

**ERRATUM TO EXERCISE A4.2 IN “AN INTRODUCTION TO
THE THEORY OF THE RIEMANN ZETA FUNCTION” (1988)
BY S. J. PATTERSON**

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ABSTRACT. The evaluation of coefficients of the Laurent series of $\Gamma(x)$ on page 135 of Patterson’s book “An introduction to the theory of the Riemann zeta function” has sign and other errors which are corrected here. A C program is listed which demonstrates the application of Taylor series to compute the Gamma and Barnes G-function. [viXra:2507.0094]

1. LAURENT SERIES OF $\Gamma(z)$

The Laurent series of Euler’s Γ -function near the pole at $z = 0$ (with residuum 1) defines coefficients $a_k = [x^k]\Gamma(z)$:

$$(1) \quad \Gamma(z) = \frac{1}{z} + a_0 + a_1z + a_2z^2 + a_3z^3 + \dots$$

Obedying the functional equation [1, 6.1.15],

$$(2) \quad z\Gamma(z) = \Gamma(z + 1)$$

the coefficients a_k resurface as coefficients of the Taylor series at $z = 1$:

$$(3) \quad \Gamma(z + 1) = 1 + a_0z + a_1z^2 + a_2z^3 + a_3z^4 + \dots = \sum_{k \geq 0} a_{k-1}z^k$$

and its derivative

$$(4) \quad \Gamma'(z + 1) = \sum_{k \geq 1} ka_{k-1}z^{k-1}$$

supplied with the definition

$$(5) \quad a_{-1} \equiv 1.$$

We may start the evaluation of the a_i with the series of the digamma function [1, 6.3.14][6, §1.17][5]

$$(6) \quad \psi(1 + z) = -\gamma + \sum_{n \geq 2} (-)^n \zeta(n) z^{n-1}$$

where ζ is the Riemann zeta function and [7, A001620]

$$(7) \quad \gamma \approx 0.577215664\dots$$

the Euler-Mascheroni constant.

Remark 1. *In numerical practice this is replaced by Chebyshev series [17, 16, 14] or Padé series [5, 4].*

Date: July 22, 2025.

2020 Mathematics Subject Classification. Primary 33B15 Secondary 26A09.

The digamma function is the derivative of the logarithm of the Γ -function [1, 6.3.1]

$$(8) \quad \psi(z)\Gamma(z) = \Gamma'(z).$$

Transition $z \rightarrow z + 1$

$$(9) \quad \psi(z+1)\Gamma(z+1) = \Gamma'(z+1)$$

and insertion of the power series of the three terms shows that a convolution of the coefficients of ψ and Γ gives Γ' , a tight coupling within the coefficients of Γ :

$$(10) \quad \left[-\gamma - \sum_{m \geq 1} (-)^m \zeta(m+1) z^m \right] \sum_{k \geq 0} a_{k-1} z^k = \sum_{k \geq 0} (k+1) a_k z^k,$$

which is Jensen's Equation (49') [9]. Equating coefficients of z^0, z^1, z^2 etc. on both sides of the equation yields

$$(11) \quad -\gamma a_{-1} = 1a_0;$$

$$(12) \quad -\gamma a_0 + \zeta(2)a_{-1} = 2a_1;$$

$$(13) \quad -\gamma a_1 + \zeta(2)a_0 - \zeta(3)a_{-1} = 3a_2;$$

$$(14) \quad -\gamma a_2 + \zeta(2)a_1 - \zeta(3)a_0 + \zeta(4)a_{-1} = 4a_3,$$

so the a_i on the right hand sides are recursively computed starting from (5) [8, 8.321.1]:

Algorithm 1.

$$(15) \quad a_i = \frac{1}{i+1} \left[-\gamma a_{i-1} + \sum_{k=2}^{i+1} (-)^k \zeta(k) a_{i-k} \right]; \quad a_{-1} = 1.$$

