On the kinetic energy $E_k = mv^2$

Tai-choon Yoon[1]

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Abstract

The kinetic energy derived from Newton's equation of motion is obtained by integrating force over the distance traveled. Objects cannot move on their own. The motion of an object is the result of action-reaction. This can be said to be the same as interaction. Force is the source of interaction, and the result of interaction. So far, kinetic energy has been defined as $E_k = \frac{1}{2} mv^2$, but it is the amount of energy corresponding to the area of a triangle, which is half of the total energy. This corresponds to the area on the x-axis in a Cartesian coordinate system. There is also an identical area on the y-axis. When these two are combined, it becomes the sum of the energy of forces in both directions, not the energy applied to a one-way force, which forces in both directions become $E_k = mv^2$.

We are familiar with the two formulas of physical quantities: $E = mc^2$ that appears in the energy-mass equivalence principle, and $E_k = \frac{1}{2}mv^2$ that appears in kinetic energy. We know that these two physical quantities should be used differently even though they are physical quantities related to velocity. It is said that this is because force acts in one direction in kinetic energy. However, according to Newton's law of action-reaction, force acts in both directions. If one tries to push one's hand into the air, the air will be pushed back depending on whether one pushes slowly or hard, but there is little reaction force required. The reason why little reaction force is required is because kinetic energy is proportional to the square of the velocity, but it reflects the result of the reaction. Therefore, kinetic energy should be modified as follows to take into account the force acting in both directions.

$$E_k = 2Fs = mv^2. \tag{1}$$

Then, the two physical quantities can be explained simultaneously with a single definition of velocity.

Newton's laws of motion are well known as the law of inertia, the law of acceleration, and the law of action-reaction.

In the first law, the law of inertia, momentum (p) is expressed as the product of the mass (m) and the velocity of an object moving at a constant velocity (v). This shows that if the velocity is constant, the momentum is also constant, so the state of motion does not change unless an external force

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[1] Electronic address: tcyoon@hanmail.net

such as acceleration acts on it,

$$p = m\vec{v}\,. \tag{2}$$

This can be obtained by differentiating the position vector $\vec{r} = r\hat{r}$ with respect to time t,

$$p = m\vec{v} = m\frac{d\vec{r}}{dt} = m\left(\frac{dr}{dt}\right)\hat{r} = m\dot{r}\hat{r}\,, \tag{3}$$

where, $\vec{v} = \dot{r} = \frac{dr}{dt}$ represents the first differentiation with respect to time t. The differentiation equation with respect to distance r may be expressed as $\acute{r}$.

The second law, the law of acceleration, is obtained by differentiating the momentum p with respect to time t again.

$$F = m\vec{a} = \frac{dp}{dt} = m\frac{d\vec{v}}{dt} = m\frac{d}{dt}(\dot{r}\hat{r}) = m\ddot{r}\hat{r}. \tag{4}$$

Here, F represents force, and work W is defined as follows:

$$W = Fs. \tag{5}$$

Here, s represents displacement, the distance that an object moves due to acceleration. Therefore, work or kinetic energy E_k is obtained by integrating the equation (4) with respect to distance s.

In general, kinetic energy E_k is derived as follows,

$$E_k = \int_0^r F\,dr = \int_0^r \left(\frac{dp}{dt}\right) dr = m\int_0^r \left(\frac{dv}{dt}\right) dr = m\int_0^v \left(\frac{dr}{dt}\right) dv = m\int_0^v v dv = \frac{1}{2}mv^2. \tag{6}$$

In the Cartesian coordinate system, a point is expressed by a coordinate, and there are countless paths from the origin to the point, including one straight line. The area between the straight line and the x-axis becomes the area of a triangle. The area surrounded by the straight line and the coordinate of the y-axis is also the same. In other words, the area surrounded by one coordinate and the Cartesian coordinate system becomes the area of a rectangle. Similarly, when an object of mass(m) moves at a velocity v, its kinetic energy is equal to the product of the mass and the area of the square to the coordinate. Therefore, the total amount of energy is not $\frac{1}{2}mv^2$ but mv^2. This indicates that it is the same as the area of the square formed by the rectangular coordinate system and a point (coordinate) regardless of the path. This is because the amount of energy is not derived by multiplying the area of the triangle and the mass, but by multiplying the mass by the square of the velocity of the moving object at the time of acceleration or at the end of acceleration.

Work occurs only during the time that the object is accelerating. If acceleration is in progress, the velocity at the time of measurement while acceleration is in progress is v, or if the velocity at the time of acceleration is completed and the object moves at the velocity of v at the time of acceleration completion, work is done until the point of acceleration completion. Therefore, if the velocity is accelerated again from the velocity v_1 to the velocity v_2, the energy consumed for acceleration at that time is as follows. From the equation (6) above, we get the following equation,

$$E_k = m\int_{v_1}^{v_2} v dv = \frac{1}{2}m({v_2}^2 - {v_1}^2). \tag{7}$$

If an object accelerates from rest velocity $v_1 = 0$ to the speed of light so that $v_2 = c$, we get $E_k = \frac{1}{2}mc^2$. This does not match Einstein's energy-mass equivalence principle. This is because there is no other formula for the energy-mass equivalence principle, but we simply define that kinetic energy is $E_k = \frac{1}{2}mv^2$. Forces do not act in one direction, such as pushing each other in action-reaction or pulling each other like two forces in attraction, but act in both directions simultaneously, so the energy applied to one force F is to be doubled, which results mv^2.

