A New Perspective on Kirchhoff's Law in Linear Sinusoidal AC Circuits

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[Abstract] In 1847, Kirchhoff proposed his current law and voltage law through the study of complex linear DC circuits. Building on the analysis of transient currents in linear sinusoidal AC circuits, this paper demonstrates that Kirchhoff's current law can serve as an approximate formula for transient analysis in low- and medium-frequency AC circuits. However, it becomes invalid for transient analysis at high frequencies. Considering that real-world linear sinusoidal AC circuits inherently possess distributed radiation capacitance and inductance, the paper further reveals that, under low- and medium-frequency sinusoidal AC signals, these distributed effects are minimal. Therefore, Kirchhoff's voltage law and current law can be reliably used as approximate formulas for both transient and steady-state analysis in such frequency ranges. Under high-frequency sinusoidal AC signals, the circuit's distributed radiation capacitance and inductance become significant and cannot be neglected. As a result, Kirchhoff's voltage law and current law are no longer valid for transient and steady-state circuit analysis.

[Keywords] Kirchhoff's law, Linear sinusoidal circuits, Distributed radiation capacitance, Distributed radiation inductance, Transient state circuits, Steady state circuits, Electromagnetic radiation.

1. Introduction

In 1847, Gustav Kirchhoff proposed two fundamental principles [1], Kirchhoff's current law and Kirchhoff's voltage law, based on the study of complex linear DC circuits. These laws form the foundation for analyzing and calculating complex electrical circuits. Kirchhoff's laws were originally applied to DC linear circuits and then later extended to AC high-frequency circuits and further developed for both transient and steady-state analyses of LC nonlinear circuits [2] [3] [4].

Over the past half-century, high-frequency circuits have evolved from radio frequency (RF) and microwave technologies to the terahertz range, with signal wavelengths reducing from meters and centimeters to millimeters. However, in the application of high-frequency technology, existing theories often fall short in adequately explaining specific engineering challenges. In many areas involving high-frequency circuits and electromagnetic radiation, such as electromagnetic compatibility , transmission line theory, and antenna design [5] [6] [7] [8] [9], classical electromagnetic theory no longer fully meets the demands of modern technical applications.

2. Kirchhoff's Current Law in Linear Sinusoidal AC Circuits

Kirchhoff's current law defines the relationship between the branch currents at any node in an electrical circuit. By convention, currents entering the node are considered positive, while those leaving are considered negative. According to Kirchhoff's current law, the algebraic sum of all currents at a node is zero. This can be expressed mathematically as:

$$\sum_{k=1}^{n} i_k = 0$$

It is demonstrated below that Kirchhoff's current law does not hold during the transient analysis of linear sinusoidal AC circuits. As illustrated in Fig. 2.1, a sinusoidal AC source is connected in series with a resistor R. The sinusoidal signal has an angular frequency ω, a wavelength λ, and a peak current of I_P. A microelement of the circuit wire, of length Δz, is intercepted. and the node Δz has only one input current $i_{in}(t)$ and one output current $i_{out}(t)$, which form the simplest possible node.

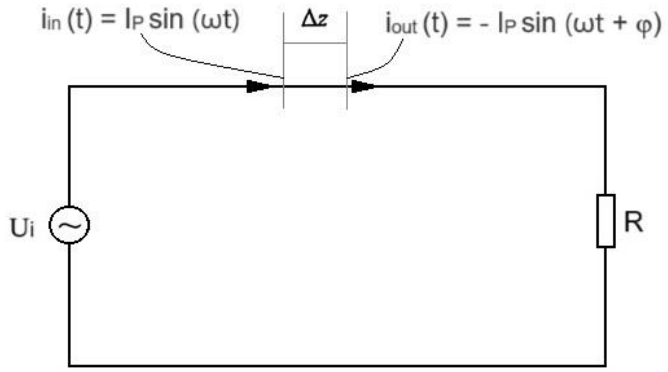

Fig. 2.1 Input and output currents at node Δz

Let the input current at node Δz at time t be defined as:

$$i_{in}(t) = I_P \sin(\omega t) \quad (2\text{-}1)$$

The output current at node Δz at time t is defined as:

$$i_{out}(t) = - I_P \sin(\omega t + \varphi), \quad (2\text{-}2)$$

where $\varphi = 2\pi(\Delta z/\lambda)$ is the phase difference.

