## Quantum Gravitation in $D$ −space dimensions Universe

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**ABSTRACT**

From Restraint Relativity it is possible to consider a corpuscle as a packet of strings. The variation of the length of this packet-the string vector- is equal to the phase speed of the corpuscle as a packet of waves times an universal constant. In a system of units where $\hbar = c = a = 1$ the string vector becomes equal to the wave vector.

Considering the interaction of the corpuscle with vacuum we deduce the link between the two packets and the values of the universal constants by referring to cosmological observations and models of black body radiation & Bohr atom. Considering electron in the first layer of Bohr atom to calculate the exchanged energy with vacuum we deduce quantified gravitational constants $G$ & $\Lambda$ to be introduced in Einstein equations of the gravitational field.

### 1-Revisited Special Relativity:

The introduction of Special Relativity by Einstein in 1905 is due only theoretical thinking that Maxwell equations of the electro-magnetic field should be the same in all inertial frames. Those equations in vacuum are:

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \qquad (1)$$

$$\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} \qquad (2)$$

Which are two equations of waves with a speed of propagation equal to $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ with:

$\varepsilon_0 = 8.85\ 10^{-12}\ Fm^{-1}$ : the permittivity of vacuum .

$\mu_0 = 4\pi 10^{-7} Hm^{-1}$ : the permeability of vacuum.

But what is vacuum? If vacuum signify that there is nothing but here there is the electromagnetic field and so there is temperature. The only response is that vacuum is when there is nothing and the temperature is zero absolute or approaches asymptotically to zero. If there is any electromagnetic field in vacuum it signify the fundamental state of the field : there is no emission of energy.

In Special Relativity based on the constancy of the speed of light in all inertial frames it is defined:

-Let's have two inertial frames $S(ct, x, y, z)$& $S'(ct', x', y', z')$ with $S'$ is moving in constant speed $v$ and the axes coincide at $t' = t = 0$ than the Lorentz transformations of space and time are as follows [1]:

$ct' = \gamma(ct - \beta x)$

$x' = \gamma(x - \beta ct)$ (3)

$y' = y$

$z' = z$

With $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ & $\beta = \frac{v}{c}$

Considering two events $A(t_A, x_A, y_A, z_A)$ & $B(t_B, x_B, y_B, z_B)$ in the frame $S$ the interval squared :

$\Delta s^2 = c^2\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$ (4)

is invariant under Lorentz transformations.

If $\Delta s^2 > 0$ : than the interval is time-like.

$\Delta s^2 = 0$ : than the interval is light-like

$\Delta s^2 < 0$ : than the interval is space-like.

Lorentz transformations can be also considered as hyperbolic rotation in the 4th space dimensions as the following:

$ct' = ct \cosh\psi - x\ sinh\psi$

$x' = -ct \sinh\psi + x\ cosh\psi$ (5)

$y' = y$

$z' = z$

With $th\psi = \beta$ , $cosh\psi = \gamma$, $sinh\psi = \gamma\beta$.

Space & time are considered as a four dimensions continuum that describes physics world with the Minkowski geometry related by relation (4).

The Minkowski geometry is not Euclidian due to the sign minus in (4).

The interval given by equation (4) is the "distance " between the events $A$ & $B$ measured in a straight line and can be considered as the worldline of a free particle moving in constant speed between the points $A$ & $B$.

The interval measured between the two points $A$ & $B$ in any arbitrary path is:

$\Delta s = \int_A^B ds$ (6)

With: $ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$ : the line element of the space-time.

For a massive particle we define the proper time $\tau$ as a parameter $t(\tau), x(\tau), y(\tau)$ & $z(\tau)$ in order to determine the position of the particle in the 4th space-time as:

$$d\tau^2 = \frac{ds^2}{c^2} \qquad (7)$$

Which mean that:

$$d\tau = \sqrt{1 - \frac{v^2}{c^2}}.dt = \frac{dt}{\gamma} \qquad (8)$$

The total elapsed proper time interval is:

$$\Delta\tau = \int_A^B \sqrt{1 - \frac{v(t)^2}{c^2}}.dt \qquad (9)$$

Where $v(t)$ is the instantaneous speed of the corpuscle along its path.

If we introduce a rest frame in which the corpuscle is in rest along its path the proper time is the time recorded by a clock moving with the corpuscle.

