# Solving ($\boldsymbol{a^3 + b^3 + c^3 = d^3 + e^3 + f^3}$) by means of algebra

**Author: Oliver Couto**

**Email: matt345@celebrating-mathematics.com**

## Abstract

In math literature (eg: ref. #3) most solutions to the above equations are dealth with using elliptic curve theory. In this paper the author has, by imposing certain conditions, provided a parametric solution of the equation by using Algebra only.

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Consider the equation:

$p^3 + q^3 + r^3 = u^3 + v^3 + w^3$ -----(1)

After transposing we get

$$(u^3 + v^3) - (p^3 + q^3) = (r^3 - w^3)$$

$$(u + v)(u^2 - uv + v^2) - (p + q)(p^2 - pq + q^2) = (r^3 - w^3) ---- (2)$$

Since equation (2) is a cubic equation inorder to make it a quadratic we eliminate a factor from both sides of the equation.

So we take, $(p + q) = (u + v) = (-w)$ & $r = 4w$

After removing the factor & simplification of (2) we get:

$$(p^2 - pq + q^2) - (u^2 - uv + v^2) = 63(w)^2 ----(3)$$

We note that equation (3), is numerically satisfied by:

$$(p, q, u, v, w) = (29, -33, 23, -27, 4)$$

Hence we parametrize equation (3) by taking:

$(p, q, u, v, w) = [(29 + kt), (-33 + t), (23 + kt), (-27 + t), (4 - kt - t)]$ ----(4)

And we get after simplication:

$$t = \frac{2(29k + 27)}{7(k+1)^2}$$

After substituting for ‘t’ at (p,q,u,v,w) in equation (4) and taking (r=4w), we get:

$$p = (261k^2 + 460k + 203)$$

$$q = -(231k^2 + 404k + 177)$$

$$r = -8(k+1)(15k + 13)$$

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$$u = (219k^2 + 376k + 161)$$

$$v = -(189k^2 + 320k + 135)$$

$$w = -2(15k + 13)(k + 1)$$

And, for (k=0) we get the numerical solution:

$$(203, -177, -104)^3 = (161, -135, -26)^3$$

And for (k=1) we get after removing common factors:

$$(231, -203, -112)^3 = (189, -161, -28)^3$$

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## References


1) Published math paper, author's Oliver Couto & Seiji Tomita, Generalized parametric solution to multiple sums of powers, Universal Journal of applied mathematics, Year July 2015, Volume 3(5),102-111, http://www.hrpub.org,
2) Wolfram Mathworld: website: mathworld.wolfram.com, section Number- -theory (Diophantine equations)
3) Website: Seiji Tomita, computation number theory: Section third powers, article # 323,233,337. http://www.maroon.dti.ne.jp/fermat/eindex.html
4) Website, (author) Oliver Couto, Number theory is: [http://www.celebrating-mathematics.com]
5) Published paper, Oliver Couto, Vixra.org, # 2502.0196,dtd 28 Feb, 2025, on Diophantine equation
6) Universal Journal of Applied Mathematics, Vol 4(3),45-65, year oct.2016, Parametric solutions to (six) nth powers equal to another (six) nth powers for n=(2,3,4,5,6,7,8,9). Author's Oliver Couto & Seiji Tomita
7) Ajai Choudhry, Number theory web site, https://sites.google.com/view/ajaichoudhry/publications
8) Jaroslaw Wroblewski , Tables of Numerical solutions for various degrees, Website, www.math.uni.wroc.pl/~jwr/eslp
9) Collection of Algebraic identities (Book), Tito Piezas, http://sites.google.com/site/tpiezas/