According to practical engineering parameters, let the node length $\Delta z = 1.0$ mm, and assume the circuit operates at a frequency of 50 Hz. At time $t = 0$, using Equation (2-1), the input current $i_{in}(0) = 0$. At this frequency, the wavelength of the sinusoidal signal is $\lambda = 6 \times 10^6$ m, resulting in a phase difference of $\varphi = 6 \times 10^{-8\circ}$. According to Equation (2-2), the output current is:

$$i_{out}(t) = - I_P \sin(\varphi)$$
$$= - 1.05 \times 10^{-9} I_P$$

The current leaving the node is not exactly equal to the current entering the node, though they are approximately the same. Therefore, Kirchhoff's current law can be used as an approximate formula under low-frequency sinusoidal AC signals.

Let the circuit operating frequency be 10 GHz. At time $t = 0$, using Equation (2-1), the input current $i_{in}(0) = 0$. At this frequency, the wavelength of the sinusoidal signal is $\lambda = 0.03$ m, resulting in a phase difference of $\varphi = 12°$. According to Equation (2-2), the output current is:

$$i_{out}(t) = - I_P \sin(\varphi)$$
$$= - 0.21 I_P$$

The current leaving the node is not equal to the current entering it. Under high-frequency sinusoidal AC signals, Kirchhoff's current law no longer holds.

If the node Δz is set to zero, the surface through which current enters becomes the same as the surface through which it exits. Therefore, Δz cannot be zero. However, it is possible to compute the rate of change of the algebraic sum of the node currents with respect to Δz as Δz approaches zero. The algebraic sum of the currents entering and leaving the node Δz at time t is:

$$i_{in}(t) + i_{out}(t) = I_P \sin(\omega t) - I_P \sin(\omega t + \varphi)$$
$$= I_P (\sin(\omega t) - \sin(\omega t) \cos(2\pi(\Delta z/\lambda)) - \cos(\omega t) \sin(2\pi(\Delta z/\lambda)))$$

As Δz approaches zero, the rate of change of the algebraic sum of the node currents with respect to Δz is given by the following limiting expression:

$$\lim_{\Delta z \to 0} \frac{I_P(\sin(\omega t) - \sin(\omega t)\cos(2\pi(\Delta z/\lambda)) - \cos(\omega t)\sin(2\pi(\Delta z/\lambda)))}{\Delta z}$$

$$= \frac{I_P(\sin(\omega t) - \sin(\omega t) - \cos(\omega t)(2\pi(\Delta z/\lambda)))}{\Delta z}$$

$$= \frac{-I_P(2\pi\Delta z/\lambda)\cos(\omega t)}{\Delta z}$$

$$= -I_P(2\pi/\lambda)\cos(\omega t) \qquad (2\text{-}3)$$

In the above derivation, when Δz tends to 0, we can take $\cos(2\pi(\Delta z/\lambda) = 1$, $\sin(2\pi(\Delta z/\lambda) = 2\pi(\Delta z/\lambda)$. When $\omega t = 0$ or $\omega t = \pi$, Formula (2-3) reaches its maximum absolute value: $I_P(2\pi/\lambda)$.

From Formula (2-3), it follows that for low-frequency sinusoidal AC signals, $2\pi/\lambda$ is approximately zero, allowing Kirchhoff's current law to be used as an approximate formula. However, for high-frequency sinusoidal AC signals, $2\pi/\lambda$ can exceed 1, rendering Kirchhoff's current law invalid.

