Friedmann's Galilean-relativity-consistent coordinate condition $g_{00}(x) = 1$ for all x was, *without any compelling argument for doing so*, incorporated into the standard Robertson-Walker metric form which is almost universally used in cosmology, i.e.,

$$(c\,d\tau)^2 = (c\,dt)^2 - (R(t))^2\{(1/(1-kr^2))(dr)^2 + r^2[(d\theta)^2 + (\sin\theta\,d\phi)^2]\}. \tag{2.7a}$$

The Galilean-relativity-consistent character of the Eq. (2.7a) Robertson-Walker metric form unsurprisingly imposes *Newtonian gravity* on cosmological models [7], a consequence of which is a Big Bang singularity wherein $R(t)$ in Eq. (2.7a) was equal to zero at a past finite time t [8]. At that past finite time t when $R(t)$

was equal to zero, *the matrix inverse of the* Eq. (2.7a) *metric form was obviously undefined*, so at that past finite time t *the affine connection (the gravitational field) was undefined, as were the curvature tensors!*

The Newtonian-gravity albatross which the Galilean-relativity-consistent character of the Eq. (2.7a) Robertson-Walker metric form imposes on cosmological models is, however, *completely unnecessary.* For example, the particular Eq. (2.7a) Robertson-Walker metric form that has $k = 0$, namely,

$$(c\,d\tau)^2 = (c\,dt)^2 - (R(t))^2\{(dr)^2 + r^2[(d\theta)^2 + (\sin\theta\,d\phi)^2]\}, \tag{2.7b}$$

is very easily coordinate-transformed to the metric form,

$$(c\,d\tau)^2 = (1/S(t))^6\,(c\,dt)^2 - (S(t))^2\{(dr)^2 + r^2[(d\theta)^2 + (\sin\theta\,d\phi)^2]\}, \tag{2.7c}$$

which satisfies Einstein's November 18, 1915 *Lorentz-covariant coordinate condition* $\det(g_{\mu\nu}(x)) = -1$ *for all* x *instead of satisfying* Friedmann's *Galilean-relativity-consistent coordinate condition* $g_{00}(x) = 1$ *for all* x, *and therefore doesn't permit either Newtonian gravity or a Big Bang singularity.*

The very simplest expanding-dust-sphere cosmological model has recently been studied in detail both in Galilean-relativity-consistent Friedmann coordinates, wherein $g_{00}(x) = 1$ for all x, and in Lorentz-covariant Einstein coordinates, wherein $\det(g_{\mu\nu}(x)) = -1$ for all x [9]. The *deceleration of cosmic expansion* in Friedmann coordinates *is changed to its acceleration* in Einstein coordinates, and the Friedmann-coordinate Big Bang *is swapped for a peak in this Einstein-coordinate inflation.*

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