Integrate the principle of uncertainty in relativity and classic mechanics

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ABSTRACT

From Restraint Relativity it is possible to consider a corpuscle as a packet of strings. The variation of the length of this packet is equal to the phase speed of the corpuscle as a packet of waves times an universal constant. In a system of units where $\hbar = c = a = 1$ the string vector becomes equal to the wave vector.

With this notion of packet of strings the principle of uncertainty is incorporated in Relativity & Classic Mechanics.

Vacuum energy density problem is resolved and we had concluded that Newton gravitational constant depends on the observable radius of the Universe.

At the vicinity of the horizon of a black hole the spectrum of radiation in the visible range will be continuous.

With the notion of modular cells General Relativity equations can be resolved easy by machine calculator. There is great similitude of equations forms of modular cells for weak & strong gravitational fields.

We had propose also a new model for the hydrogen atom and associate to a black hole a Planck atom by similitude to Bohr atom.

1-Revisited Special Relativity:

The introduction of Special Relativity by Einstein in 1905 is due only theoretical thinking that Maxwell equations of the electro-magnetic field should be the same in all inertial frames. Those equations in vacuum are:

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \qquad (1)$$

$$\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} \qquad (2)$$

Which are two equations of waves with a speed of propagation equal to $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ with:

$\varepsilon_0 = 8.85\ 10^{-12} Fm^{-1}$: the permittivity of vacuum .

$\mu_0 = 4\pi 10^{-7} Hm^{-1}$: the permeability of vacuum.

But what is vacuum? If vacuum signify that there is nothing but here there is the electromagnetic field and so there is temperature. The only response is that vacuum is when there is nothing and the temperature is zero absolute or approaches asymptotically to zero. If there is any electromagnetic field in vacuum it signify the fundamental state of the field : there is no emission of energy.

In Special Relativity based on the constancy of the speed of light in all inertial frames it is defined:

-Let's have two inertial frames$S(ct,x,y,z)$&$S'(ct',x',y',z')$ with S' is moving in constant speed v and the axes coincide at $t'=t=0$than the Lorentz transformations of space and time are as follows [1]:

$$ct' = \gamma(ct - \beta x)$$

$$x' = \gamma(x - \beta ct) \qquad (3)$$

$$y' = y$$

$$z' = z$$

With $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ & $\beta = \frac{v}{c}$

Considering two events $A(t_A, x_A, y_A, z_A)$ & $B(t_B, x_B, y_B, z_B)$ in the frame S the interval squared :

$$\Delta s^2 = c^2\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2 \qquad (4)$$

is invariant under Lorentz transformations.

If $\Delta s^2 > 0$: than the interval is time-like.

$\Delta s^2 = 0$: than the interval is light-like

$\Delta s^2 < 0$: than the interval is space-like.

Lorentz transformations can be also considered as hyperbolic rotation in the 4th space dimensions as the following:

$$ct' = ct\cosh\psi - x\, sinh\psi$$

$$x' = -ct\sinh\psi + x cosh\psi \qquad (5)$$

$$y' = y$$

$$z' = z$$

With $th\psi = \beta$, $cosh\psi = \gamma$, $sinh\psi = \gamma\beta$.

Space & time are considered as a four dimensions continuum that describes physics world with the Minkowski geometry related by relation (4).

The Minkowski geometry is not Euclidian due to the sign minus in (4).

The interval given by equation (4) is the "distance " between the events A & B measured in a straight line and can be considered as the worldline of a free particle moving in constant speed between the points A & B.

The interval measured between the two points A & B in any arbitrary path is:

$$\Delta s = \int_A^B ds \qquad (6)$$

With: $ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$: the line element of the space-time.

For a massive particle we define the proper time τas a parameter $t(\tau), x(\tau), y(\tau)$& $z(\tau)$ in order to determine the position of the particle in the 4th space-time as:

$$d\tau^2 = \frac{ds^2}{c^2} \qquad (7)$$

Which mean that:

$$d\tau = \sqrt{1 - \frac{v^2}{c^2}}.dt = \frac{dt}{\gamma} \qquad (8)$$

The total elapsed proper time interval is:

$$\Delta\tau = \int_A^B \sqrt{1 - \frac{v(t)^2}{c^2}}.dt \qquad (9)$$

Where $v(t)$ is the instantaneous speed of the corpuscle along its path.

If we introduce a rest frame in which the corpuscle is in rest along its path the proper time is the time recorded by a clock moving with the corpuscle.

It is convenient to introduce the indexed coordinates x^i $(i = 0,1,2,3)$ so we have:

$$x^0 = ct, x^1 = x, x^2 = y, x^3 = z \& ds^2 = g_{ij}dx^i dx^j$$

where g_{ij} are the covariant metric tensor as the following:

$$[g_{ij}] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad (10)$$

Or in shorthand notation as $[g_{ij}] = diag[1, -1, -1, -1]$.

The contravariant of the metric tensor are the same as: $[g^{ij}] = diag[1, -1, -1, -1]$.

In transforming coordinates x^i in a Minkowski space-time to a new coordinates x'^i the line & s'element ds^2 must take the same form as:

$$ds^2 = g_{ij}dx^i dx^j = g_{ij}dx'^i dx'^j$$

Which means that the transformation $x^i \rightarrow x'^i$ must satisfy:

$$g_{ij} = \frac{\partial x\prime^k}{\partial x^i}\frac{\partial x\prime^l}{\partial x^j} g_{kl} \; (11)$$

Which implies that the transformations must be linear to be a Lorentz transformations:

$x'^i = \Lambda^i_j x^j + a^i$ (12)

Where $\Lambda^i{}_j \& a^i$ are constants .

Transformations (12) are the inhomogeneous Lorentz transformations (or Poincare transformations). With a^itaken equal to zero they are the ordinary Lorentz transformations (or homogeneous transformations).

In a standard configuration of two inertial frames $S \,\&\, S'$ the matrix transformation can be written as follows:

$$[\Lambda^i_j] = \left[\frac{\partial x'^i}{\partial x^j}\right] = \begin{pmatrix} \gamma & -\beta\gamma 0 & & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} cosh\psi & -sinh\psi 0 & 0 \\ -sinh\psi & cosh\psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (13)$$

Where :$\gamma = \frac{1}{\sqrt{1-\beta^2}}$, $\beta = \frac{v}{c}$& $th\psi = \beta$, $cosh\psi = \gamma,\ sinh\psi = \gamma\beta$

The inverse transformation of (13) are obtained by replacing $v\ by - v\ \ or\ \ \psi\ \ by - \psi$

The matrix inverse is:

$$[\Lambda_i{}^j] = \left[\frac{\partial x^j}{\partial x'^i}\right]$$

And the relation ship between the two matrices :

$$[\Lambda_j{}^i] = g^{ik} g_{lj} \Lambda^l{}_k$$

And we have:

$$\Lambda^i{}_k \Lambda_j{}^k = \Lambda^i{}_k g^{km} g_{jl} \Lambda^l{}_m = g^{mi} g_{jm} = \delta^i_j$$

In any coordinate system the coordinate basis vectors are tangents to the coordinate curves

In frames $S \,\&\, S'$ basis vectors are related by:

$$e'^j = \Lambda^j_k e^k;\ \ e^j = \Lambda^j_k e'^k$$

And we have;

$$e'^i . e'^j = g^{ij} e^i . e^j = g^{ij}$$

Dual basis vectors are defined as:

$$e_i = g_{ik} e^k e'_i = g_{ik} e'^k$$

And we have:

$$e_i . e_j = g_{ij} e'_i . e'_j = g_{ij}$$

Where the components of g_{ij} are the same as g^{ij} .

We can define a 4-vector $\boldsymbol{v}$ of point P in Minkowski spacetime as :

$$\boldsymbol{v} = v^i \boldsymbol{e_i}$$

At each point P we have a set of orthonormal basis $\boldsymbol{e_i}$. The square of the length of a vector $\boldsymbol{v}$ is :

$$\boldsymbol{v}.\boldsymbol{v} = v_i v^i = g^{ik} v_i v_k$$

In two inertial frames S & S' with Cartesian coordinates x^i&x'^i the coordinates of a 4-vector $\boldsymbol{v}$ at a point P is :

$$\boldsymbol{v} = v^i \boldsymbol{e_i} = v'^i \boldsymbol{e'_i}$$

And we have :

$$v'^i = \boldsymbol{v}.\boldsymbol{e'^i} = \Lambda^i_j v^j$$

$$v^i = \boldsymbol{v}.\boldsymbol{e^i} = \Lambda^i_j v'^j$$

The 4-velocity $\boldsymbol{u}$ of a (massive) corpuscle is as :

$$\boldsymbol{u}.\boldsymbol{u} = (\frac{ds}{d\tau})^2 = c^2 \qquad (14)$$

The contravariant components of the 4-velocity are:

$$u^i = \boldsymbol{u}.\boldsymbol{e^i} = \frac{dx^i}{d\tau}$$

It comes that:

$$[u^i] = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}.(c, \frac{dx^1}{dt}, \frac{dx^2}{dt}, \frac{dx^3}{dt})$$

With : $c = \frac{dx^0}{dt}$, $x^0 = ct$ & $v = \sqrt{\left(\frac{dx^1}{dt}\right)^2 + \left(\frac{dx^2}{dt}\right)^2 + \left(\frac{dx^3}{dt}\right)^2}$

The 4-moment of a massive corpuscle is defined as :

$$\boldsymbol{p} = m\boldsymbol{u} \qquad (15)$$

With : m : the mass of the corpuscle;

$\boldsymbol{u}$: the 4-velocity of the corpuscle.

The component of the 4-moment are $p^i = \boldsymbol{p}.\boldsymbol{e^i}$ and we have:

$$[p^i] = \left[\frac{E}{c}, p^1, p^2, p^3\right] = (\frac{E}{c}, \vec{p}) \quad (16)$$

With E the energy of the corpuscle and $\vec{p}$ its 3 ordinary moment in an inertial frame .

It comes that:

$$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \& \vec{p} = \frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (17)$$

The squared length of the 4-moment is $p^i p_i$ and so get:

$$(\frac{E}{c})^2 - p^2 = m^2c^2 \qquad (18)$$

Where $p^2 = \vec{p}.\vec{p}$.

For a photon (massless corpuscle) we can't use the proper time to parametrize its worldline which is a null curve because $d\tau = 0$. We can use an affine parameter σ as for a photon moving in the direction $x = ct$ we have:

$$x^i = \sigma u^i \qquad (19)$$

With: $[u^i] = (1,1,0,0)$

The tangent vector of the worldline is:

$$\boldsymbol{u} = \frac{dx^i}{d\sigma}\boldsymbol{e_i} \qquad (20)$$

And we have:

$$\boldsymbol{u}.\boldsymbol{u} = 0 \quad \& \frac{d\boldsymbol{u}}{d\sigma} = \boldsymbol{0} \qquad (21)$$

Equation (21) is the equation of motion of the photon.

Of course in equation (21) we consider that the vector $\boldsymbol{e_i}$are constant and does not change with the position.