Example 1. *The first examples are [7, A090998, A385960, A385961, A385962]*

$$(16) \mu_0 = -\gamma \approx -0.5772156649015329;$$

$$(17) \mu_1 = \frac{1}{2}[\gamma^2 + \zeta(2)] \approx 0.9890559953279726;$$

$$(18) \mu_2 = \frac{1}{3!}[-\gamma^3 - 3\gamma\zeta(2) - 2\zeta(3)] \approx -0.9074790760808863;$$

$$(19) \mu_3 = \frac{1}{4!}[\gamma^4 + 6\gamma^2\zeta(2) + 8\gamma\zeta(3) + 3\zeta(2)^2 + 6\zeta(4)] \approx 0.9817280868344002;$$

$$(20) \mu_4 = \frac{1}{5!}[-\gamma^5 - 10\gamma^3\zeta(2) - 20\gamma^2\zeta(3) - (15\zeta(2)^2 + 30\zeta(4))\gamma - 20\zeta(2)\zeta(3) - 24\zeta(5)] \approx -0.9819950689031452;$$

2. CORRIGENDUM

The main result: On page 135 of Patterson's book [13], there is

- a sign error in front of $\zeta(2)$ in (17),
- a sign error in front of $3\gamma\zeta(2)$ and a missing factor in front of $\zeta(3)$ in (18),
- two sign errors and a wrong factor for $\gamma^2\zeta(2)$ in (19),
- two sign errors and a missing mixed term $\zeta(2)\zeta(3)$ in (20).

3. RECIPROCAL Γ -FUNCTION

The expansion coefficients c_k of [1, 6.1.34]

$$(21) \quad \frac{1}{\Gamma(z)} = \sum_{k \geq 1} c_k z^k = \frac{1}{\frac{1}{z} + a_0 + a_1 z + a_2 z^2 + \dots} = \frac{z}{1 + a_0 z + a_1 z^2 + a_2 z^3 + a_3 z^4 + \dots}$$

are obtained by series reversion [1, 3.6.25], which means comparison of like powers of z of both sides of

$$(22) \quad \left[\sum_{k \geq 1} c_k z^k \right] \left[\sum_{l \geq 0} a_{l-1} z^l \right] = z.$$

Resummation $\sum_{k \geq 1} \sum_{l \geq 0} = \sum_{m=1}^{\infty} \sum_{k=1}^m$ at $k+l=m$ and solving for c_m yields the recurrence

Algorithm 2.

$$(23) \quad c_m = - \sum_{k=1}^{m-1} c_k a_{m-k-1}, \quad m > 1.$$

Algorithm 3. *This can be simplified to [11, 5.7.2][8, 8.321.2]*

$$(24) \quad c_m = \frac{1}{m-1} \left[\gamma c_{m-1} - \sum_{i=2}^{m-1} (-)^i \zeta(i) c_{m-i} \right], \quad m > 3.$$

Example 2. [7, A001620,A385965,A385966]

$$(25) \quad c_1 = 1;$$

$$(26) \quad c_2 = \gamma \approx 0.577215664901533;$$

$$(27) \quad c_3 = \frac{1}{12}[-\pi^2 + 6\gamma^2] \approx -0.655878071520254;$$

$$(28) \quad c_4 = \frac{1}{12}[4\zeta(3) - \pi^2\gamma + 2\gamma^3] \approx -0.042002635034095;$$

$$(29) \quad c_5 = \frac{1}{1440}[\pi^4 - 60\pi^2\gamma^2 + 60\gamma^4 + 480\gamma\zeta(3)] \approx 0.166538611382291;$$

$$(30) \quad c_6 = \frac{1}{1440}[288\zeta(5) - 20\pi^2\gamma^3 + \pi^4\gamma + 12\gamma^5 - 40\pi^2\zeta(3) + 240\zeta(3)\gamma^2] \approx -0.042197734555544;$$

Numerical values have been tabulated up to $k = 40$ by Wrench [18, Table 5], reproduced in Table 1.

Splitting off a factor $z+1$ on the right hand side of (21),

$$(31) \quad \frac{1}{z\Gamma(z)} = (1+z) \sum_{k \geq 0} b_k z^k,$$

defines coefficients b_k that have been tabulated up to $k = 38$ by Wrench [18, Table 4]. The computation of the Γ -function with Wrench's equation (20)

$$(32) \quad \ln[z\Gamma(z)] = -\gamma z + \sum_{k \geq 2} (-)^k \zeta(k) z^k / k, \quad |z| < 1$$

is demonstrated in the C-function `Gamma()` in the appendix.