It is convenient to introduce the indexed coordinates $x^i$ $(i = 0,1,2,3)$ so we have:

$x^0 = ct, x^1 = x, x^2 = y, x^3 = z$ & $ds^2 = g_{ij}dx^i dx^j$

where $g_{ij}$ are the covariant metric tensor [1] as the following:

$$[g_{ij}] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad (10)$$

Or in shorthand notation as $[g_{ij}] = diag[1, -1, -1, -1]$.

The contravariant of the metric tensor are the same as: $[g^{ij}] = diag[1, -1, -1, -1]$.

In transforming coordinates $x^i$ in a Minkowski spacetime to a new coordinates $x'^i$ the line & s'element $ds^2$ must take the same form as:

$$ds^2 = g_{ij}dx^i dx^j = g_{ij}dx'^i dx'^j$$

Which means that the transformation $x^i \to x'^i$ must satisfy:

$$g_{ij} = \frac{\partial x'^k}{\partial x^i}\frac{\partial x'^l}{\partial x^j} g_{kl} \qquad (11)$$

Which implies that the transformations must be linear to be a Lorentz transformations:

$$x'^i = \Lambda^i{}_j x^j + a^i \qquad (12)$$

Where $\Lambda^i{}_j$ & $a^i$ are constants .

Transformations (12) are the inhomogeneous Lorentz transformations (or Poincare transformations). With $a^i$taken equal to zero they are the ordinary Lorentz transformations (or homogeneous transformations).

In a standard configuration of two inertial frames $S$ & $S'$ the matrix transformation can be written as follows:

$$[\Lambda^i{}_j] = \left[\frac{\partial x'^i}{\partial x^j}\right] = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} cosh\psi & -sinh\psi & 0 & 0 \\ -sinh\psi & cosh\psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad (13)$$

Where : $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ , $\beta = \frac{v}{c}$ & $th\psi = \beta$ , $cosh\psi = \gamma$, $sinh\psi = \gamma\beta$

The inverse transformation of (13) are obtained by replacing $v\ by - v\ \ or\ \ \psi\ \ by - \psi$

The matrix inverse is:

$$[\Lambda_i{}^j] = \left[\frac{\partial x^j}{\partial x'^i}\right]$$

And the relation ship between the two matrices :

$$[\Lambda^i{}_j] = g^{ik} g_{lj} \Lambda_k{}^l$$

And we have:

$$\Lambda^i{}_k \Lambda_j{}^k = \Lambda^i{}_k g^{km} g_{jl} \Lambda^l{}_m = g^{mi} g_{jm} = \delta^i_j$$

In any coordinate system the coordinate basis vectors are tangents to the coordinate curves

In frames $S$ & $S'$ basis vectors are related by:

$$e'^j = \Lambda^j{}_k\, e^k \qquad\qquad e^j = \Lambda_k{}^j\, e'^k$$

And we have;

$$e'^i . e'^j = g^{ij} \qquad\qquad e^i . e^j = g^{ij}$$

Dual basis vectors are defined as:

$$e_i = g_{ik} e^k \qquad\qquad e'_i = g_{ik} e'^k$$

And we have:

$$e_i . e_j = g_{ij} \qquad\qquad e'_i . e'_j = g_{ij}$$

Where the components of $g_{ij}$ are the same as $g^{ij}$ .

We can define a 4-vector $\boldsymbol{v}$ of point $P$ in Minkowski spacetime as :

$$\boldsymbol{v} = v^i \boldsymbol{e_i}$$

At each point $P$ we have a set of orthonormal basis $\boldsymbol{e_i}$ . The square of the length of a vector $\boldsymbol{v}$ is :

$\boldsymbol{v}.\boldsymbol{v} = v_i v^i = g^{ik} v_i v_k$

In two inertial frames $S$ & $S'$ with Cartesian coordinates $x^i$ & $x'^i$ the coordinates of a 4-vector $\boldsymbol{v}$ at a point $P$ is :

$\boldsymbol{v} = v^i \boldsymbol{e_i} = v'^i \boldsymbol{e'_i}$

And we have :

$v'^i = \boldsymbol{v}.\boldsymbol{e'^i} = \Lambda^i{}_j v^j$

$v^i = \boldsymbol{v}.\boldsymbol{e^i} = \Lambda_j{}^i v'^j$

The 4-velocity $\boldsymbol{u}$ of a (massive ) corpuscle is as :