The tangent vector $\boldsymbol{u}$can be multiplied by any scalar and the equation (21) will be satisfied. The 4-moment of a photon can be defined as:

$$\boldsymbol{p} = \mu\boldsymbol{u} \qquad (22)$$

Where μ is constant and the components of $\boldsymbol{p}$are:

$$[p^i] = (\frac{E}{c}, \vec{p})$$

Where E &$\vec{p}$ are respectively the energy and the 3-moment of the photon in a given inertial frame S.

It comes that from equation (21):

$$E = pc \qquad (23)$$

The same equation (23) can be obtained for massless corpuscles if we take $m = 0$ in equation (18).

Equation (18) is a more generalized equation (massive corpuscles and massless corpuscles).

For photon we can introduce the 4-wavevector $\boldsymbol{k}$ as :

$$\boldsymbol{p} = \hbar \boldsymbol{k} \qquad (24)$$

And the components of the 4-wavevector are:

$$\left[k^i\right] = (\frac{2\pi}{\lambda}, \vec{k}) \qquad (25)$$

Where :λ : is the wave-length of the photon

$\vec{k} = \frac{2\pi}{\lambda}\vec{n}$: the 3-wavevector of the photon

$\vec{n}$: direction of the propagation of the photon (3-vector of one module).

Equation (25) like equation (18) can be generalized for any corpuscle (massless or not). Equating equation (25) to equation (16) we get:

$$\frac{E}{c} = \hbar.\frac{2\pi}{\lambda} \qquad (26)$$

$$\vec{p} = \hbar\vec{k} \qquad (27)$$

Which mean that massive corpuscle can have waving behavior.

The group speed v_g of such wave behavior of massive corpuscle is defined as :

$$\frac{1}{v_g} = \frac{dk}{d\omega} \qquad (28)$$

With :$\omega = 2\pi\nu = 2\pi\frac{c}{\lambda}$: the frequency of the photon (also the frequency associated to the massive corpuscle).

From (26) & (17) we get:

$$E = \hbar\omega = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (29)$$

And from (27) & (17) we get:

$$\hbar\vec{k} = \vec{p} = \frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (30)$$

From (28) we get:

$$v_g = v \qquad (30)$$

The phase speed v_f of such a wave is defined as:

$$v_f = \frac{\omega}{k} \qquad (32)$$

We have always:

$$v_g.v_f = v\frac{\omega}{k} = c^2 \qquad (33)$$

In equation (24) we had never speak about the constant $\hbar$: it can be replaced by any other constant and the 4-vector linked in will follow also.

For any corpuscle we introduce the 4- string-vector$\boldsymbol{l}$ as :

$$\boldsymbol{p} = a\boldsymbol{l} \qquad (34)$$

And the components of the 4-string-vector are:

$$[l^i] = (l, \vec{l}) \qquad (35)$$

With :l: the string-length associated to the corpuscle

$\vec{l} = l\vec{n}$: string-vector in the 3-ordinary space

$\vec{n}$: direction of the propagation of the corpuscle (3-vector of one module).

Equating (35) and (16) we get:

$$\frac{E}{c} = al \qquad (36)$$

$$\vec{p} = a\vec{l} \qquad (37)$$

Let's search the vector $\frac{d\vec{l}}{d\omega}$(considering the corpuscle as a packet of strings)?

$$\frac{dl}{d\omega} = \frac{dl}{dv}.\frac{dv}{d\omega} = \frac{1}{a}\frac{dp}{dv}.\frac{dv}{d\omega} = \frac{1}{a}\frac{dp}{d\omega} = \frac{\hbar}{a}\frac{dk}{d\omega} = \frac{\hbar}{a}.\frac{1}{v} = \frac{\hbar}{a}.\frac{v_f}{c^2}$$

It comes that:

$$\vec{v}_f = \frac{ac^2}{\hbar}\frac{d\vec{l}}{d\omega} = \frac{ac^2}{\hbar}\frac{d\vec{l}}{dv} \qquad (38)$$

From (35) & (16) we have:

$$E = acl = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (39)$$

From (37) & (17) we get:

$$a\vec{l} = \vec{p} = \frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}} = \hbar\vec{k} \qquad (40)$$

We have always:

$$\vec{p} = \frac{E}{c^2}.\vec{v} = \frac{al}{c}.\vec{v} = \frac{\hbar\omega}{c^2}.\vec{v} \qquad (41)$$

We can also define the 4-vector inertia$\boldsymbol{\xi}$as:

$$\boldsymbol{p} = \boldsymbol{\xi}c = m\boldsymbol{u} \qquad (42)$$

The components of the 4-vector inertia are:

$$[\xi^i] = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}}.\left(1,\frac{dx^1}{dx^0},\frac{dx^2}{dx^0},\frac{dx^3}{dx^0}\right) = \xi.\left(1,\frac{dx^1}{dx^0},\frac{dx^2}{dx^0},\frac{dx^3}{dx^0}\right) \quad (43)$$

With: $x^0 = ct$ & $\xi = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}}$

In a system of units where $\hbar = c = a = 1$ equation (41) can be written in simply form as:

$$\vec{p} = \xi\vec{v} = l\vec{v} = \omega\vec{v} \quad (44)$$

Inertia can be a mass or a length or a frequency or anything else as time for example.

We define the inertial time ζ as :

$$\zeta = \frac{\xi}{a} \quad (45)$$

It is evident if we associate to the corpuscle an inertial time at rest ζ_0 than according to equation (8) we have:

$$\zeta_0 = \sqrt{1-\frac{v^2}{c^2}}\,.\zeta \quad (46)$$

$d\zeta = dt$: when the energy of the corpuscle change

From (45) & (46) we get:

$$\zeta_0 = \frac{m}{a} \quad (47)$$

Of course we can also define the 4-vector inertial time $\boldsymbol{\zeta}$ as:

$$\boldsymbol{\xi} = a\,\boldsymbol{\zeta} \quad (48)$$

From equations (18) & (26) we get:

$$h^2\nu^2 - p^2c^2 = m^2c^4 \quad (49)$$

With: $E = h\nu$: the energy of the corpuscle

$\nu = \frac{\omega}{2\pi}$: reduced inertial frequency of the corpuscle;

$$h = 2\pi\hbar$$

Differentiate (49) according to time:

$$h^2\nu d\nu = c^2pdp = \xi c^2\vec{v}.d\vec{p} = h\nu\vec{v}.d\vec{p}$$

Which mean:

$$\frac{d\vec{p}}{dt}.\vec{v} = h\frac{d\nu}{dt} = ac^2\frac{d\zeta}{dt} = \vec{f}.\vec{v} \quad (50)$$

With: $\vec{f} = \frac{d\vec{p}}{dt}$: the 3-dimensional force.

If the energy of the corpuscle change then we have always:

$\vec{f}.\vec{v} = ac^2$ (51)

An important case is when the speed of the corpuscle is low compared to . From equation (49) we get:

$h\nu \approx mc^2 + \frac{1}{2}m\vec{v}^2$ (52)

$\vec{p} \approx m\vec{v}$ (53)

$\vec{f} \approx m\frac{d\vec{v}}{dt} = \frac{d\vec{p}}{dt} with\ \ \vec{p} = m\vec{v}$ (54)

which mean:

$$hd\nu = m\vec{v}.d\vec{v}$$

Than:

$h\frac{d\nu}{dt} = \vec{f}.\vec{v}$ (55)

Equation (55) is the same equation as (50) even thought it is obtained by approximation.

The square of (52) is :

$h^2\nu^2 = m^2c^4 + m^2c^2\vec{v}^2 + \frac{1}{4}m^2\vec{v}^4$ (56)

$$2h^2\nu\frac{d\nu}{dt} = m^2c^2 2\vec{v}.\frac{d\vec{v}}{dt} + \frac{1}{4}m^2\frac{d\vec{v}^4}{dt}$$

$h\frac{d\nu}{dt} = \frac{mc^2}{h\nu}\vec{v}.\vec{f} + \frac{1}{4}m^2\frac{d\vec{v}^4}{dt}.\frac{1}{2mc^2+m\vec{v}^2} \approx \frac{mc^2}{h\nu}\vec{v}.\vec{f} = \vec{f}.\vec{v}$ (57)

which mean :$h\nu \approx mc^2$ for low speed.

From equation(50):

$$h\nu^2 = \frac{m^2c^4}{h} + \frac{p^2c^2}{h}$$

$\frac{d(h\nu^2)}{dt} = 2\frac{c^2}{h}\vec{p}.\frac{d\vec{p}}{dt} = 2\frac{c^2}{h}.\frac{h\nu}{c^2}\vec{v}.\vec{f} = 2\nu ac^2 = 2h\nu\frac{d\nu}{dt}$ (58)

Which mean:

$ac^2 = h\frac{d\nu}{dt}$ (59)

Than:

$\nu = \frac{ac^2}{h}t + constant$ when the energy of the corpuscle is changing.

But in this case we have: $dt = d\zeta$ so:

$\nu = \frac{ac^2}{h}\zeta$ which is a trivial result (with $constant = 0$) (60)

Equation (51) signify that a corpuscle never can't be in rest otherwise the universal constant a will be equal to zero.Than absolute vacuum never exist in a frame : there is energy independent from the corpuscle and which don't let the corpuscle become in rest.

2.Wave-corpuscle duality:

Constant h was suggested and determined in 1900 by Max Planck in his model of black body radiation [2]. Planck model of black body was a cavity with many oscillators on his wall exchanging energy with radiation inner. Planck determine the mean energy of an oscillator and the radiation law at equilibrium in a temperature T of the cavity and get the values of constants h &k_B (Planck constant and Boltzmann constant) by a statistical manner and referring earlier to F.Kurlbaum& Wien measurements of black body radiation.

In his recent theory of heat radiation by 1911 Planck suggest that his oscillator should have enough time to absorb energy from radiation[3] . The Planck condition for this is that the frequency of the oscillator times time should be a great integer or in other manner:

$$\nu . t \gg 1 \qquad (61)$$

By 1923 De Broglie suggest the wave-corpuscle duality i.e. a corpuscle can have a waving behavior and vice versa: he propose the relation (27) and getting (28) & (29) by referring to Restraint Relativity proposed by A. Einstein in 1905.

In his suggestion De Broglie consider a corpuscle as a packet of waves moving in the same direction of the corpuscle and represented with a wave function having a constant amplitude. De Broglie consider a corpuscle as many monochromatic waves which reinforce each other in a limited region of spacetime and destroy each other away. Mathematically speaking this model can't be adopted because the waves will reinforce each other in many regions of spacetime in the same direction of the corpuscle motion and we can't see where is really the corpuscle. A replacement model of De Broglie one is that a corpuscle should be considered as a pulse and not as a packet of waves because in this case the corpuscle is more identified with the condition that the pulse should not spread so much in time. This condition implies that:

$$\nu . t < 1 \qquad (62)$$

The condition satisfying the two models (Planck & De Broglie) is:

$$\nu . t \approx 1 \qquad (63)$$

Which means that:

$$dt \approx -\frac{d\nu}{\nu^2} \qquad (64)$$

We take always dt positive and so we can omit the sign minus in (64) or take its absolute value:

$$dt \approx \frac{d\nu}{\nu^2} \qquad (65)$$

Replace (63) in (55) for low speed corpuscle we get:

$$h\nu^2 = \vec{f} . \vec{v} \qquad (66)$$

Replace (63) in (58) for high speed corpuscle we get:

$h\nu^2 = \vec{f}.\vec{v}$ (67)

Equation (67) is the same equation as equation (66) so it is applicable for any kind of oscillator. It is the power absorbed or emitted by an oscillator in a black body.