k	c_k
1	1.00000000000000000000000000000000e+00
2	5.77215664901532860606512090082402431e-01
3	-6.55878071520253881077019515145390481e-01
4	-4.20026350340952355290039348754298187e-02
5	1.66538611382291489501700795102105236e-01
6	-4.21977345555443367482083012891873913e-02
7	-9.62197152787697356211492167234819898e-03
8	7.21894324666309954239501034044657271e-03
9	-1.16516759185906511211397108401838867e-03
10	-2.15241674114950972815729963053647806e-04
11	1.28050282388116186153198626328164323e-04
12	-2.01348547807882386556893914210218184e-05
13	-1.25049348214267065734535947383309224e-06
14	1.13302723198169588237412962033074494e-06
15	-2.05633841697760710345015413002057284e-07
16	6.11609510448141581786249868285534287e-09
17	5.00200764446922293005566504805999130e-09
18	-1.18127457048702014458812656543650558e-09
19	1.04342671169110051049154033231225019e-10
20	7.78226343990507125404993731136077723e-12
21	-3.69680561864220570818781587808576624e-12
22	5.10037028745447597901548132286323180e-13
23	-2.05832605356650678322242954485523742e-14
24	-5.34812253942301798237001731872793995e-15
25	1.22677862823826079015889384662242243e-15
26	-1.18125930169745876951376458684229783e-16
27	1.18669225475160033257977724292867407e-18
28	1.41238065531803178155580394756670904e-18
29	-2.29874568443537020659247858063369926e-19
30	1.71440632192733743338396337026725707e-20
31	1.33735173049369311486478139512226802e-22
32	-2.05423355176667278932502535135573380e-22
33	2.73603004860799984483150990433098201e-23
34	-1.73235644591051663905742845156477980e-24
35	-2.36061902449928728734345073542753101e-26
36	1.86498294171729443071841316187866690e-26
37	-2.21809562420719720439971691362686038e-27
38	1.29778197494799366882441448633059417e-28
39	1.18069747496652840622274541550997152e-30
40	-1.12458434927708809029365467426143951e-30

TABLE 1. Coefficients c_k in (21).

4. DOUBLE GAMMA-FUNCTION

Barnes' Double-Gamma-Function G satisfies [11, §5.17][12]

$$(33) \quad \Gamma(x)G(x) = G(x+1), \quad G(1) = 1.$$

It defines a logarithmic derivative $\varphi(x) = (d/dx) \log G(x)$

$$(34) \quad \varphi(x)G(x) = G'(x)$$

satisfying [3, (2.17)][2, §12][10, (29)]

$$(35) \quad \varphi(x) = (x-1)(\psi(x) - 1) + \varphi(1), \quad \varphi(1) = \frac{\ln(2\pi) - 1}{2}.$$

Remark 2. *Special values are $G(1/2) \approx 0.603244$ [7, A087014], a local maximum $\varphi(x) = 0$, $G(x) \approx 1.0730$ at $x \approx 1.39147$ [7, A245081, A245082], and a local minimum $\varphi(x) = 0$ at $x \approx 2.5576$ [7, A245083].*

A followup of (6) is the Taylor expansion

$$(36) \quad \varphi(x) = \varphi(1) - (\gamma+1)(x-1) + \zeta(2)(x-1)^2 - \zeta(3)(x-1)^3 + \zeta(4)(x-1)^4 - \dots$$

Taylor coefficients g_k of G are fixed as

$$(37) \quad G(1+x) = 1 + \sum_{k \geq 1} g_k x^k.$$

Insertion into (34) yields the analog of (10):

$$(38) \quad [\varphi(1) - (\gamma+1)x + \zeta(2)x^2 - \zeta(3)x^3 + \zeta(4)x^4 - \dots](1 + g_1x + g_2x^2 + g_3x^3 + \dots) = g_1 + 2g_2x + 3g_3x^2 + \dots$$

Equating equal powers of x on both sides, the recurrence follows an obvious pattern:

$$(39) \quad \varphi(1) = g_1;$$

$$(40) \quad \varphi(1)g_1 - (\gamma+1) = 2g_2;$$

$$(41) \quad \varphi(1)g_2 - (\gamma+1)g_1 + \zeta(2) = 3g_3;$$

$$(42) \quad \varphi(1)g_3 - (\gamma+1)g_2 + \zeta(2)g_1 - \zeta(3) = 4g_4;$$

Algorithm 4.

$$(43) \quad g_k = \frac{1}{k}[\varphi(1)g_{k-1} - (\gamma+1)g_{k-2} + \sum_{l=2}^{k-1} (-)^k \zeta(k)g_{k-l}], \quad g_0 \equiv 1.$$