$\boldsymbol{u}.\boldsymbol{u} = (\frac{ds}{d\tau})^2 = c^2$

The contravariant components of the 4-velocity are:

$u^i = \boldsymbol{u}.\boldsymbol{e^i} = \frac{dx^i}{d\tau}$

It comes that:

$$[u^i] = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}.(c, \frac{dx^1}{dt}, \frac{dx^2}{dt}, \frac{dx^3}{dt})$$

With : $c = \frac{dx^0}{dt}$, $x^0 = ct$ & $v = \sqrt{\left(\frac{dx^1}{dt}\right)^2 + \left(\frac{dx^2}{dt}\right)^2 + \left(\frac{dx^3}{dt}\right)^2}$

The 4-moment of a massive corpuscle is defined as :

$\boldsymbol{p} = m\boldsymbol{u}$ (14)

With : $m$ : the mass of the corpuscle;

$\boldsymbol{u}$ : the 4-velocity of the corpuscle.

The component of the 4-moment are $p^i = \boldsymbol{p}.\boldsymbol{e^i}$ and we have:

$[p^i] = \left[\frac{E}{c}, p^1, p^2, p^3\right] = (\frac{E}{c}, \vec{p})$ (15)

With $E$ the energy of the corpuscle and $\vec{p}$ its 3 ordinary moment in an inertial frame $S$ .

It comes that:

$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$ & $\vec{p} = \frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}$ (16)

The squared length of the 4-moment is $p^i p_i$ and so get:

$(\frac{E}{c})^2 - p^2 = m^2c^2$ (17)

Where $p^2 = \vec{p}.\vec{p}$ .

For a photon (massless corpuscle) we can't use the proper time to parametrize its worldline which is a null curve because $d\tau = 0$. We can use an affine parameter $\sigma$ as for a photon moving in the direction $x = ct$ we have:

$$x^i = \sigma u^i \qquad (18)$$

With: $[u^i] = (1,1,0,0)$

The tangent vector of the worldline is:

$$\boldsymbol{u} = \frac{dx^i}{d\sigma}\boldsymbol{e_i} \qquad (19)$$

And we have:

$$\boldsymbol{u}.\boldsymbol{u} = 0 \quad \& \quad \frac{d\boldsymbol{u}}{d\sigma} = \boldsymbol{0} \qquad (20)$$

Equation (20) is the equation of motion of the photon.

Of course in equation (20) we consider that the vector $\boldsymbol{e_i}$ are constant and does not change with the position.

The tangent vector $\boldsymbol{u}$ can be multiplied by any scalar and the equation (20) will be satisfied. The 4-moment of a photon can be defined as:

$$\boldsymbol{p} = \mu\boldsymbol{u} \qquad (21)$$

Where $\mu$ is constant and the components of $\boldsymbol{p}$ are:

$$[p^i] = (\frac{E}{c}, \vec{p})$$

Where $E$ & $\vec{p}$ are respectively the energy and the 3-moment of the photon in a given inertial frame $S$.

It comes that from equation (20):

$$E = pc \qquad (22)$$

The same equation (22) can be obtained for massless corpuscles if we take $m = 0$ in equation (17).

Equation (17) is a more generalized equation (massive corpuscles and massless corpuscles).

For photon we can introduce the 4-wavevector $\boldsymbol{k}$ as :

$$\boldsymbol{p} = \hbar\boldsymbol{k} \qquad (23)$$

And the components of the 4-wavevector are:

$$[k^i] = (\frac{2\pi}{\lambda}, \vec{k}) \qquad (24)$$

Where : $\lambda$ : is the wave-length of the photon

$\vec{k} = \frac{2\pi}{\lambda}\vec{n}$ : the 3-waveevector of the photon

$\vec{n}$ : direction of the propagation of the photon ( 3-vector of one module).

Equation (24) like equation (17) can be generalized for any corpuscle (massless or not). Equating equation (24) to equation (15) we get:

$$\frac{E}{c} = \hbar.\frac{2\pi}{\lambda} \qquad (25)$$

$$\vec{p} = \hbar\vec{k} \qquad (26)$$

Which mean that massive corpuscle can have waving behavior.