In summary, Kirchhoff's current law can be used as an approximate formula for transient analysis in low- and medium-frequency AC circuits. However, in high-frequency AC circuits, it is no longer valid for such analysis.

3. Kirchhoff's Voltage Law in Linear Sinusoidal AC Circuits

Kirchhoff's voltage law describes the relationship between the voltages across the elements of any closed loop in an electric circuit. Assuming that voltage rises are treated as positive and voltage drops as negative, the algebraic sum of the voltages around any closed loop is zero. This can be expressed mathematically as:

$$\sum_{k=1}^{n} U_k = 0$$

The following is an in-depth analysis of Kirchhoff's voltage law in a linear sinusoidal AC circuit. As shown in Fig. 3.1, the circuit consists of a sinusoidal AC voltage source U_i with angular frequency ω, connected in series with a resistor R. The connecting wires ABCD and EFGH are uniform conductors with low conductivity, each having a DC resistance of R_0 (in Ω/m).

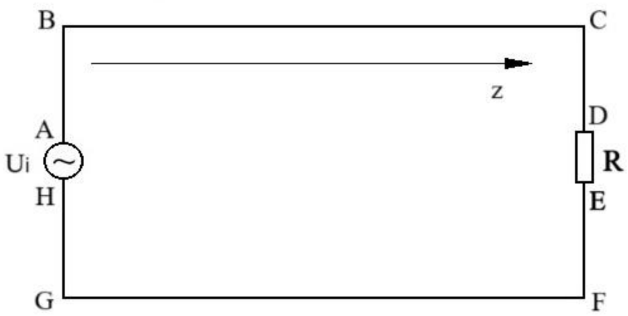

Figure 3.1 Resistive linear sinusoidal AC circuit

Assuming that the sinusoidal AC circuit in Figure 3.1 has no distributed capacitance or inductance, Kirchhoff's voltage law gives the following relationship:

$$U_i - U_{BA} - U_{CB} - U_{DC} - U_{ED} - U_{FE} - U_{GF} - U_{HG} = 0. \qquad (3\text{-}1)$$

Here, U_i, U_{BA}, U_{CB}, U_{DC}, U_{ED}, U_{FE}, U_{GF}, and U_{HG} represent the RMS values of the sinusoidal AC voltages. For example, U_{BA} denotes the voltage at point B with respect to point A, and similarly for the others. U_{ED} is the voltage across the resistor R. Equation (3-1) represents a steady-state analysis of the circuit, and it can be extended to the transient analysis of sinusoidal signals as well.

In Equation (3-1), no restriction is placed on the frequency of the sinusoidal AC signal. This leads to **Corollary I**: Under the assumption that distributed capacitance and distributed inductance are negligible, Kirchhoff's voltage law holds in purely resistive linear sinusoidal high-frequency circuits. It is evident that Corollary I applies to both transient and steady-state analyses of such circuits.

However, in high-frequency circuits, the connection lines inevitably exhibit non-negligible distributed capacitance and inductance. As a result, a circuit that behaves as a purely resistive linear circuit under DC conditions becomes a nonlinear circuit with significant capacitive and inductive characteristics when subjected to high-frequency signals.

Recent research [7] has identified two categories of capacitors, storage capacitors and radiation capacitors, and two categories of inductors, storage inductors and radiation inductors. The capacitors and inductors used in LC resonant circuits must be of the storage type.

In high-frequency circuits, the distributed capacitance of connecting lines is mainly distributed radiation capacitance, which radiates electric field energy into the surrounding free space. Similarly, the distributed inductance is mainly distributed radiation inductance, which radiates magnetic field energy into the surrounding free space. There is no energy exchange between the distributed radiation capacitance and inductance, meaning that LC resonance cannot occur between them.

In the following analysis, the high-frequency circuit shown in Figure 3.1 is examined with consideration of distributed parameters. A segment of the connecting line BC, with length Δz, is intercepted for analysis. The equivalent circuit of this wire segment, incorporating distributed parameters, is illustrated in Figure 3.2.