Example 3.

$$(44) \quad g_1 = \varphi(1) \approx 0.4189385332046727417803297364;$$

$$(45) \quad g_2 = \frac{\ln^2(2)}{8} - \frac{\ln 2 \ln \pi}{4} + \frac{\ln^2 \pi}{8} - \frac{3}{8} - \frac{\gamma}{2} \approx -0.7008530851489250875957714314;$$

$$(46) \quad g_3 = \frac{\ln^3(2)}{48} - \frac{\ln^2 2}{16} - \frac{\ln 2 \ln \pi}{8} + \frac{\ln^3 \pi}{48} - \frac{\ln^2 \pi}{16} + \frac{11}{8} + \frac{\gamma}{4} + \frac{\pi^2}{18} + \frac{\ln^2 2 \ln \pi}{16} + \frac{\ln 2 \ln^2 \pi}{16} - \frac{3 \ln(2\pi)}{16} - \frac{\gamma(\ln 2 + \ln \pi)}{4} \approx 0.23018776205422844354581487236.$$

The computation of the G -function via (37) for positive real values based on the first 40 of these g_k coefficients is illustrated in the C-function `BarnesG()` in the program in the appendix.

The Taylor series for $G(x)$ follows from the Taylor series for $G(1+x)$ by solving (33) for $G(x)$ with (21):

$$(47) \quad G(x) = \frac{G(1+x)}{\Gamma(x)} \equiv \sum_{j \geq 1} h_j x^j = \left[\sum_{l \geq 0} g_l x^l \right] \left[\sum_{k \geq 1} c_k x^k \right].$$

$$(48) \quad \therefore h_j = \sum_{k=1}^j g_{j-k} c_k; \quad h_0 = g_0 = 1.$$

Example 4. *The list of the h_j starts*

$$(49)_1 = 1;$$

$$(50)_2 = \varphi(1) + \gamma \approx 0.9961541981062056023868418264880;$$

$$(51)_3 = \frac{\varphi(1)^2}{2} - \frac{\gamma}{2} + \varphi(1)\gamma - \frac{\pi^2}{12}$$

$$+ \frac{\gamma^2}{2} \approx -1.1149132726725708898650525079570;$$

$$(52)_4 = \frac{\varphi(1)^3}{6} - \frac{\varphi(1)\gamma}{2} - \frac{\varphi(1)}{2} + \frac{\pi^2}{18} + \frac{\gamma\varphi(1)^2}{2} - \frac{\gamma^2}{2} - \frac{\gamma}{2} - \frac{\varphi(1)\pi^2}{12} + \frac{\varphi(1)\gamma^2}{2}$$

$$- \frac{\pi^2\gamma}{12} + \frac{\zeta(3)}{3} + \frac{\gamma^3}{6} \approx -0.491130849766198815922858839744.$$

