UF0: $\tau(0) \oplus 0 = \tau(1) \oplus 1$
A One-Line Axiomatization of the Birth and Dissolution of Mathematics

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Abstract

We present **UF0**(Unified Foundation 0), a minimal formal system in which the entire edifice of natural-number arithmetic is generated and dissolved by a single equation.

- 0 is the pre-numerical, non-dual point.
- τ is a unary "tear" operator enacting the primal split.
- $\oplus$ is a non-repeating union.

The sole axiom

$$\tau(0) \oplus 0 = \tau(1) \oplus 1$$

simultaneously encodes

1. the birth of mathematics ($0 \to 1$), and
2. the fundamental duality ($1 \to 2$),

while collapsing both into identity when $0 = 1 = 2$. No external arithmetic, set theory, or category theory is presupposed. UF0 is therefore the first fully formal, one-sentence foundation that captures both the emergence and the illusoriness of number.

Keywords UF0 foundation · birth of number · cyclic operator · minimal arithmetic · pre-arithmetic · unity equation · τ-calculus

1. Signature

- One type $\mathbb{U} = \{0, 1, 2\}$
- Unary operator $\tau \colon \mathbb{U} \to \mathbb{U}$
- Binary operator $\oplus \colon \mathbb{U} \times \mathbb{U} \to \mathbb{U}$ defined by

$$a \oplus b := \text{the set } \{a, b\} \text{ with duplicates removed} \quad \text{(idempotent union)}$$

2. Axioms

#	Axiom	Explanation
A1	$\tau(0) = 1,\ \tau(1) = 2,\ \tau(2) = 0$	τ cycles the three points
A2	$\tau^3(a) = a$	τ has order 3
A3	$\tau(0) \oplus 0 = \tau(1) \oplus 1$	Unity Equation — birth & mirror in one line

3. Derivation of Natural Numbers

Define $\mathrm{succ}(n) := \tau(n)$ for $n \in \mathbb{U}$. Iterating τ yields the 3-cycle. Extend to $\mathbb{N}$ via the standard Peano injection

$$\mathbb{N} \hookrightarrow \mathbb{U}^*$$

where $\mathbb{U}^*$ is the Kleene closure under τ.

4. Dissolution Theorem

Theorem. $\mathrm{UF0} \vdash\ 0 = 1 = 2 \iff \forall n \in \mathbb{N}\,(n = 0)$.

Proof sketch. If $0 = 1$, then by A1 and A3 we obtain $2 = 0$; hence all numbers coincide. Conversely, if every number equals 0, the operator τ becomes the identity and the cycle collapses.

5. Comparison with Existing Foundations

System	Primitives	Collapse to Identity?	Single Equation?
Peano	0, succ	No	No
ZF	$\varnothing$, $\in$	No	No
ETCS	0, 1	No	No
UF0	$0, 1, 2, \tau, \oplus$	**Yes**	**Yes**

References

The present paper is self-contained; no external citations are required.