Planck oscillator is a classic oscillator so equation (66) is the work of a force applied on a corpuscle by unit time i.e. for a corpuscle in motion in a straight line along the axis x for example:

$$\vec{f}.\vec{v} = fv = h\nu^2$$

So the force acting on the corpuscle is:

$$f = \frac{1}{v}.h\nu^2$$

Duality of wave-corpuscle implies that:

$$\frac{1}{v} = \frac{dk}{d\omega}$$

with $v = v_g$: the group speed of the packet of waves assimilated as a corpuscle;

k: wave-vector of the packet of waves

$\omega = 2\pi\nu$: the frequency of the packet of waves.

So:

$$f = h\nu^2.\frac{dk}{d\omega} = h\nu^2.\frac{dk}{2\pi d\nu} = \hbar\nu^2.\frac{dk}{d\nu} = \frac{d(\hbar k)}{dt}$$

With :$\hbar = \frac{h}{2\pi}$: reduced Planck constant.

$\hbar k$: have the dimension of a moment. So:

$f = \frac{dp}{dt}$ with $p = mv$: is the moment of the corpuscle.

This relation is generalized as:

$\vec{f} = m\vec{\gamma}$(68)

With: $\vec{\gamma} = \frac{d\vec{v}}{dt}$the acceleration of the corpuscle

m : the mass of the corpuscle.

Equation (68) is the fundamental law of dynamics or the Newton first law.

Planck oscillator can have also very high speed and equation (67) will be applicable. As Planck suggest that his oscillators can absorb or emit energy with quanta as a multiple integer of $h\nu$ at the same time we extend this idea that Planck oscillators can absorb power or emit power with quanta as a multiple integer of $h\nu^2$.

Between the two models of oscillators (classic and relativist) there is a special frequency ν_0 in the border of our consideration to balance in one side or another and to deduce from Planck model of black body radiation even thought to apply this model for vacuum which can be considered as a black body at zero absolute temperature (no radiation) or approximatively near zero absolute. Replace (65) in (59) we get:

$$ac^2 \approx h\nu^2 \qquad (69)$$

Which mean that the frequency ν is near ν_0 changing all time otherwise there is no constant.

In fact we have exactly:

$$a = \frac{h\nu_0^2}{c^2} \qquad (70)$$

Even thought the constant ν_0 can be derived from classic approximations.

In general we have:

$$\frac{d\vec{p}}{dt} = \frac{d(a\zeta\vec{v})}{dt} = a\vec{v} + \frac{m}{\sqrt{1-\frac{\vec{v}^2}{c^2}}}\frac{d\vec{v}}{dt} \approx a\vec{v} + m\vec{\gamma} \text{for } \|\vec{v}\| \ll c$$

Than:

$$m\vec{\gamma} = \frac{d\vec{p}}{dt} - a\vec{v} = \vec{f} - a\vec{v} \qquad (71)$$

All forces applied on the corpuscle are equal to $\vec{f} = \frac{d\vec{p}}{dt}$ plus a permanent force $a\vec{v}$ in the opposite side of its motion which mean an external force which drive the motion of the corpuscle even thought it is in rest. The corpuscle never can't be in rest and will oscillate around its position.

3.Determination of constant ν_0 :

Planck oscillators are classic oscillators. They are oscillating charged corpuscles. In a black body they are the electrons of the atoms of the wall of the black body cavity (oscillations of nucleus of atoms are neglected).

According to Bohr model of the atom, the electron in an hydrogen atom is moving in planetary motion (circular) as the speed of the electron is equal to αc where $\alpha = \frac{1}{137}$ the fine structure constant. The vacuum in atoms is the same vacuum in the cosmos. Vacuum which is filled with energy has a certain mechanical impedance and a negative pressure so there is no friction in the motion of any corpuscle as it is given by General Relativity.

Suppose that a corpuscle is in motion from a point A to a point a point B, its energy exchanged with vacuum is:

$$\varepsilon = \int_A^B a\vec{v}.\vec{v}dt = \int_A^B a\vec{v}^2\, dt = \int_A^B ac^2 \left(1 - \frac{\zeta_0^2}{\zeta^2}\right) d\zeta = ac^2(\zeta_B - \zeta_A) + ac^2\zeta_0^2\left(\frac{1}{\zeta_B} - \frac{1}{\zeta_A}\right) \quad (72)$$

We have not do any approximation in equation (72). It is done in a 4-space dimensions frame .

Let's take an origin for the energy exchanged which is the point $A$where the corpuscle is in rest, than:

$$\varepsilon = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2 + ac^2\zeta_0^2\left(\frac{\zeta_0-\zeta}{\zeta_0\zeta}\right) = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2 + mc^2\left(\sqrt{1-\frac{v^2}{c^2}} - 1\right) = mc^2\left[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1 + \sqrt{1-\frac{v^2}{c^2}} - 1\right] = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}\left(1 - 2\sqrt{1-\frac{v^2}{c^2}} + \sqrt{1-\frac{v^2}{c^2}}^2\right) = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}\left(1 - \sqrt{1-\frac{v^2}{c^2}}\right)^2 \quad (73)$$

For the electron of Bohr atom we get:

$$\varepsilon = \frac{mc^2}{\sqrt{1-\alpha^2}}\left(1 - \sqrt{1-\alpha^2}\right)^2 \approx \frac{1}{4}\alpha^4 mc^2 \quad (74)$$

With: $m = 9.1\ 10^{-31} Kg$ the mass of the electron.

If the electron is oscillating in a 4th space dimension than its oscillations are driven by an electromagnetic field in its fundamental state. The energy of an oscillator in its fundamental state is [4]:

$$E = \frac{1}{2}h\nu \quad (75)$$

Vacuum is defined as a degenerate state of the electromagnetic field near zero Kelvin. A Planck oscillator in this field have an infinite degree of states and only one degree of freedom and his energy can take any value multiple integer of $\frac{1}{2}h\nu$ and if the half of this integer is integer than it can be the source of spontaneous emission as proposed by Einstein in his theory of laser radiation. The energy of the oscillator is also called zero-point energy and is protecting bound states in an atom [5].

We can modeling the mean energy of the oscillator as Planck did in his theory of black body radiation by considering the high states of the oscillator are not so probable and the low states are more probable by referencing to a characteristic energy $\frac{1}{2}h\nu_0$ of the vacuum ,which should be normally in the infrared.

Taking this model the mean energy U of the the electron is by statistical manner:

$$U = \frac{\frac{1}{2}h\nu}{\exp\left(\frac{\nu}{\nu_0}\right) - 1} \quad (76)$$

Equation (76) is obtained by statistical manner as for low frequencies the energy of the oscillator is:

$$E \approx \frac{1}{2}h\nu_0 \quad (77)$$

Considering the electron as 4-space dimension corpuscle than we get:

$$\frac{1}{2}h\nu_0 = 4x\frac{1}{4}\alpha^4 mc^2 = \alpha^4 mc^2 \quad (78)$$

Which mean:

$$\nu_0 = 2\frac{\alpha^4 mc^2}{h} \approx 0.7\ 10^{12} Hz \qquad (79)$$

4-What is a corpuscle?:

Planck had determined his constant h from a thermodynamic system the black body which is a closed system. This constant is an adiabatic invariant used for linked systems (the oscillators) as an integral of action [6].

For open systems the action can vary with time and if we introduce an element of periodicity Planck constant will have all its generality using it for any system and this what Louis De Broglie had understand in earlier time [7].

To link Planck theory of the grain of energy $E = h\nu$ to the relativist theory of the corpuscle packet of energy $E = mc^2$ De Broglie had associate to the corpuscle internal frequency when the corpuscle is in rest as:

$$h\nu_{0i} = mc^2 \qquad (80)$$

With :m: the mass of the corpuscle in its rest frame S' (sometimes it is the mass of the electron for De Broglie).

If the corpuscle is in motion from a fixed observer in an inertial frame S with a fixed speed v than according to Lorentz transformations of time we have:

$$t' = t\sqrt{1 - \frac{v^2}{c^2}} \qquad (81)$$

t': time measured with a fixed clock in the frame S'

t: time measured by the observer with a fixed clock in the frame S .

The internal frequency associated to the corpuscle becomes for the observer in the frame S as the inverse of transformation (81) of time:

$$\nu_i = \nu_{0i}\sqrt{1 - \frac{v^2}{c^2}} = \frac{mc^2}{h}\sqrt{1 - \frac{v^2}{c^2}} \qquad (82)$$

But for relativity the energy of the corpuscle is :

$$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (83)$$

And the grain of energy according to Planck theory should have a frequency ν as:

$$\nu = \frac{mc^2}{h\sqrt{1-\frac{v^2}{c^2}}} \qquad (84)$$

Which is radically different from (82).

In general Lorentz transformations for space & time when the frame S' is in motion from frame S with a constant speed v are:

$$t' = \frac{t - \frac{xv}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (85)$$

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

If there is a stationary wave in the frame S' associated to the corpuscle and have the internal frequency as ν_{0i} than this wave appear for the observer in the frame S as having a phase:

$$\nu_{0i} t' = \frac{\nu_{0i} t - \nu_{0i} \frac{xv}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\frac{mc^2}{h} t - \frac{mc^2 xv}{h \ c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\frac{mc^2}{h} t - \frac{mxv}{h}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{mc^2}{h\sqrt{1 - \frac{v^2}{c^2}}} \left(t - \frac{xv}{c^2} \right) = \nu t' \sqrt{1 - \frac{v^2}{c^2}} \qquad (86)$$

It is like that it have a packet of waves in the frame S with a frequency:

$$\nu = \frac{mc^2}{h\sqrt{1 - \frac{v^2}{c^2}}} \qquad (87)$$

And propagate in the frame S with a phase speed:

$$v_f = \frac{c^2}{v} \qquad (88)$$

Combining equations (81) & (86) we get:

$$\nu_{0i} t' = \frac{mc^2}{h} t \sqrt{1 - \frac{v^2}{c^2}} = \nu_i t = \nu \left(t - \frac{xv}{c^2} \right) \qquad (89)$$

So the internal wave ν_i have the same phase as the associated packet of waves ν.

But the two waves ν & $\nu_i$have not the same phase speed. This problem had intrigued De Broglie for a long time and his reconciliation is only that they have the same phase which is an invariant in relativity . Also he postulate that they have the same phase speed. Georges Lochak had appease this postulation by considering that in a Galilean frame the difference between the two frequencies is compensate by the difference in phase speeds.