The group speed $v_g$ of such wave behavior of massive corpuscle is defined as :

$$\frac{1}{v_g} = \frac{dk}{d\omega} \qquad (27)$$

With : $\omega = 2\pi\nu = 2\pi\frac{c}{\lambda}$ : the frequency of the photon (the frequency associated to the massive corpuscle).

From (25) & (16) we get:

$$E = \hbar\omega = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (28)$$

And from (26) & (16) we get:

$$\hbar\vec{k} = \vec{p} = \frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (29)$$

From (27) we get:

$$v_g = v \qquad (30)$$

The phase speed $v_f$ of such a wave is defined as:

$$v_f = \frac{\omega}{k} \qquad (31)$$

We have always:

$$v_g.v_f = v\frac{\omega}{k} = c^2 \qquad (32)$$

In equation (23) we had never speak about the constant $\hbar$ : it can be replaced by any other constant and the 4-vector linked in will follow also.

For any corpuscle we introduce the 4- string-vector $\boldsymbol{l}$ as :

$$\boldsymbol{p} = a\boldsymbol{l} \qquad (33)$$

And the components of the 4-string-vector are:

$$[l^i] = (l, \vec{l}) \qquad (34)$$

With : $l$: the string-length associated to the corpuscle

$\vec{l} = l\vec{n}$ : string-vector in the 3-ordinary space

$\vec{n}$ : direction of the propagation of the corpuscle ( 3-vector of one module).

Equating (34) and (15) we get:

$$\frac{E}{c} = al \qquad (35)$$

$$\vec{p} = a\vec{l} \qquad (36)$$

Let's search the vector $\frac{d\vec{l}}{d\omega}$ (considering the corpuscle as a packet of strings)?

$$\frac{dl}{d\omega} = \frac{dl}{dv}.\frac{dv}{d\omega} = \frac{1}{a}\frac{dp}{dv}.\frac{dv}{d\omega} = \frac{1}{a}\frac{dp}{d\omega} = \frac{\hbar}{a}\frac{dk}{d\omega} = \frac{\hbar}{a}.\frac{1}{v} = \frac{\hbar}{a}.\frac{v_f}{c^2}$$

It comes that:

$$\vec{v}_f = \frac{ac^2}{\hbar}\frac{d\vec{l}}{d\omega} \qquad (37)$$

From (35) & (16) we have:

$$E = acl = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (38)$$

From (36) & (16) we get:

$$a\vec{l} = \vec{p} = \frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}} = \hbar\vec{k} \qquad (39)$$

We have always:

$$\vec{p} = \frac{E}{c^2}.\vec{v} = \frac{as}{c}.\vec{v} = \frac{\hbar\omega}{c^2}.\vec{v} \qquad (40)$$

We can also define the 4-vector inertia $\boldsymbol{\xi}$ as:

$$\boldsymbol{p} = \boldsymbol{\xi}c = m\boldsymbol{u} \qquad (41)$$

The components of the 4-vector inertia are:

$$[\xi^i] = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}}.\left(1, \frac{dx^1}{dx^0}, \frac{dx^2}{dx^0}, \frac{dx^3}{dx^0}\right) = \xi.\left(1, \frac{dx^1}{dx^0}, \frac{dx^2}{dx^0}, \frac{dx^3}{dx^0}\right) \qquad (42)$$

With: $x^0 = ct$ & $\xi = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}}$

In a system of units where $\hbar = c = a = 1$ equation (40) can be written in simply form as:

$$\vec{p} = \xi\vec{v} = l\vec{v} = \omega\vec{v} \qquad (43)$$

Inertia can be a mass or a length or a frequency or anything else as time for example.