APPENDIX A. C IMPLEMENTATION OF $\Gamma(x)$ AND $G(x)$

```
#include <math.h>
#include <stdio.h>
#include <stdlib.h>

#ifndef M_PI1
/* Pi https://oeis.org/A000796
*/
# define M_PI1          3.141592653589793238462643383279502884L
#endif

/* G(1/2) https://oeis.org/A087014
*/
#define M_G1_2 0.6032442812094462061914292245347L

/** Evaluation of Euler's Gamma-function.
* The argument x is reduced to the interval [0,1/2] with
* the aid of the functional equation and reflection formula, and
* then evaluated with the Taylor series of Wrench, Math. Comp. 22 (1968) 617-626, eq (20).
* @param x real-valued argument
* @return Gamma(x)
* @author Richard J. Mathar
* @since 2025-07-19
*/
long double Gamma(long double x)
{
    /* list of -gamma =zeta[1] and zeta[k] = (-1)^k*RiemannZeta(k)/k for k>=2
    */
    static long double zeta[] =
    {
        0.,
        -5.77215664901532860606512090082402431e-01L, /* https://oeis.org/A001620 */
        8.22467033424113218236207583323012592e-01L, /* https://oeis.org/A072691 */
        -4.00685634386531428466579387170483330e-01L, /* https://oeis.org/A386403 */
        2.70580808427784547879000924135291974e-01L, /* https://oeis.org/A098198 */
    }
}
```

```

-2.07385551028673985266273097291406834e-01L, /* https://oeis.org/A386404 */
1.69557176997408189952419654965153420e-01L, /* https://oeis.org/A259928 */
-1.44049896768846118119971078549970966e-01L,
1.25509669524743042422335654813581557e-01L,
-1.11334265869564690490872529914712451e-01L,
1.00099457512781808533714595890031900e-01L,
-9.09540171458290422326092984114972673e-02L,
8.33538405461090040248864998373116379e-02L,
-7.69325164113521914728270643481813385e-02L,
7.14329462953613360592327532217953796e-02L,
-6.66687058824204680329034485673763374e-02L,
6.25009551412130407419832857179772938e-02L,
-5.88239786586845823389572706055037076e-02L,
5.5557676274036111022142478691456620e-02L,
-5.26316793796166607336276661556734264e-02L,
5.00000476981016936398056576019341711e-02L,
-4.7619070301422279907839579390287795e-02L,
4.54545562932046694424086365294630329e-02L,
-4.34782660530402593613510029473356035e-02L,
4.16666691503412104691449838516753293e-02L,
-4.00000011921401405860912074425482028e-02L,
3.84615390346751857063477397945579460e-02L,
-3.70370373129893255494603515548520063e-02L,
3.57142858473333580281591805292572850e-02L,
-3.44827586849193008107947933242727569e-02L,
3.3333333643775810806556060957254898e-02L,
-3.22580645311504163388186582999652687e-02L,
3.1250000072759744802390796625045486e-02L,
-3.03030303065580455068789453866453751e-02L,
2.94117647075943447317360884968363769e-02L,
-2.85714285722601100127134570534084071e-02L,
2.77777777781819978303067217843313774e-02L,
-2.70270270272236745901366886760681316e-02L,
2.63157894737799468301941750313220312e-02L,
-2.56410256410722817859053093832714059e-02L,
2.5000000000227373696006597232063314e-02L
};

if ( x == 0.)
    return nanl("");
else if ( x == 1. || x== 2.)
    return 1.L ;
else if ( x == 3. )
    return 2.L ;
else if ( x > 1.)
{
    /* reduce argument to region x<1
    * (x-1)(x-2)(x-3)..(x-n)Gamma(x-n)=Gamma(x)
    */
    const int n = x ;
    if ( x == n)
        return (x-1.0L)*Gamma(x-1.0L) ;
    long double f = x-1.0L ;
    for (int i=2 ; i<=n ; i++)
        f *= x-i ;
    return f*Gamma(x-n) ;
}
else if ( x < -1.)
{
    /* lift negative argument to region 0<x<1
    * x(x+1)(x+2)..(x+n)Gamma(x)=Gamma(x+n+1)
    */
    const int n = -x ;
    long double f = x ;
    for (int i=1 ; i<=n ; i++)
        f *= x+i ;
    return Gamma(x+n+1)/f ;
}
else if ( x > 0.5)
{
    /* reflection formula for 1/2 < x< 1 to keep x small
    */