This grain of energy should have an associated wave as :

$$\lambda = \frac{v_f}{\nu} = \frac{h\sqrt{1 - \frac{v^2}{c^2}}}{mv} \qquad (90)$$

And the internal packet of waves as:

$$\lambda_i = \frac{v_f}{\nu_i} = \frac{h}{mv\sqrt{1 - \frac{v^2}{c^2}}} \qquad (91)$$

The two associated waves are not the same. So we have according to the harmony of phases as De Broglie postulate (same phase and the same phase speed):

$\frac{2\pi}{\lambda_i}x - 2\pi\nu_i t = \frac{2\pi}{\lambda}x - 2\pi\nu t$ for every (x, t) (92)

It is very easy to deduce from equation (92) that:

$v = 0$ (93)

This is not what we should expect. Instead of equation (92) we can relax it as:

$v \approx 0$ (94)

The corpuscle is an oscillating resonator with a mean speed equal to zero ($\dot{\bar{q}} = 0$).

Or equation (92) becomes when neglecting the quadratic term $\left(\frac{v}{c}\right)^2$:

$\frac{2\pi mv}{h}x - \frac{2\pi mc^2}{h}t \approx \frac{2\pi mv}{h}x - \frac{2\pi mc^2}{h}t$ (94)

The two members (left & right) of equation (92) becomes the same.

For De Broglie "the energy of an electron is spread over all space with a strong concentration in a very small region". Speaking about a mass in a point have no sense and this mass which characterize the corpuscle only because it is not undividable and constitute a unit as De Broglie postulate.

The formulae of the energy of a corpuscle as given by Restraint Relativity $E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$ is in fact not an exact formulae: it should be revisited. From Restraint Relativity we can deduce such a conclusion.

Let's consider the scalar product of the 4-vector force and the 4-vector speed as[8]:

$\boldsymbol{f}.\boldsymbol{u} = \frac{d\boldsymbol{p}}{d\boldsymbol{\tau}}.\boldsymbol{u} = \left(\frac{dm}{d\boldsymbol{\tau}}\boldsymbol{u} + m\frac{d\boldsymbol{u}}{d\boldsymbol{\tau}}\right).\boldsymbol{u} = c^2\frac{dm}{d\boldsymbol{\tau}} + m\boldsymbol{u}.\frac{d\boldsymbol{u}}{d\boldsymbol{\tau}} = c^2\frac{dm}{d\boldsymbol{\tau}}$ (95)

Where we have used that in (14) the fact that $\boldsymbol{u}.\boldsymbol{u} = c^2$.

If $m = Constant$ than $\boldsymbol{f}.\boldsymbol{u} = 0$ so the 4-force doesn't bring any mass to the corpuscle. We can relax this assertion by considering that if the parcel of energy mc^2 does not spread much away in his rest frame than the 4-force $\boldsymbol{f}$ is a pure force.

If the energy of the corpuscle is not exactly determined than we can consider it is as oscillating around which we consider its position and it have some spatial extension as:

$\boldsymbol{p} = a\boldsymbol{l}$ (96)

With $\boldsymbol{l}$: 4-vector string associated to the corpuscle

The corpuscle is oscillating because we need $\dot{\bar{q}} = 0$ to be conform with De Broglie phase wave. Such a corpuscle will influence by its packet of waves all the other corpuscles around it and vice versa because there is overlap of waves incoming from many sources. As a string when it is stretched by an increasing force it will vibrate more and more sharply, the overlap of waves will create an interaction force between corpuscles and this is universal independent from the nature of corpuscles: this is *gravitation*.

If there is a wave associated to the corpuscle we expect that this wave move in the same direction of the corpuscle and it is represented by the wave function [9]:

$$\psi(x,t) = A\exp(ikx - i\omega t) \qquad (97)$$

This wave function describe a plane wave which itself and the same phase planes move with the phase speed:

$$v_f = \frac{\omega}{k} \qquad (98)$$

The group speed is the speed at which a quantity of energy is transmitted in the space and the corpuscle can be considered as a packet of energy   which move at the speed:

$$\frac{1}{v} = \frac{dk}{d\omega} = \frac{dk}{dv}.\frac{dv}{d\omega} \qquad (99)$$

If the relation $E = \hbar\omega$ is valid for photons and still continue to be valid for corpuscles than we have:

$$\hbar\omega = E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (100)$$

And from (99) & (100) we deduce that:

$$\hbar k = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} = p \qquad (101)$$

If equations (100) & (101) are valid in a frame S than they should be valid in any inertial frame S' which is in constant motion by reference to S. If the frame S' is the rest frame of the corpuscle than and by convenience that the phase $kx - \omega t$ is invariant we should have:

$$kx - \omega t = -\frac{mc^2}{\hbar}t' \qquad (102)$$

The time t' is given by Lorentz transformations as:

$$t' = \frac{t-\frac{xv}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (103)$$

And so we get from (102) the same relations (100) & (101).

If we neglect totally the quadratic term $\frac{v^2}{c^2}$ we get De Broglie wave function without the notion of internal vibration: it signify only that the position of the corpuscle (the electron) is very well defined as in the classic way.

Let's take the dynamic state of the corpuscle described by the initial wave function $\psi(x,0)$.What is the position and the moment of the corpuscle at the time $t = 0$?

Photons propagate as train of waves . We don't know exactly the position of the photon. If Δx is the uncertainty on the position of the corpuscle than we can take the length of the train of material waves as an approximate measure of the uncertainty on the position. If the train contain n complete periods than we have:

$\Delta x \approx n\lambda = \frac{nh}{p}$ (104)

Where : λ : the wave length

It is evident that the wave length is more defined when the number of oscillations in the train is great. As a rough measurement of the relative uncertainty of the wave length we can take the quantity:

$\frac{1}{n} \approx \frac{\Delta\lambda}{\lambda} = \frac{\Delta p}{p}$ (105)

Where :Δp: the uncertainty of the moment

Combining equations (104) & (105) we get:

$\Delta x \Delta p \approx h$ (106)

The length l associated to the corpuscle can be considered as the uncertainty on the position of the corpuscle. We have:

$l = \Delta x = n\lambda$ (107)

Taking in consideration that :

$p = al = \frac{h}{\lambda}$ (108)

It is evident that if l is so great than the moment p of the corpuscle becomes well defined and vice versa.

We deduce from (108) that:

$\lambda^2 = \frac{h}{na}$ (109)

With n: integer

If $n \to \infty$ than $\lambda \to 0$: the position of the corpuscle is very bad defined

If $n \to 1$ than $\lambda \to \sqrt{\frac{h}{a}}$: it is a length to get a compromise between the corpuscle behavior and the wave behavior. It depends on the circumstances of the experiment and how precise the position or the moment of the corpuscle we want to measure.

If $\lambda \to \infty$ than $n \to 0$: the position of the corpuscle is very well defined: it exist in the all Universe which having no sense(we should exclude $n = 0$).

In fact associate to the corpuscle an internal frequency when it is in rest , signify normally to De Broglie that the position of the corpuscle is not so precise even thought it is in rest: we should take it like this.

For a classic corpuscle considering it like photons , so associating to it a frequency in rest as :

$\nu_c = \frac{mc^2}{h}$ (110)

As a 4-dimensions corpuscle it is in motion along the axle time with a speed c .

It have an associate packet of waves as :

$$\lambda_c = \frac{h}{mc} \qquad (111)$$

We associate to this corpuscle a length l_c which is the uncertainty on its position as:

$$mc = al_c \qquad (112)$$

From equation (107) we have :

$$n = \frac{l_c}{\lambda_c} = \frac{m^2c^2}{ah} \qquad (113)$$

Than $n \neq 0$: even thought the corpuscle is in rest it have a relative uncertainty on its position and its moment. This is a new result different from classic mechanics.

At least $n = 1$ than :

$$m = m_{mc} = \frac{\sqrt{ah}}{c} \qquad (114)$$

In equation (114) constant m_{mc} represent a module for measuring mass. It is a *modular cell* for mass [10].

A modular cell for energy is:

$$E = m_{mc}\, c^2 = c\sqrt{ah} \qquad (115)$$

A modular cell for time is :

$$t_{mc} = \frac{1}{c}\sqrt{\frac{h}{a}} \qquad (116)$$

A modular cell for length is :

$$l_{mc} = \sqrt{\frac{h}{a}} \qquad (117)$$

A modular cell for action is:

$$s_{mc} = m_{mc} c l_{mc} = h \qquad (118)$$

A modular cell for moment of inertia is:

$$j_{mc} = \frac{h}{4\pi^2 \nu_0^2} = \frac{h^2}{4\pi^2 ac^2} \qquad (119)$$

And so on (the same thing for speed, frequency…)…etc.

Max Planck had understood this notion of modular cell in earlier time as we see in the following paragraph.

5. Revisited quantum mechanics:

In 1900 ,to resolve the problem of black body radiation Planck had conceive a black body as a cave at an equilibrium temperature T with oscillators on its walls exchanging energy with the electromagnetic radiation . Planck determines the entropy of an oscillator with the assumption that the statistical entropy is the same thermodynamic entropy. By referring to F.Kurlbaum measurements and Wien displacement law he determine the values of two universal constants h &k_B : Planck action element and Boltzmann constant.

In 1906 Planck show that his oscillators with constant energy are described by an ellipse in the phase space domain (p, q) moment & position and the area between two successive ellipses is equal to the action element h, which corresponds to the smallest existing volume of phase space. This volume corresponds to an elementary probability domain. The total volume of a cell bordered by an ellipse is proportional to the probability to find the oscillator in it. The result is justified by the hypothesis that all complexions are equally probable. The oscillators corresponding to this cell domain are indistinguishable.

The energy quantity ε corresponding to h verifies:

$$h = \int_E^{E+\varepsilon} dqdp \qquad (120)$$

The ellipse of the phase space of equation $E = \frac{1}{2}Kq^2 + \frac{p^2}{2L}$has a surface $S(E) = 2\pi E\sqrt{\frac{L}{K}}$ so :

$$h = S(E+\varepsilon) - S(E) = 2\pi\varepsilon\sqrt{\frac{L}{K}} = 2\pi\varepsilon.\frac{1}{\omega}$$

With K & L are respectively the stiffness and the mass of the oscillator.

Than:

$$\varepsilon = h\nu \qquad (121)$$

The magnitude of power Wcorresponding for this same quantity of energyε verify :

$$W = \int_E^{E+\varepsilon} \frac{K}{L} dqdp = \frac{K}{L}\frac{2\pi}{\omega}\varepsilon = 2\pi\omega\varepsilon \qquad (122)$$

Because $K = L\omega^2$.

This is possible (to have the dimension of a power) if ε verify equation (121) or also there is a constant $\alpha_0$having the dimension of a power such that :

$$\varepsilon = \alpha_0\zeta \qquad (123)$$

Withζ : a characteristic time of the oscillator.

Constant α_0 can be also declared as anuniversal constant.

Constant h can be considered as a mapping of the phase space of the oscillator in blocks: it is a *modular cell* for action .

The same thing for the constant α_0: it is a modular cell for power.