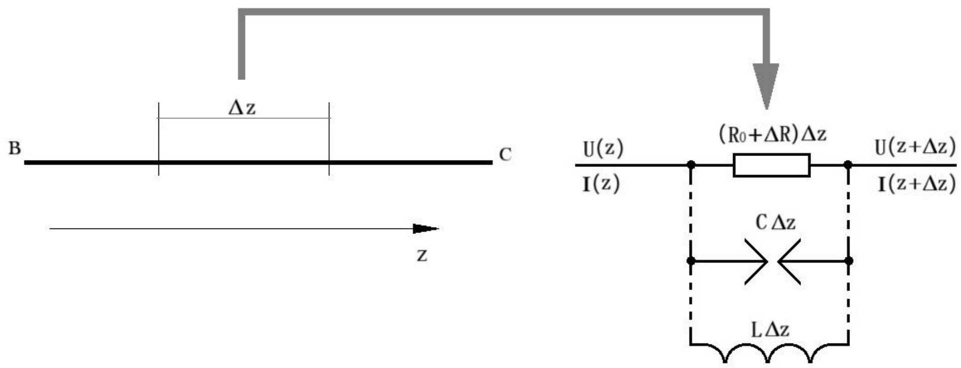

Fig. 3.2 Equivalent circuit of the wire Δz with distributed parameters

Owing to the skin effect, the resistance of high-frequency circuits increases. Let the resistance of the wire segment Δz be $(R_0 + \Delta R)\ \Delta z$, the distributed radiation capacitance be $C\Delta z$, and the distributed radiation inductance be $L\Delta z$. Let U (z) and U (z+ Δz) denote the RMS voltages, and I(z) and I(z+ Δz) the RMS currents. The phase difference between U(z) and I(z) is φ, while the phase difference between U(z+ Δz) and I(z+ Δz) is φ+Δφ.

The thermal loss power in the resistor $(R_0 + \Delta R)\Delta z$ is denoted by P_R; the electric field radiation loss power in the distributed capacitance $C\Delta z$ is P_C; and the magnetic field radiation loss power in the distributed inductance $L\Delta z$ is P_L. These loss components, P_R, P_C, and P_L, are distinct and cannot be transformed into one another.

According to the principle of energy conservation, the following equation holds:

$$U(z)\, I(z) \cos\varphi - U(z+ \Delta z)\, I(z+ \Delta z) \cos(\varphi+\Delta\varphi) = P_R + P_C + P_L. \quad (3\text{-}2)$$

In Equation (3-2), if the distributed radiation inductance and capacitance of the wire segment Δz are neglected, considering only its resistance $(R_0 + \Delta R)\Delta z$, then the wire segment behaves as a purely resistive linear element. In this case, $\varphi = \Delta\varphi = 0$, $I(z) = I(z+ \Delta z)$, allowing Equation (3-2) to be simplified as:

$$U_R(z)\, I_R(z) - U_R(z+ \Delta z)\, I_R(z) = P_R. \quad (3\text{-}3)$$

Equation (3-3) represents a purely resistive linear circuit. According to Corollary I, Kirchhoff's voltage law applies: the voltage difference across the segment Δz is equal to the purely resistive voltage drop $U_R(z) - U_R(z +\Delta z)$. Consequently, **Corollary II** can be stated as follows: if the voltage difference across Δz is not equal to the purely resistive voltage drop $U_R(z) - U_R(z + \Delta z)$, then Kirchhoff's voltage law does not hold. Corollary II is valid for both transient and steady-state analyses of the circuit.