We define the inertial time $\zeta$ as :

$$\zeta = \frac{\xi}{a} \qquad (44)$$

It is evident if we associate to the corpuscle an inertial time at rest $\zeta_0$ than according to equation (8) we have:

$$\zeta_0 = \sqrt{1 - \frac{v^2}{c^2}}\,.\zeta \qquad (45)$$

$d\zeta = dt$ : when the energy of the corpuscle change (46)

From (44) & (45) we get:

$$\zeta_0 = \frac{m}{a} \qquad (47)$$

Of course we can also define the 4-vector inertial time $\boldsymbol{\zeta}$ as:

$$\boldsymbol{\xi} = a\,\boldsymbol{\zeta} \qquad (48)$$

From equations (17) & (25) we get:

$$h^2\nu^2 - p^2c^2 = m^2c^4 \qquad (49)$$

With: $E = h\nu$ : the energy of the corpuscle

$\nu = \frac{\omega}{2\pi}$ : reduced inertial frequency of the corpuscle;

$h = 2\pi\hbar$

Differentiate (49) according to time:

$$h^2\nu d\nu = c^2 p dp = \xi c^2\vec{v}.d\vec{p} = h\nu\vec{v}.d\vec{p}$$

Which mean:

$$\frac{d\vec{p}}{dt}.\vec{v} = h\frac{d\nu}{dt} = ac^2\frac{d\zeta}{dt} = \vec{f}.\vec{v} \qquad (50)$$

With: $\vec{f} = \frac{d\vec{p}}{dt}$ : the 3-dimensional force.

If the energy of the corpuscle change then we have always:

$$\vec{f}.\vec{v} = ac^2 \qquad (51)$$

An important case is when the speed of the corpuscle is low compared to $c$ . From equation (49) we get:

$$h\nu \approx mc^2 + \frac{1}{2}m\vec{v}^2 \qquad (52)$$

$\vec{p} \approx m\vec{v}$ (53)

$\vec{f} \approx m\frac{d\vec{v}}{dt} = \frac{d\vec{p}}{dt}$ $with\, \vec{p} = m\vec{v}$ (54)

which mean:

$hd\nu = m\vec{v}.d\vec{v}$

Than:

$h\frac{d\nu}{dt} = \vec{f}.\vec{v}$ (55)

Equation (55) is the same equation as (50) even thought it is obtained by approximation.

The square of (52) is :

$h^2\nu^2 = m^2c^4 + m^2c^2\vec{v}^2 + \frac{1}{4}m^2\vec{v}^4$ (56)

$2h^2\nu\frac{d\nu}{dt} = m^2c^2 2\vec{v}.\frac{d\vec{v}}{dt} + \frac{1}{4}m^2\frac{d\vec{v}^4}{dt}$

$h\frac{d\nu}{dt} = \frac{mc^2}{h\nu}\vec{v}.\vec{f} + \frac{1}{4}m^2\frac{d\vec{v}^4}{dt}.\frac{1}{2mc^2+m\vec{v}^2} \approx \frac{mc^2}{h\nu}\vec{v}.\vec{f} = \vec{f}.\vec{v}$ (57)

which mean :$h\nu \approx mc^2$ for low speed.

From equation(50):

$h\nu^2 = \frac{m^2c^4}{h} + \frac{p^2c^2}{h}$

$\frac{d(h\nu^2)}{dt} = 2\frac{c^2}{h}\vec{p}.\frac{d\vec{p}}{dt} = 2\frac{c^2}{h}.\frac{h\nu}{c^2}\vec{v}.\vec{f} = 2\nu ac^2 = 2h\nu\frac{d\nu}{dt}$ (58)

Which mean:

$ac^2 = h\frac{d\nu}{dt}$ (59)

Than:

$\nu = \frac{ac^2}{h}t + constant$ when the energy of the corpuscle is changing.

But in this case we have: $dt = d\zeta$ so:

$\nu = \frac{ac^2}{h}\zeta$ which is a trivial result (with $constant = 0$) (60)

Equation (51) signify that a corpuscle never can't be in rest otherwise the universal constant $a$ will be equal to zero. Than absolute vacuum never exist in a frame $S$ : there is energy independent from the corpuscle and which don't let the corpuscle become in rest.

**2.Wave-corpuscle duality:**

Constant $h$ was suggested and determined in 1900 by Max Planck in his model of black body radiation [2]. Planck model of black body was a cavity with many oscillators on his wall exchanging energy with radiation inner. Planck determine the mean energy of an oscillator and the radiation law at equilibrium in a temperature $T$ of the cavity and get the values of constants $h$ & $k_B$ (Planck constant and Boltzmann constant) by a statistical manner and referring earlier to F. Kurlbaum & Wien measurements of black body radiation.