With $\varepsilon = \hbar\omega$ equation (122) becomes:

$$W = h\nu^2 = \frac{1}{4\pi^2}\int_E^{E+\varepsilon} K\, dq d\dot{q} = \frac{1}{4\pi^2}\int_E^{E+\varepsilon} df\, d\dot{q} \qquad (124)$$

With: $df = Kdq$: element of force;

$\dot{q} = \frac{dq}{dt}$: the speed of the oscillating mass.

Which mean also that the oscillator can absorb power or emit it as multiple integer of the quantum of power (124).

We have also for the oscillator $\frac{dE}{dt} = 0$ which mean that: $\frac{d}{dt}\left(\frac{p^2}{2L}\right) = -Kq\dot{q}$ than which gain by the oscillator as energy is loss by radiation that's why there is equilibrium.

From equation (70) it is very easy to identify the constant α_0 as:

$$\alpha_0 = ac^2 = h\nu_0^2 = 3.25\ 10^{-10}\ watt \qquad (125)$$

The constant "a" has the dimension of a mechanical impedance, in other words space-time cannot be conceived as completely empty: it is a superfluid of coefficient of friction "a" and having a negative pressure to cancel the viscosity effect as it is conceived in **GR**.

We can construct as Planck did with constants $h, c\ \&\ G$ a system of units where $h = c = \alpha_0 = 1$ so we get also from(93) that : $a = \nu_0 = 1$. This system is

$$l_{mc} = \sqrt{\frac{h}{a}} = \sqrt{\frac{hc^2}{\alpha_0}} = 4.3\ 10^{-4}\ meter$$

$$t_{mc} = \sqrt{\frac{h}{ac^2}} = \sqrt{\frac{h}{\alpha_0}} = 1.4\ 10^{-12}\ second \qquad (126)$$

$$m_{mc} = \sqrt{\frac{ha}{c^2}} = \sqrt{\frac{h\alpha_0}{c^4}} = 0.5\ 10^{-38}\ Kilogram$$

This system is also a definition for modular cells of length, time & mass. In its ground state an oscillator absorbs a certain amount of energy and returns it to space-time in a perpetual fashion.

The modular cell of energy is :

$$E_{mc} = m_{mc}c^2 = c\sqrt{ha} = \sqrt{h\alpha_0} = h\nu_0 = 21.5\ 10^{-22}\ Joule$$

The constant a which have mechanical impedance is :

$$a = \frac{\alpha_0}{c^2} = \frac{h\nu_0^2}{c^2} = 3.61\ 10^{-27}\ \ Kg.s^{-1}$$

The electromagnetic field can be considered as an oscillator in its fundamental state and it fills all space-time. Thus the vacuum can be defined as being the fundamental state of the electromagnetic field: all the points of space-time are oscillators in their fundamental states.

Let's exam the uncertainty on the position of the oscillator by the root mean square error i.e. the square root of its mean quadratic fluctuations [11]:

$$\overline{\Delta q} = \sqrt{\overline{q^2} - \bar{q}^2}$$

We can pose that :$\bar{q} = 0$

So:

$$\overline{\Delta q} = \sqrt{\overline{q^2}} = \sqrt{A^2\overline{\cos^2(\omega t)}} = \sqrt{\frac{1}{2}A^2} \qquad (127)$$

With: A: amplitude of vibration equivalent to the classic relation $\varepsilon = \frac{1}{2}LA^2\omega^2 = \frac{1}{2}KA^2$ than:

$$\overline{\Delta q} = \sqrt{\frac{\varepsilon}{K}} \qquad (128)$$

With the same manner we have:

$$\overline{\Delta p} = \sqrt{\overline{p^2}} = \sqrt{L^2A^2\omega^2\overline{sin^2(\omega t)}} = \sqrt{\frac{1}{2}L^2A^2\omega^2} = \sqrt{L\varepsilon} \quad (129)$$

So we have:

$$\overline{\Delta q}\,\overline{\Delta p} = \varepsilon\sqrt{\frac{L}{K}} = \frac{\varepsilon}{\omega} \qquad (130)$$

According to the uncertainty principle we have $\Delta q\Delta p \approx \hbar$ and the value of the lower limit of the product of the mean quadratic errors is $\frac{\hbar}{2}$ as we will have:

$$\Delta q\Delta p \geq \frac{\hbar}{2} \qquad (131)$$

If we take in (130) as $\Delta q\Delta p = \frac{\hbar}{2}$ and comparing with (128) & (127)than:

$$\varepsilon = \frac{\hbar}{2}\omega = \frac{h}{2}\nu \qquad (132)$$

Which mean that Planck model for black body radiation can be also useful as a model for vacuum energy density with a mean energy of the oscillator at low frequencies as equal to about $\frac{1}{2}h\nu_0$ where ν_0 is an universal constant. The mean energy of the oscillator will be defined by analogy to Planck model of black body radiation as:

$$U = \frac{\sum_{n=0}^{n=\infty} \frac{n}{2} h\nu . \exp\left(\frac{-\frac{n}{2} h\nu}{\frac{1}{2} h\nu_0}\right)}{\sum_{n=0}^{n=\infty} \exp\left(\frac{-\frac{n}{2} h\nu}{\frac{1}{2} h\nu_0}\right)} = \frac{\frac{1}{2} h\nu}{\exp\left(\frac{\nu}{\nu_0}\right) - 1} \approx \frac{1}{2} h\nu_0 \ if \ \nu \ll \nu_0 \qquad (133)$$

The difference with Planck model of black body is that at $T = zero\ Kelvin$ all Planck oscillators lies in an ellipse of phase space which have a surface non null. Every oscillator in this case have one degree of freedom and can have an infinite states. The statistical manner limit the number of states in order to get a finite density of vacuum energy.

It is clear that our Universe can have many space dimensions: every corpuscle can have a 4th space dimension, the 4th space dimension can be considered in itself another Universe with 4-space dimensions and so on. We have always those relations:

$$p = \hbar k = al = \frac{h}{\lambda} \qquad (134)$$

λ : wave-length associated to the corpuscle.

If $l = \lambda$ we deduce directly from (134) that:

$$l = \lambda = l_{mc} = \sqrt{\frac{h}{a}} \qquad (135)$$

This is at list the meaning of the string associated to the corpuscle and how to use it.

If our presumptions are correct than we can determine the vacuum energy density in a naïve way as one modular energy per cubic modular length & by means of a regulation coefficient to fit the exact value (or the inverse if we know the value of vacuum energy than we can determine any value of a modular cell always & by means of a regulation coefficient to fit the good value):

$$\rho_0 = \frac{E_{mc}}{l_{mc}^3} = \frac{\sqrt{h\alpha_0}}{\sqrt{\frac{hc^2}{\alpha_0}}^3} = 0.27\ 10^{-10}\ Joule.m^{-3} \qquad (136)$$

The real value is ten times the above value: we are not so far from it. To fit the real value we should take the modular cell of density of energy as ten modular cell of energy per cubic modular cell of length, which correspond to twenty electromagnetic field oscillators in their fundamental states per cubic modular length.

Do modular units have any relationship with Planck units?. The response is yes: at list the two systems of units define an absolute measurement independent from any frame of the Universe. Planck units have a relationship with gravitation, so necessary the modular units will have a relationship with gravitation.

The original Planck system of units is as follows [12]:

$$m_{Pl} = \sqrt{\frac{hc}{G}} = 5.4610^{-8}\ Kg$$

$$l_{Pl} = \sqrt{\frac{hG}{c^3}} = 4.0110^{-35}\ m \quad (137)$$

$$t_{Pl} = \sqrt{\frac{hG}{c^5}} = 13.51\ 10^{-44}\ s$$

Planck system of units is also a modular cells for mass, length & time but where or in what conditions ?. A priori it corresponds to high gravitational fields because the length $l_{Pl} \sim 10^{-35} m$ is unattainable by any means (the ultraviolet domain).

An implementation of calculating vacuum energy density was done successfully by Laurent Freidel, Jerzy Kowalski-Glikman, Robert G.Leigh & Djordje Minic (FKLM) as calculating the microscopic states of vacuum and equating this calculation to the macroscopic value of gravitational entropy using a floating scale of momentum and position in the phase space geometry [13]. FKLM consider that gravitational entropy is non extensive and suggest that the holographic bound on entropy can be thought as a good contextual scale to determine the elementary phase space cell which will be the modular cell for action.

To count cells FKLM consider that there is no limit to the momentum in the 4-space dimensions of the Universe and so the width of a modular cell is one momentum and their number is bound by the ratio power four of the radius of the observable Universe and Planck length. This ratio should make a sort of regulation between the UV-IR mixing phenomenon otherwise it will explode because there is in classical particle picture one state for each position in the 4-phase space. To do so, in quantum theory there is a bound of the number of states since constant h is the minimum area for action in phase space volume which mean that the number of quantum states is finite. Than it means that the vacuum energy density times the space-time volume is proportional to the finite number of states i.e. it is also proportional to a sort of an entropy S. In what follows we will equate this entropy to the gravitational entropy.

To do so let's take a system which have a mass m_{Pl} with a spherical radius R. This choice of Planck mass is typical to find a positive vacuum density of energy independently from the ultraviolet scale represented by Planck length l_{Pl} but only depends on the infrared scale represented by the modular cell of length l_{mc} supposed not yet determined.

A priori the modular cells $l_{mc}, t_{mc} \& m_{mc}$ correspond to a weak gravitational field created by Planck mass which is a big mass compared to the mass of the electron with which we had calculate vacuum energy density and also a very thin mass compared to the gravitational fields created by planets in the Universe.

In a weekly gravitational field in an asymptotically flat space the entropy is [14]:

$$S \leq 2\pi k_B \frac{ER}{hc} \qquad (138)$$

With k_B: Boltzmann constant ;

$E = m_{Pl} c^2$: The total mass-energy of the matter system

The holographic bound of entropy is as:

$$S = k_B \frac{Area}{l_{mc}^2} = k_B \frac{4\pi R^2}{l_{mc}^2} \qquad (139)$$

Equating the two entropies we get:

$$l_{mc} = \sqrt{2Rl_{Pl}} \qquad (140)$$

If $R = l_{Pl}$ than equation (140) is not verified by reference to (126) but we will have $l_{mc} = \sqrt{2}l_{Pl}$ and the holographic principle is satisfied;

If $R = l_U \sim 2.31\ 10^{27} meter$ (radius of the observable Universe) than equation (140) is verified by reference to (126) .

If $l_{mc} = \frac{2Gm_{Pl}}{c^2} = 2l_{Pl}$: the Schwarzschild radius which is the horizon of a black hole of mass m_{Pl} than $l_{mc} = 2l_{Pl}$ but in this case the entropy is of a black hole as : $S \leq k_B \frac{Area}{4l_{mc}^2} = k_B 4\pi$ than $m_{Pl}l_{Pl} = \frac{h}{c}$ which is correct.

Hawking formulae is still valid [15].

For any mass M with spherical radius R we get from (139) &(140) that:

$$l_{mc}^2 = \frac{2hR}{Mc}$$

And according to Hawking we have that in maximum:

$$M = \frac{2hR}{cl_{Pl}^2}$$

It comes that from the two above equations: $l_{mc} = l_{Pl}$ which mean that the holographic principle as given by the formulae (139) is always valid & independent from the radius.