```

```

        return M_PiI/sin(M_PiI*x)/Gamma(1.L-x) ;
    }
    else
    {
        /* Wrench Math Comp 1968 eq (20).
        * Accumulate the powers with Horner scheme backwards
        * through the list of zeta[]
        */
        int i = sizeof(zeta)/sizeof(long double)-1 ;
        long double val = zeta[i] ;
        for(; i > 0 ; i--)
            val = val*x+zeta[i-1] ;
        return expl(val)/x ;
    }
} /* Gamma */

/** Evaluation of Barnes G (double-gamma) function.
 * The argument x is reduced to the interval [0.5,1.4] with the
 * aid of the doubling formula and then evaluated using the roughly 40
 * first coefficients of the Taylor series around x=1.
 * @param x real-valued positive argument
 * @return G(x)
 * @author Richard J. Mathar
 * @since 2025-07-19
 */
long double BarnesG(long double x)
{
    /* list of expansion coefficients G(1+x)= sum_{k >=0} g[k]*x^k
    */
    static long double g[] =
    {
        1.L,
        4.189385332046727417803297364056176e-01L, /* https://oeis.org/A122914 */
        -7.008530851489250875957714314012163e-01L,
        2.301877620542284435458148723670193e-01L,
        1.722250875280789989163086400293195e-01L,
        -1.730051712749474434838808472155208e-01L,
        4.891558248172620385489138662872530e-02L,
        1.776338651299101608747736412822872e-02L,
        -2.094857489513651363463115860696725e-02L,
        7.553356816234341968130706295057217e-03L,
        3.155355619216939268909041122629250e-05L,
        -1.342643086942326207010896379225567e-03L,
        6.730876946047402107889482278636407e-04L,
        -1.291790361959763180663296174650732e-04L,
        -3.313687662547405966666359980182819e-05L,
        3.316880349005663079711923719812491e-05L,
        -1.136398091495942456735717401021501e-05L,
        1.254808922328232128002000326886695e-06L,
        7.307867193999494898667343831534017e-07L,
        -4.639200308816481046222604976789908e-07L,
        1.281945788032240213912157426008722e-07L,
        -9.300145078159181602039101425361011e-09L,
        -8.322094239502112501438282505538277e-09L,
        4.376441459287180760011218939096202e-09L,
        -1.085888012321594744636765321963967e-09L,
        7.190075732108255896823394849936488e-11L,
        6.073960639943675912346784799310504e-11L,
        -3.019045859122299951156590061624047e-11L,
        7.285123846921800039785202976190580e-12L,
        -5.908422824752644585253825661242362e-13L,
        -3.025482883218481405143266736324748e-13L,
        1.583143380208163076239959656962341e-13L,
        -3.938647610372438112537285328856724e-14L,
        4.209054155968583018976424938731895e-15L,
        1.006080732269768525832583244151471e-15L,
        -6.401895499229033822513508986706591e-16L,
        1.717874493874452064898120991702527e-16L,
        -2.344643296697054736019440701732451e-17L,
        -1.757710868698839585403598853058713e-18L,
        1.98499226326453542621043281514081e-18L,
    }
}

```

```

-6.016205927979519993729771000768044e-19
};

if ( x == 0.)
    return 0.L ;
else if ( x == 1. || x== 2. || x == 3.)
    return 1.L ;
else if ( x < 0.)
    return nanl(""); ;
else if ( x >= 1.4L)
{
    /* reduce argument to region x<1.4 by doubling formula
    */
    const long double xhalf = x/2.L ;
    long double gg = BarnesG(xhalf)*BarnesG(xhalf+0.5L)/M_G1_2 ;
    gg *= gg ;
    gg *= powl(4.0L, (xhalf-1.0L)*(xhalf-0.5L))
        *powl(M_Pi1,-xhalf)*Gamma(xhalf) ;
    return gg ;
}
else if ( x < 0.6L)
{
    /* reduce argument to region x>0.6 by (inverted) doubling formula,
    * i.e., calculating the G(2x)and G(x+1/2) where 2x and x+1/2
    * are closer to 1 (the origin of the Taylor expansion).
    */
    long double gg = BarnesG(2.0L*x)/Gamma(x)
        *powl(4.0L, (x-1.0L)*(0.5L-x))
        *powl(M_Pi1,x) ;
    gg = sqrtl(gg) ;
    return gg *M_G1_2 / BarnesG(x+0.5L) ;
}
else
{
    /* Accumulate the powers with Horner scheme backwards
    * through the list of g[], G(1+x)= sum_k g[k]*x^k.
    * This x in the formula is one less than the x in the argument.
    */
    x -= 1.0L ;
    int i = sizeof(g)/sizeof(long double)-1 ;
    long double val = g[i] ;
    for(; i > 0 ; i--)
        val = val*x+g[i-1] ;
    return val ;
}

} /* BarnesG */

#ifdef TEST
/** Test function for the computation of Gamma(x) and Barnes G(x).
* To be compiled with
* gcc -O2 -o Gamma -DTEST Gamma.c -lm
* There is a single command line argument, the value of x.
* The program prints x, Gamma(x) and G(x) to standard output.
* Note that the number of digits printed exceeds the typical precision
* on IEEE computers, so even using an argument like 1.0 on the command
* line will echo a different value in that standard output.
*/
int main(int argc, char *argv[])
{
    if (argc < 2)
    {
        printf("usage: %s x\n",argv[0]) ;
        return 1;
    }
    long double x= strtold(argv[1],NULL) ;
    printf("%.25Le %.25Le %.25Le\n",x,Gamma(x),BarnesG(x)) ;
    return 0 ;
}
#endif

```

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