In the circuit shown in Fig. 3.2, the DC resistance $R_0\Delta z$ is significantly larger than both the distributed resistance $\Delta R\Delta z$ and the capacitive and inductive impedances associated with the distributed radiation capacitance and inductance. Therefore, the phase differences φ and Δφ can be approximated as zero, allowing Eq. (3-2) to be simplified as follows:

$$U(z)I(z) - U(z+ \Delta z)\, I(z+ \Delta z) = (P_R + P_C + P_L) / \cos\varphi \quad (3\text{-}4)$$

From Eq. (3-3) and (3-4):

$$U(z)\, I(z) - U(z+ \Delta z)\, I(z+ \Delta z) > U_R(z)\, I_R(z) - U_R(z+ \Delta z)\, I_R(z). \quad (3\text{-}5)$$

Equation (3-5) indicates that for the Δz wire segment in Fig. 3.2, under high-frequency signals, the presence of non-negligible distributed radiation capacitance and inductance causes the actual voltage difference $U(z) - U(z+ \Delta z)$ to generally exceed the purely resistive voltage drop $U_R(z) - U_R(z+ \Delta z)$. According to Corollary II, if the voltage difference across Δz does not equal purely resistive voltage drop, then Kirchhoff's voltage law does not hold. Therefore, the law is invalid for high-frequency sinusoidal circuits, and this conclusion applies to both transient and steady-state analysis.

Equation (3-5) also shows that for the Δz wire segment in Fig. 3.2, under high-frequency sinusoidal signals, the presence of non-negligible distributed radiation capacitance and inductance causes the currents at the two ends of the wire to differ. In general, $I(z+ \Delta z) < I(z)$, indicating a loss of current along the segment. As a result, Kirchhoff's current law does not hold in such cases. This conclusion applies to both transient and steady-state analyses of high-frequency sinusoidal circuits.

In summary, the circuit shown in Fig. 3.1, a linear circuit containing only resistors, does not satisfy Kirchhoff's voltage law and Kirchhoff's current law under high-frequency sinusoidal signals, owing to the presence of significant distributed radiation capacitance and inductance. However, at low- and medium-frequency, where these distributed effects are minimal, Kirchhoff's current law and Kirchhoff's voltage law can be applied as approximate formulas for engineering analysis. These conclusions are valid for both transient and steady-state circuit analysis.

4. Conclusion

In 1847, Kirchhoff introduced his current law and voltage law through his study of complex linear DC circuits. According to the analysis of transient currents in linear sinusoidal AC circuits, this paper demonstrates that Kirchhoff's current law can serve as an approximate formula for transient analysis in low- and medium-frequency AC circuits. However, at high-frequency, Kirchhoff's current law is no longer valid for transient circuit analysis.

Since real-world linear sinusoidal AC circuits inherently possess distributed radiation capacitance and inductance, this paper further demonstrates that under low- and medium-frequency sinusoidal signals, these effects are minimal. As a result, Kirchhoff’s voltage law and Kirchhoff’s current law can be applied as approximate formulas for both transient and steady-state circuit analysis. However, at high-frequency, the distributed radiation capacitance and inductance become significant and cannot be ignored, rendering Kirchhoff’s voltage law and Kirchhoff’s current law invalid for transient and steady-state analysis.

The above conclusions are based on linear circuits containing only resistive elements. However, they can be extended to nonlinear circuits that include LCs, though this extension is beyond the scope of this paper.

Owing to the presence of non-negligible distributed radiation capacitance and inductance, high-frequency circuits do not exhibit only current energy flow within the connecting wires, but also electric field radiation energy flow (from the distributed radiation capacitance) and magnetic field radiation energy flow (from the distributed radiation inductance). These radiated energy flows diverge into the surrounding free space and do not form closed loops. According to the principle of energy conservation, electromagnetic radiation results in energy loss, leading to a reduction in current along the wires. Kirchhoff's current law no longer holds. At the same time, electromagnetic radiation results in energy loss, leading to the voltage drop of each wire segment is greater than the assumed pure resistance, and the algebraic sum of the voltages around the closed loop is less than 0. Kirchhoff's voltage law no longer holds.

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