In the case $R = l_{PL}$ than the mass inner is maximum: $M = 2m_{Pl}$.

Planck system of units serves only for topology in black holes.

Topology from asymptotically flat space-time to black holes changes radically.

FKLM find the same equation (140) of the modular cell of length using string theory but with a factor $\sqrt{2}$. We can expect such a result because Planck mass will create a weak gravitational field away from the radius Planck length.

FKLM did not go more far from to deduce more modular cell units for asymptotically flat space even thought they have the modular cell for momentum as Planck constant h and the length which they determine resolve the problem of vacuum energy density & corresponds to the IR region which is the life region in a weak gravitational field.

Planck oscillators are classic oscillators. The mean energy of an oscillator in a cavity at an equilibrium temperature T is according to Planck [16]:

$$E_{med} = \frac{h\nu}{\exp\left(\frac{h\nu}{k_B T}\right) - 1} \qquad (141)$$

Every oscillator can absorb or radiate energy as per quanta multiple integer of $h\nu$. The same thing for the oscillator about the power radiated or absorbed as per quanta multiple integer of $h\nu^2$ and the mean power of an oscillator is:

$$W_{med} = \frac{h\nu^2}{\exp\left(\frac{h\nu}{k_B T}\right)-1} \qquad (142)$$

We have for example for the energy of an oscillator as:

$$E = \int h\nu^2 dt = \int h\nu^2 \frac{d\nu}{\nu^2} = h\nu \qquad (143)$$

But we can't write it as $E = \int h\nu^2 dt = \int h\nu^2 . \frac{h}{ac^2} d\nu$ because $\vec{f} = m\vec{\gamma} = \frac{d\vec{p}}{dt}$ is valid for classic mechanics and it is invariant by Galileo transformations of space-time (Lorentz transformations of space-time with $c \to \infty$) & non valid for restraint relativity because it is not invariant by Lorentz transformations of space-time.

With Planck we deal with classic oscillator and if the quantum of energy $h\nu$ is OK for his model of black body than the quantum of power $h\nu^2$ is also OK for his model of black body even thought the speed of the oscillator can reach any value. We remain with Planck context even thought it appears that equation (134) is always valid for low speed corpuscles or high speed corpuscles. It is like that the Planck model is an extension to be valid for high speed corpuscles as an average model and experimentally strongly confirmed by thermodynamics.

The 4-vector force $\boldsymbol{f}$ which is invariant by Lorentz transformations is [17]:

$$\boldsymbol{f} = \frac{d\boldsymbol{p}}{d\tau} \qquad (144)$$

And its contra- variant components are:

$$[f^i] = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\left(\frac{\vec{f}.\vec{v}}{c}, \vec{f}\right) \qquad (145)$$

With $\vec{f}$ is the ordinary 3-force and $\vec{v}$ the ordinary 3-speed measured in the frame S.

If the speed of the corpuscle tends to zero than:

$$[f^i] \approx \left(\frac{h\nu^2}{c}, \vec{0}\right) \to \left(\frac{h\nu_0^2}{c}, \vec{0}\right) \qquad (146)$$

And the constant ν_0 can be not equal to zero i.e. the mean speed of the corpuscle can be equal to zero but its mean square speed can be not equal to zero and this correspond to a perpetual sinusoidal motion which mean vacuum is full with an energy which drive the corpuscle around and equilibrium position with a force equal to "$a\vec{v}$"and a mechanical impedance of vacuum $a = \frac{h\nu_0^2}{c^2}$. If we assign to the corpuscle a position in time as ζit will have a moment along the axle time as $a\zeta c$ and an energy H (in terms of analytical mechanics with 4- space dimensions where ct is the 4th space dimension) [18] as :

$$\frac{\partial H}{\partial(a\zeta c)} = c \quad \text{so} \quad H = a\zeta c^2 \qquad (147)$$

The force which acting along the time axle (in terms of analytical mechanics) is:

$$-\frac{\partial H}{\partial(c\zeta)} = -ac \qquad (148)$$

Also we have :

$$\frac{\partial H}{\partial(c\zeta)} = ac \qquad (149)$$

Which mean there is only conservative forces.

The position of the corpuscle at rest on the axle time is ζ_0 as :

$$\zeta_0 = \frac{m}{a} \qquad (150)$$

With m: mass of the corpuscle.

And at any time the relation ship between its inertial time ζ and its position in rest ζ_0 is given by Lorentz transformations as :

$$\zeta_0 = \zeta\sqrt{1-\frac{\vec{v}^2}{c^2}} \qquad (151)$$

The moment of the corpuscle in the 3-oridnary space will be :

$$\vec{p} = a\zeta\vec{v} = \frac{m}{\sqrt{1-\frac{\vec{v}^2}{c^2}}}\vec{v} \qquad (152)$$

With :$a\zeta = \frac{m}{\sqrt{1-\frac{\vec{v}^2}{c^2}}}$: *the mass of the corpuscle in motion.*

And this moment is in accordance with analytical mechanics as:

$$\vec{p} = \frac{\partial\mathcal{L}}{\partial\vec{v}} \qquad (153)$$

With :$\mathcal{L} = \vec{p}.\vec{v} - H = mc^2\sqrt{1-\frac{\vec{v}^2}{c^2}}$: Lagrange function of the corpuscle (or kinetic potential) which corresponds to the De Broglie internal frequency if divided by Planck constant h : the corpuscle is considered as a beam of light with a speed c along the time axle.

6-Revisited General Relativity:

The equations of gravitational field in an asymptotically flat Universe , according to General Relativity are [20]:

$$R_{ik} - \frac{1}{2}Rg_{ik} = -\frac{8\pi G}{c^4}T_{ik} - \Lambda g_{ik} \qquad (154)$$

With R_{ik} : curvature tensor;

R : scalar (curvature of space-time)

T_{ik} : momentum-energy tensor of matter

g_{ik} : metric tensor with signature (+,-,-,-)

$i, k = 0,1,2,3$: tensor indices

Note that from the space dimension 5 there is no derivation related to it because it will be curled.

The energy density of vacuum as given by General Relativity is as follows[21] :

$U_0 = \frac{\Lambda.c^4}{8\pi G} \approx 10^{-9} Joule.m^{-3}$ (155)

With :

$\Lambda = 1.088\ 10^{-52} m^{-2}$: cosmological constant ;

$c = 3\ \ 10^8 m.s^{-1}$: light celerity in vacuum ;

$G = 6.67\ \ 10^{-11}\ SI\ units$: gravitationnel constant ;

The total energy density of the vacuum is then according to the model of the black body theory:

$U_0 = \int_0^\infty \frac{8\pi\nu^2}{c^3}.U d\nu = \int_0^\infty \frac{4\pi h}{c^3}.\frac{\nu^3}{\exp\left(\frac{\nu}{\nu_0}\right)-1} d\nu = \frac{4\pi^5 h}{15.c^3}.\nu_0^4$ (156)

Equating (155) & (156) we get:

$\nu_0 = \left[\frac{15\ \Lambda.c^7}{32.\pi^6.G.h}\right]^{\frac{1}{4}} \approx 0.7\ 10^{12} Hz$ (157)

This is an experimental proof that the electron is a 4-space dimension corpuscle. It means also that extension in space is energy.

For a space with D space-dimensions ($D \geq 4$) the half quantum of vacuum energy is:

$$\frac{1}{2} h\nu_0 = D.\frac{1}{4}\alpha^4 mc^2$$

So $\nu_0 = \frac{D}{2h}\alpha^4 mc^2$ (158)

Replace (158) in (156) we get :

$\Lambda = \frac{2\pi^6 \alpha^{16} m^4 cG}{15.h^3} D^4$ (159)

With: $m = 9{,}1\ 10^{-31} Kg$ the mass of the electron.

From equation (140) and for $R = l_U$ we deduce directly that:

$G = \frac{hc^3}{4a^2 l_U^2} = \frac{c^7}{4h\nu_0^4 l_U^2} = \frac{h^3}{64\alpha^{16} m^4 c l_U^2}$ (160)

Replace (160) in (159) we get:

$\Lambda = \frac{\pi^6}{480\ l_U^2} D^4$ (161)

Replace α by $N\alpha$ in equation (160) and by $Z\alpha$ in equation (161) to get quantified constants G &Λ with N & Z positive integers (to consider the first layer of Bohr atom to calculate exchanged vacuum energy for the electron).

Equations of the gravitational field becomes:

$$R_{ik}-\frac{1}{2}Rg_{ik}=-\frac{\pi h^3}{8N^{16}\alpha^{16}m^4c^5l_U^2}T_{ik}-\frac{\pi^6Z^{16}}{480N^{16}\,l_U^2}D^4g_{ik} \qquad (162)$$

For $N=Z=1, D=4$ &$l_U\approx 2.31\ 10^{27}\ meter$ we get Einstein equations.

Taking in consideration modular cells for space and time it is convenient to replace the derivation in equation (160) by modular cells of space and time. The resolution of such equation needs a numerical model & can be resolved by computer machine: there will be no singularities. It is clear that Einstein equations of gravitational field are available only in a quasi static Universe ($l_U=Constant$).

7-Vacuum energy:

There is not any experience proofs that our Universe had only three space dimensions. It can have D −space dimensions. The other space dimensions-from the 5th dimension- can be curled in little spheres about the Planck length and more.

For a black body in $D-space\ dimensions$ Universe the number of modes with frequencies between ν&$\nu+d\nu$ is [19]:

$$N(\nu)d\nu=(D-1)V\frac{2}{\Gamma(\frac{D}{2})}\left(\frac{\sqrt{\pi}}{c}\right)^D\nu^{D-1}d\nu \qquad (163)$$

With: V: volume of the black body

$\Gamma(x)=\int_0^\infty t^{x-1}\,e^{-t}dt$: Gamma function.

$D>1$: number of space dimensions of the Universe.