The integration from the equation (4) has a slightly different aspect. The result of the integration of this differential equation can be obtained by multiplying by $\dot{r}$ and integrating with respect to the time t instead of the distance r traveled.

Multiplying by $2\dot{r}$ and rearranging the equation (4), we get the integral equation with respect to time t, which is integrated as follows,

$$E_k = 2\int_0^r F\,dr = 2m\int_0^r \ddot{r}dr = 2m\int_0^r \ddot{r}\left(\frac{dr}{dt}\right)dt = m\int_0^r 2\ddot{r}\dot{r}\,dt = m\left(\frac{dr}{dt}\right)^2 = mv^2. \quad (8)$$

It can be argued that dividing the above equation by 2 gives the same result as (6). However, Newton's third law, the law of action-reaction, supports this. When the force acting at a point is F_1 and the reaction force is F_2, the two forces are in opposite directions, so they can be written as follows. The same applies when two objects are pulling each other while maintaining balance.

$$\overrightarrow{F_1} + \left(-\overrightarrow{F_2}\right) = \vec{0}\,. \quad (9)$$

It is correct that the sum of the two forces is $\vec{0}$. However, this contains a flaw in interpretation. That is because when two objects interact, the state of being stationary without any force and the state of being balanced by the interaction by pulling each other or the state of being balanced by the repulsive force by pushing each other are interpreted as the same.

It can be expressed precisely as follows.

$$2\vec{F} = \overrightarrow{F_1} + \left|-\overrightarrow{F_2}\right|\,. \quad (10)$$

This indicates that when one of the forces acting on the right is $\vec{0}$, the other is also to become $\vec{0}$, indicating the state of being stationary before the action. No object can move on its own. Acceleration is required when a stationary object moves. If it moves at a velocity v after a time has passed, it moves at a velocity v after accelerating for a certain amount of time since it started. Acceleration requires energy. This is explained by the law of acceleration and the law of inertia. Action and reaction are required to move. To move faster, more acceleration is required. Therefore, all moving objects undergo acceleration or deceleration, and energy is required each time. The energy required at that time is obtained by multiplying the mass of the moving object by the square of the velocity.

The equation (8) also appears in the cross product of vectors.

The cross product of vectors $\vec{A}$ and $\vec{B}$ is used to express the area of a parallelogram whose angle between the two vectors is θ. That is,

$$\vec{A} \times \vec{B} = |A||B| \sin\theta. \qquad (11)$$

Therefore, when $\theta = \frac{\pi}{2}$, it represents the area of a rectangle or square. In other words, the vector whose path is $\vec{A} \rightarrow \vec{B}$ and the vector whose path is $\vec{B} \rightarrow \vec{A}$ are the same vectors with different directions, so the magnitude indicated by the square of the vector is equal to the area of the rectangle, or the area of the square when $|A| = |B|$.

The equation (8) also appears in the Einstein field equation,

$$R_{ik} - \frac{1}{2} g_{ik} R = \kappa T_{ik}. \qquad (12)$$

The solution when $\kappa = 0$ is called the Schwarzschild Solution, and since the energy tensor is 0, it is also called the Schwarzschild Exterior Solution. Fortunately, he heuristically derived out the escape velocity $v = \sqrt{\frac{2GM}{R}}$ as the integration constant, which provides $v^2 = \frac{2GM}{R}$. This is the same as the solution when $\kappa T_r{}^r = 4GM\sqrt{g^{\theta\theta}}$.

This is obtained by dividing the mass of a moving object from its kinetic energy. Here, v is the escape velocity, G is the gravitational constant, M is the mass of the system, and R is the radius of the system. If the speed of light is c, then it is expressed as $\frac{v^2}{c^2} = \frac{2GM}{Rc^2}$.

If we substitute Newton's law of universal gravitation $F = -\frac{GMm}{r^2}\hat{r}$ into the equation (4), the escape velocity is expressed as follows,

$$-\frac{GMm}{r^2} = m\ddot{r}. \qquad (13)$$

Multiplying both sides by $2\dot{r}$ and integrating, we get

$$v^2 = \frac{2GM}{r}. \qquad (14)$$

This is the same result as the Schwarzschild solution.

In equation (12), the solution when $\kappa T^r{}_r = 3H^2$ can be written as $HR = v$ or $H^2R^2 = v^2 = \frac{2GM}{R}$. This can be understood as the Hubble constant since v represents the escape velocity in celestial bodies and H is the reciprocal of time in the cosmological universe.

Equation (8) can be written as follows in Lagrangian form,

$$L = V - T = 2m\int_v^c v dv = mc^2\left(1 - \frac{v^2}{c^2}\right). \qquad (15)$$

Here, L is the Lagrangian, V = potential energy, T = kinetic energy. When the kinetic energy becomes $v = 0$, the total energy becomes equal to the potential energy, so the law of conservation of energy is deserved. $\left(1 - \frac{v^2}{c^2}\right)$ is a major topic in the Lorentz transformation and Einstein's field equations.

References

[1] https://en.wikipedia.org/wiki/Newton's laws of motion

[2] https://en.wikipedia.org/wiki/Kinetic energy