In his recent theory of heat radiation by 1911 Planck suggest that his oscillator should have enough time to absorb energy from radiation[3] . The Planck condition for this is that the frequency of the oscillator times time should be a great integer or in other manner:

$$\nu . t \gg 1 \qquad (61)$$

By 1923 De Broglie suggest the wave-corpuscle duality i.e. a corpuscle can have a waving behavior and vice versa: he propose the relation (27) and getting (28) & (29) by referring to Restraint Relativity proposed by A. Einstein in 1905.

In his suggestion De Broglie consider a corpuscle as a packet of waves moving in the same direction of the corpuscle and represented with a wave function having a constant amplitude. De Broglie consider a corpuscle as many monochromatic waves which reinforce each other in a limited region of spacetime and destroy each other away. Mathematically speaking this model can't be adopted because the waves will reinforce each other in many regions of spacetime in the same direction of the corpuscle motion and we can't see where is really the corpuscle. A replacement model of De Broglie one is that a corpuscle should be considered as a pulse and not as a packet of waves because in this case the corpuscle is more identified with the condition that the pulse should not spread so much in time. This condition implies that:

$$\nu . t < 1 \qquad (62)$$

The condition satisfying the two models (Planck & De Broglie) is:

$$\nu . t \approx 1 \qquad (63)$$

Which means that:

$$dt \approx -\frac{d\nu}{\nu^2} \qquad (64)$$

We take always $dt$ positive and so we can omit the sign minus in (64) or take its absolute value:

$$dt \approx \frac{d\nu}{\nu^2} \qquad (65)$$

Replace (63) in (55) for low speed corpuscle we get:

$$h\nu^2 = \vec{f} . \vec{v} \qquad (66)$$

Replace (63) in (58) for high speed corpuscle we get:

$$h\nu^2 = \vec{f} . \vec{v} \qquad (67)$$

Equation (67) is the same equation as equation (66) so it is applicable for any kind of oscillator. It is the power absorbed or emitted by an oscillator in a black body.

Planck oscillator is a classic oscillator so equation (66) is the work of a force applied on a corpuscle by unit time i.e. for a corpuscle in motion in a straight line along the axis $x$ for example:

$$\vec{f}.\vec{v} = fv = h\nu^2$$

So the force acting on the corpuscle is:

$$f = \frac{1}{v}.h\nu^2$$

Duality of wave-corpuscle implies that:

$$\frac{1}{v} = \frac{dk}{d\omega}$$

with $v = v_g$ : the group speed of the packet of waves assimilated as a corpuscle;

$k$: wave-vector of the packet of waves

$\omega = 2\pi\nu$ : the frequency of the packet of waves.

So:

$$f = h\nu^2.\frac{dk}{d\omega} = h\nu^2.\frac{dk}{2\pi d\nu} = \hbar\nu^2.\frac{dk}{d\nu} = \frac{d(\hbar k)}{dt}$$

With : $\hbar = \frac{h}{2\pi}$ : reduced Planck constant.

$\hbar k$ : have the dimension of a moment. So:

$f = \frac{dp}{dt}$ with $p = mv$ : is the moment of the corpuscle.

This relation is generalized as:

$$\vec{f} = m\vec{\gamma} \qquad (68)$$

With: $\vec{\gamma} = \frac{d\vec{v}}{dt}$ the acceleration of the corpuscle

$m$ : the mass of the corpuscle.

Equation (68) is the fundamental law of dynamics or the Newton first law.

Planck oscillator can have also very high speed and equation (67) will be applicable. As Planck suggest that his oscillators can absorb or emit energy with quanta as a multiple integer of $h\nu$ at the same time we extend this idea that Planck oscillators can absorb power or emit power with quanta as a multiple integer of $h\nu^2$ .

Between the two models of oscillators (classic and relativist) there is a special frequency $\nu_0$ in the border of our consideration to balance in one side or another and to deduce from

Planck model of black body radiation even thought to apply this model for vacuum which can be considered as a black body at zero absolute temperature (no radiation) or approximatively near zero absolute. Replace (65) in (59) we get:

$$ac^2 \approx h\nu^2 \qquad (69)$$

Which mean that the frequency $\nu$ is near $\nu_0$ changing all time otherwise there is no constant $a$ .

In fact we have exactly:

$$a = \frac{h\nu_0^2}{c^2} \qquad (70)$$

Even thought the constant $\nu_0$ can be derived from classic approximations.