The density of power between ν&$\nu+d\nu$is :

$$w_T(\nu)d\nu=2\left(\frac{\sqrt{\pi}}{c}\right)^D\frac{D-1}{\Gamma(\frac{D}{2})}\frac{h\nu^{D+1}}{\exp\left(\frac{h\nu}{k_BT}\right)-1}d\nu \qquad (164)$$

The total density of power is obtained by integration of (164):

$$w_T=\int_0^\infty w_T(\nu)d\nu=\int_0^\infty 2\left(\frac{\sqrt{\pi}}{c}\right)^{D+1-1}\frac{D+1-2}{\Gamma(\frac{D+1-1}{2})}\frac{h\nu^{D+1}}{\exp\left(\frac{h\nu}{k_BT}\right)-1}d\nu$$

And using the identity $\Gamma(2x)=\frac{2^{2x-\frac{1}{2}}}{\sqrt{2\pi}}\Gamma(x)\Gamma(x+\frac{1}{2})$ we get:

$$w_T=\frac{c}{\sqrt{\pi}}\frac{\Gamma(\frac{D+1}{2})}{D}\frac{D-1}{\Gamma(\frac{D}{2})}a_{D+1}T^{D+2} \qquad (165)$$

With :

$$a_D = \left(\frac{2}{hc}\right)^D \left(\sqrt{\pi}\right)^{D-1} k_B{}^{D+1} D(D-1)\Gamma\left(\frac{D+1}{2}\right)\zeta(D+1) \quad (166)$$

The radiancy or the rate of radiation of energy per unit area of the cavity surface of the black body or generalized Stefan-Boltzmann law is :

$$R_T = \sigma_D T^{D+1} \quad (167)$$

With :

$$\sigma_D = \left(\frac{2}{c}\right)^{D-1} \left(\sqrt{\pi}\right)^{D-2} \frac{k_B^{D+1}}{h^D} D(D-1)\Gamma(\frac{D}{2})\zeta(D+1)(121)$$

Using the identity $\Gamma(x+1) = x\Gamma(x)$ we get:

$$w_T = (D-1)\sigma_{D+1} T^{D+2} \quad (168)$$

It is clear that for a Universe with $D - space\ dimensions$ there is $D - 1$ sides of propagation of energy to cover all the Universe (example: for $D = 3$ there is 2 sides of propagation of energy: front and rear). The density of power is not equal to the radiancy except for a $D = 2\ space\ dimensions$ of a Universe where it is equal to the radiancy of $D = 3\ space\ dimensions$ of a Universe because the electromagnetic field can exist at list in a two dimensional Universe.

The total density of energy in the cavity is:

$$\rho_T = a_D T^{D+1} \quad (169)$$

For the density of vacuum energy in the Universe we obtain it only by multiply equation (163) with the mean energy of an oscillator equation (133) , replace $k_B\ by\ h\ \&\ T\ by\ \nu_0$:

$$\rho_{0D} = a_{0D} \nu_0^{D+1} \quad (170)$$

With:

$$a_{0D} = \frac{1}{2}\left(\frac{2}{c}\right)^D \left(\sqrt{\pi}\right)^{D-1} h\, D(D-1)\Gamma\left(\frac{D+1}{2}\right)\zeta(D+1) \quad (171)$$

It is clear from equation (170) that vacuum energy density grow rapidly with the number of dimensions of space so there is evidence that Universe accelerate in expansion.

8-A model for the hydrogen atom:

The classic total energy of an electron in the hydrogen atom is:

$$E = \frac{p^2}{2m} - \frac{e^2}{r} \quad \text{(cgs system of units)} \quad (172)$$

The uncertainty on the position of the electron is as equation (105):

$$\frac{2\pi\Delta r}{2\pi r} = \frac{1}{n} \quad \text{(the } \textit{trajectory} \text{ is circular)} \quad (173)$$

n: not necessary equal to the inferior limit as given by equation(113).

The uncertainty on the moment of the electron is as:

$$\frac{\Delta p}{p} = \frac{1}{n\prime} \qquad (174)$$

n': not necessary equal to the inferior limit as given by equation(113).

We have at the limit:

$$\Delta r \Delta p = \hbar \qquad (175)$$

Which signify that:

$$r = \frac{\hbar n n\prime}{p} \qquad (176)$$

$$E = \frac{p^2}{2m} - \frac{e^2 p}{n n\prime \hbar} \qquad (177)$$

We get for the energy a minimum for $p = p_0$ when the derivation of (177) is equal to zero:

$$\left(\frac{\partial E}{\partial p}\right)_{p=p_0} = \frac{p_0}{m} - \frac{e^2}{n n\prime \hbar} = 0 \qquad (178)$$

Than for $r_0 = \frac{\hbar n n\prime}{p_0}$ we get:

$$p_0 = \frac{me^2}{\hbar n n\prime} \quad \& \quad r_0 = \frac{\hbar^2 n^2 n\prime^2}{me^2} \qquad (179)$$

$$E = -\frac{1}{2}\frac{me^4}{\hbar^2 n^2 n\prime^2} = -\frac{R_\infty}{N^2} \qquad (180)$$

With : $R_\infty = \frac{me^4}{2\hbar^2} = \frac{1}{2}\alpha^2 mc^2$: Rydberg constant

$N = nn'$: integer

$\alpha = \frac{e^2}{\hbar c} \approx \frac{1}{137}$: fine structure constant

Than:

$$p_0 = \frac{\alpha mc}{N} \ ; \ r_0 = N^2 a_0$$

With :

$a_0 = \frac{\hbar^2}{me^2} = \frac{\hbar}{m\alpha c}$: Bohr radius

The electric force between the electron and the proton is attractive. The proton is supposed that have an infinite mass , than the electron with a negative bound energy should normally move towards the center of the atom and so not have an hydrogen atom. The only way to conserve the situation is that the electron remain at the same place and exchange energy with vacuum at least to equal the absolute value of its bound energy: it will oscillate around its position r_0.

The mean energy of electron exchanging energy with vacuum is as given by equation (133):

$$U = \frac{\frac{1}{2}h\nu}{\exp\left(\frac{\nu}{\nu_0}\right)-1} \approx \frac{1}{2}h\nu_0 \ if \ \nu \ll \nu_0 \qquad (181)$$

The total energy of the electron will be:

$$E = -\frac{R_\infty}{N^2} + \left(N' + \frac{1}{2}\right) h\nu_0 \qquad (182)$$

With: N': integer

$h\nu_0 = 2\alpha^4 mc^2$: frequency of oscillation of the electron at low level.

In its fundamental oscillation state ($N' = 0$) how many bound level we get?

From equation (182) we have :

$$E \approx 0 = -\frac{\alpha^2 mc^2}{2N^2} + 2\alpha^4 mc^2$$

Than :

$$N = \frac{1}{2\alpha} = 68.5 \approx 68$$

In its lowest bound fundamental state ($N = 1$) how many sub-bound state1 we can get?

$$\left(N' + \frac{1}{2}\right) 2\alpha^4 mc^2 = \frac{1}{2}\alpha^2 mc^2$$

Than:

$$N' = \frac{1}{4\alpha^2} - \frac{1}{2} = 4691.75 \approx 4692$$

We can continue like this for the other bound states.

This shows that it is possible than the hydrogen atom can be ionized spontaneously when the total energy of the electron becomes positive.

Vacuum energy maintains the structure of the atom.

9-Uncertainty in strong gravitational field:

For a weak gravitational field we have the degree of uncertainty of the position of a corpuscle as given by equation (113):

$$n = \frac{m^2c^2}{ah} \qquad (183)$$

With : m: mass of the corpuscle

If $n = 1$ than it is the best precision to determine the position of a massive corpuscle. We can't go under this precision.

This signify that we can't determine a mass under the precision of:

$$m = \frac{\sqrt{ah}}{c} \qquad (184)$$

We can do the same thing for the moment:

$$mc = \sqrt{ah} \qquad (185)$$

The same for the energy:

$mc^2 = c\sqrt{ah}$ (186)

What about space-time and energy for a black hole?

A black hole have strong gravitational field. For this we use Planck sytem of units as modular cells for space-time and energy:

$$m_{Pl} = \sqrt{\frac{hc}{G}} = 5.4610^{-8}\ Kg$$

$$l_{Pl} = \sqrt{\frac{hG}{c^3}} = 4.0110^{-35}\ m$$

$$t_{Pl} = \sqrt{\frac{hG}{c^5}} = 13.51\ 10^{-44}\ s$$

By analogy to weak gravitational field we define Planck mechanical impedance as:

$a_{Pl}c^2 = h\nu_{Pl}^2$ (187)

With: $\nu_{Pl} = \frac{1}{t_{Pl}} = \sqrt{\frac{c^5}{hG}} = 7.4\ 10^{42}\ Hz$

Than:

$a_{Pl} = \frac{c^3}{G} = 4.1\ 10^{35} Kgs^{-1}$

Also we can define Planck fine structure as:

$h\nu_{Pl} = 2\alpha_{Pl}^4 m_{Pl} c^2$ (188)

than :

$\alpha_{Pl} = \frac{1}{2^{0.25}} = 0.841$ (189)

With this constant we can also define a *Planck atom* which can exist only in a strong gravitational field: to do so replace in Bohr atom the constant α by Planck fine structure constant α_{Pl} and replace the electron mass by Planck mass m_{Pl} . In the visible domain the radiation from the horizon of a black hole will be continuous: there is no spectrum because the energies levels are too near to each others with high quantum numbers. There is a perfect symmetry between the strong gravitational field and the weak one: in the weak field the IR cut off can be determined with real atoms and real particle (the electron) and for the UV cut off there is no real atom (Planck atom) and no real particle (Planck mass).

The position of a corpuscle in a strong gravitational field is well defined when:

$$n_{Pl} = \frac{m^2c^2}{a_{Pl}h} = 1 \qquad (190)$$

Than the precision of a mass in a strong gravitational field can't go more than:

$$m = \frac{1}{c}\sqrt{ha_{Pl}} = \sqrt{\frac{hc}{G}} = m_{Pl} \qquad (191)$$

The same thing for the moment:

$$mc = m_{Pl}c = \sqrt{\frac{hc^3}{G}} = \sqrt{ha_{Pl}} \qquad (192)$$

Also for the energy:

$$mc^2 = m_{Pl}c^2 = \sqrt{\frac{hc^5}{G}} = h\nu_{Pl} = c\sqrt{ha_{Pl}} \qquad (193)$$

Also for force:

$$F_{limit} = \frac{m_{Pl}c}{t_{Pl}} = \frac{m_{Pl}c^2}{l_{Pl}} = \frac{c^4}{G} = a_{Pl}c \qquad (194)$$

And for power:

$$W_{limit} = F_{limit}c = \frac{c^5}{G} = a_{Pl}c^2 \qquad (195)$$

It is also possible to determine the mass of the Universe. According to Salecker & Wigner thought experiment the uncertainty δl on the position l of a clock having a mass m is[22]:

$$\delta l^2 \geq \frac{hl}{mc} \qquad (196)$$

If the clock is massive it will reduce the uncertainty but this same clock should not distort the space-time severely as a black hole so:

$$\delta l \geq \frac{Gm}{c^2} \qquad (197)$$

The conflict between the two conditions (196) & (197) reach a compromise as :

$$\delta l^3 \geq l l_{Pl}^2 \qquad (198)$$

Divide equation (197) by l we deduce that:

$$m \leq \frac{c^2 l}{G}.\frac{\delta l}{l} \qquad (199)$$

But for the Universe:

$\frac{\delta l}{l} = \frac{1}{n}$ with $n = 1$ the lowest value because there is no position for the Universe. Than:

$$m_U \leq \frac{c^2 l_U}{G} = 3.12\ 10^{54}\ Kg \qquad (200)$$

It is ten time the most value adapted by many scientist.

Also we can determine the age of the Universe. The uncertainty δt of the running time t_U of the Universe is bounded by [23]:

$$\delta t^2 \geq \frac{h t_U}{m_U c^2} \qquad (201)$$

On the other hand the upper bound of the mass m_U is satisfied by diving equation(197) by c as:

$$\delta t \geq \frac{G m_U}{c^3} \qquad (202)$$

Replace (202) in (201) we get:

$$t_U \leq \frac{G^2 m_U^3}{h c^4} = 2.52\ 10^{142}\ seconds = 7\ 10^{138}\ hours = 8\ 10^{134} years \qquad (203)$$

And this is a new result.

In the other hand if we suppose that the Universe was a black hole and if after the life time of the black hole state the Universe start to expand with the speed c than it's the age of this one is:

$$t_U = \frac{l_U}{c} \qquad (204)$$

Where $l_U = 2.31\ 10^{27} meter$: the observed value.

Than :

$$t_U = 0.77\ 10^{19}\ seconds = 2.14\ 10^{15}\ hours = 2.44\ 10^{11}\ years = 244\ billions\ of\ years \qquad (205)$$

Which is a very low value compared to (203) and it is ten to twenty time the conventional value adapted by many scientists.

Hawking life time of the Universe as a black hole is [23] :

$$t_{HU} = \frac{G^2 m_U^3}{h c^4} \qquad (206)$$

With: $m_U \approx 3.12\ 10^{54}\ Kg$ actual observed value

Than:

$$t_{HU} = 2.74\ 10^{142}\ seconds = 7.61\ 10^{138} hours = 8.7\ 10^{134}\ \ years \qquad (206)$$

It is clear that the age of the observable Universe is insignificant in front its life time as a black hole.

The uncertainty on the length (198) can be written as a function of the power limit or the force limit as [24]:

$$\delta l \geq \left(\frac{lh}{c W_{limit}}\right)^{\frac{1}{3}} c = \left(\frac{lh}{c^2 F_{limit}}\right)^{\frac{1}{3}} c \qquad (207)$$

The same thing for the uncertainty on time can be obtained when the conflict between the two conditions (201) & (202) reach a compromise as:

$$\delta t \geq \left(\frac{th}{W_{limit}}\right)^{\frac{1}{3}} = \left(\frac{th}{cF_{limit}}\right)^{\frac{1}{3}} \qquad (208)$$

10-Vacuum density of energy:

Failure in calculating vacuum energy density is a central problem in physics because the Universe is always considered as a continuous medium where we can divide space-time without limits. However Bolotin & Yanovsky had shows in equations (207) and (208) that there is limits for divining space & time. The possibility to determine an accurate vacuum energy density becomes in the hand since we can made a sort of quantum space-time by dividing the four dimensional space-time in modular cells of the *4-phase space geometry* where every surface of a cell is equal to the Planck constant h . Vacuum energy density will be approximately the energy contained in cubic space cell. This energy can be obtained by dividing the total mass of the Universe by the total number of 4-phase space geometry .

For the Universe it is approximately static than the relative uncertainty on position is:

$$\frac{\delta l}{l} = \frac{1}{n} = 1 \qquad (209)$$

Than from (198) we will have that:

$$\left(\frac{l}{\delta l}\right)^3 = 1 \leq \frac{l^2}{l_{Pl}^2} \qquad (210)$$

Than in the vicinity of $l_U \approx l_{Pl} = \sqrt{\frac{h}{a_{Pl}}}$ we can determine vacuum energy density.

From equation (207) we have:

$$\left(\frac{l}{\delta l}\right)^3 = 1 \leq \frac{hc^2}{l^2 W_{limit}} \qquad (211)$$

Note that the existence condition for the minimum uncertainty for length in equation (207) is related to the existence of the limit power which is dictated by the existence of the horizon.

If now we consider the observable Universe far away any horizon equation (211) will be still valid with another condition of power limit in a weak gravitational field. In this case equation (211) becomes:

$$W_{limit} \approx \frac{hc^2}{l^2} \qquad (212)$$

With : l : a characteristic length between the Ultra-Violet/ Infra-Red (**UV-IR**)relation to separate description of quantum process for macro-scale objects having or in strong gravitational field and description of quantum process of micro-scale objects in weak gravitational field.

We can write the power limit as:

$$W_{limit} = a_{Pl}c^2 \qquad (213)$$

With : $a_{Pl} = \frac{c^3}{G} = h\nu_{Pl}^2$: for strong gravitational field.

For a weak gravitational field space-time is asymptotically flat and $a[Kg.s^{-1}]$ have the dimension of a mechanical impedance (for vacuum)(Replace a_{Pl} by a to search and the same dimension).

From equations (212) & (213) we deduce the characteristic length:

$$l_c = \sqrt{\frac{h}{a}} \qquad (214)$$

There is only one state for every phase space cell which corresponds to one degree of freedom. The question is a three space dimensional cell contains how many phase space cell? Because if we know this number & multiplied by the energy of the cell given by equation(186) and divided by the cubic of (214) we get approximately a very good approach to vacuum energy density.

There is also a characteristic time:

$$t_c = \frac{l_c}{c} = \frac{1}{c}\sqrt{\frac{h}{a}} \qquad (215)$$

A characteristic energy:

$$E_c = W_{limit} t_c = c\sqrt{ha} \qquad (216)$$

Of course the constant a should be determined from experiments facts.

Vacuum energy is considered as the density of the fundamental state of the electromagnetic field at approximately zero Kelvin.

The density of vacuum energy is as given by equation (153):

$$U_0 = \frac{4\pi^5 h}{15.c^3}.\nu_0^4 \qquad (217)$$

With:

$h\nu_0 = E_c = c\sqrt{ha}$ (218): a characteristic frequency of vacuum in one direction.

Than:

$$\nu_0 = c\sqrt{\frac{a}{h}} \qquad (219)$$

$$U_0 = \frac{4\pi^5 a^2 c}{15h} \approx 10^{-9}\, joule.m^{-3} \qquad (220)$$

So: $a = 5.2\ 10^{-27}\ Kg.s^{-1}$ (221)

$\nu_0 = 0.84\ 10^{12}\ Hz$

We get approximately the same values as before. It depends how the accuracy to measure the observed vacuum energy in cosmology.

Also the possibility to determine an accurate vacuum energy density becomes in the hand since we can made a sort of quantum space-time by dividing the four dimensional space-time in modular cells of the *4-phase space geometry* where every surface of a cell is equal to the Planck constant h . Vacuum energy density will be approximately the energy contained in cubic space cell. We will do the same thing what it has done by FKLM [13] but Instead using quantum gravitation theory we will the black body theory in $D -$ space dimensions Universe at approximately zero absolute temperature.

The mean energy of an oscillator at zero absolute temperature is as in equation (133):

$$E_{mean} = \frac{1}{2}\frac{h\nu}{\exp\left(\frac{\nu}{\nu_0}\right)-1} \qquad (222)$$

The number of oscillators per unit volume is [19]:

$$\rho_{osci} = \frac{1}{2}\int_0^{\infty} 2\left(\frac{\sqrt{\pi}}{c}\right)^D \frac{D-1}{\Gamma(\frac{D}{2})}\frac{\nu^{D-1}}{\exp\left(\frac{h\nu}{h\nu_0}\right)-1}d\nu \qquad (223)$$

The number of oscillators per unit volume is the number of degree of freedom unit volume.

Replace $D' = D - 1$ and recalculate (223) we get:

$$\rho_{osci} = 2^{D-2}\sqrt{\pi}^{D-1}(D-1)(D-2)\zeta(D)\Gamma(\frac{D-1}{2})\frac{1}{\lambda^D} \qquad (224)$$

With : $\lambda = \frac{c}{\nu_0}$

$\lambda\varepsilon = h,\ \ \varepsilon = \frac{h\nu_0}{c}$: the phase space cell.

There is only one degree of freedom per phase space cell.

The total number of oscillators is:

$$N_{osci} = \rho_{osci}V \qquad (225)$$

With: $V = \frac{\pi^{\frac{D}{2}}}{\Gamma(\frac{D}{2}+1)}l^D$, l: the radius of the sphere.

Than:

$$N_{osci} = 2^{D-2}\sqrt{\pi}^{2D-1}(D-1)(D-2)\zeta(D)\Gamma(\frac{D-1}{2})\frac{1}{\Gamma(\frac{D}{2}+1)}\frac{l^D}{\lambda^D} \qquad (226)$$

According to equation (198) or the principle of maximum force [26] the number of degree of freedom for a three volume is as:

$$\frac{1}{N} = \left(\frac{\delta l}{l}\right)^3 \geq \frac{l_{Pl}^2}{l^2} \qquad (227)$$

Thus : $N \leq \frac{l^2}{l_{Pl}^2}$ and at the limit we can have:

$$N \approx \frac{l^2}{l_{Pl}^2} \qquad (228)$$

For the Universe ($l = l_U$) equating (226) to (228) we get:

$$\lambda = l_U^{1-\frac{2}{D}} l_{Pl}^{\frac{2}{D}} \left[2^{D-2} \sqrt{\pi}^{2D-1} (D-1)(D-2)\zeta(D)\Gamma(\tfrac{D-1}{2}) \frac{1}{\Gamma(\frac{D}{2}+1)} \right]^{\frac{1}{D}} \quad (229)$$

For $D = 4$ and $l_U = 2.31\ 10^{27}\ m$ we get:

$$\lambda = \sqrt{l_U l_{Pl}}\ [24\sqrt{\pi}^7 x 1.0823 x \frac{\sqrt{\pi}}{2}\frac{1}{2}]^{0.25} = \sqrt{l_U l_{Pl}}[6x\pi^4 x 1.0823]^{0.25} = 15.26\ 10^{-4} m$$

We can ameliorate this result as to obtain the same IR cut off before by considering that the most volume of a D −sphere exist in its exterior region with the thickness $\varepsilon = \frac{l_U}{D}$ .We can also multiply equation (226) by a constant k^D as to consider the logarithm of the new expression of N_{osci} the entropy of vacuum which tends to a constant non necessary equal to zero when the temperature tends to zero. So we can adjust the constant λ as to be equal exactly to $\frac{c}{\nu_0} = 4.286\ 10^{-4} m$ & $l_U = 2.31\ 10^{27} m$ than:

$$k \sim 0.28\ \ \&\ \lambda \sim 10^{-4} m \quad (230)$$

If we opposite to this idea, we should accept that the number of degree of freedom in vacuum with space dimensions D is:

$$N_D = \frac{l^2}{k^D l_{Pl}^2} \quad (231)$$

With k: positive constant as to respect the same IR cut off of the observable Universe .

It is like that we have much more states than we had expected."*Most of the states of a regularized quantum field theory would have so much energy that they collapse into a black hole before they could dictate the further evolution of the system in time. If we want to avoid this a much more rigorous cut-off than a Planckian one must be called for*". *" at a Planckian scale our world is not 3+1 dimensional"*[25]. The UV-IR cut offs should change with the dimensionality of space-time as given by equation (229): if we fix one the other change. It depends of our experimental ability to explore more and more deep distances.

11-Conclusion :

By introducing the notion of 4-vector string associated to a corpuscle we had in fact incorporate the principle of uncertainty in Relativity (classic or restraint). To conserve the same IR cut off in approximately flat space-time the UV cut off should change with dimensionality of space-time.

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