In general we have:

$$\frac{d\vec{p}}{dt} = \frac{d(a\zeta\vec{v})}{dt} = a\vec{v} + \frac{m}{\sqrt{1-\frac{\vec{v}^2}{c^2}}}\frac{d\vec{v}}{dt} \approx a\vec{v} + m\vec{\gamma} \quad \text{for } \|\vec{v}\| \ll c$$

Than:

$$m\vec{\gamma} = \frac{d\vec{p}}{dt} - a\vec{v} = \vec{f} - a\vec{v} \qquad (71)$$

All forces applied on the corpuscle are equal to $\vec{f} = \frac{d\vec{p}}{dt}$ plus a permanent force $a\vec{v}$ in the opposite side of its motion which mean an external force which drive the motion of the corpuscle even thought it is in rest. The corpuscle never can't be in rest and will oscillate around its position.

**3.Determination of constant $\nu_0$ :**

Planck oscillators are classic oscillators. They are oscillating charged corpuscles. In a black body they are the electrons of the atoms of the wall of the black body cavity (oscillations of nucleus of atoms are neglected).

According to Bohr model of the atom, the electron in an hydrogen atom is moving in planetary motion (circular) as the speed of the electron is equal to $\alpha c$ where $\alpha = \frac{1}{137}$ the fine structure constant. The vacuum in atoms is the same vacuum in the cosmos. Vacuum which is filled with energy has a certain mechanical impedance and a negative pressure so there is no friction in the motion of any corpuscle as it is given by General Relativity.

Suppose that a corpuscle is in motion from a point $A$ to a point a point $B$, its energy exchanged with vacuum is:

$$\varepsilon = \int_A^B a\vec{v}.\vec{v}dt = \int_A^B a\vec{v}^2\,dt = \int_A^B ac^2\left(1 - \frac{\zeta_0^2}{\zeta^2}\right)d\zeta = ac^2(\zeta_B - \zeta_A) + ac^2\zeta_0^2\left(\frac{1}{\zeta_B} - \frac{1}{\zeta_A}\right) \qquad (72)$$

We have not do any approximation in equation (72). It is done in a 4-space dimensions frame .

Let's take an origin for the energy exchanged which is the point $A$ where the corpuscle is in rest, than:

$$\varepsilon = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2 + ac^2\zeta_0^2\left(\frac{\zeta_0-\zeta}{\zeta_0\zeta}\right) = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2 + mc^2\left(\sqrt{1-\frac{v^2}{c^2}} - 1\right) = mc^2\left[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1 + \sqrt{1-\frac{v^2}{c^2}} - 1\right] = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}\left(1 - 2\sqrt{1-\frac{v^2}{c^2}} + \sqrt{1-\frac{v^2}{c^2}}^2\right) = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}\left(1-\sqrt{1-\frac{v^2}{c^2}}\right)^2 \quad (73)$$

For the electron of Bohr atom we get:

$$\varepsilon = \frac{mc^2}{\sqrt{1-\alpha^2}}\left(1-\sqrt{1-\alpha^2}\right)^2 \approx \frac{1}{4}\alpha^4 mc^2 \quad (74)$$

With: $m = 9{,}1\ 10^{-31}Kg$ the mass of the electron.

If the electron is oscillating in a 4th space dimension than its oscillations are driven by an electromagnetic field in its fundamental state. The energy of an oscillator in its fundamental state is [4]:

$$E = \frac{1}{2}h\nu \quad (75)$$

The mean energy $U$ of the electromagnetic field which drive the electron is :

$$U = \frac{\frac{1}{2}h\nu}{\exp\left(\frac{\nu}{\nu_0}\right)-1} \quad (76)$$

Equation (76) is obtained by statistical manner as for low frequencies the energy of the oscillator is:

$$E \approx \frac{1}{2}h\nu_0 \quad (77)$$

Considering the electron as 4-space dimension corpuscle than we get:

$$\frac{1}{2}h\nu_0 = 4x\frac{1}{4}\alpha^4 mc^2 = \alpha^4 mc^2 \quad (78)$$

Which mean:

$$\nu_0 = 2\frac{\alpha^4 mc^2}{h} \approx 0.7\ 10^{12}\ Hz \quad (79)$$

## 4 -Experimental evidence:

### 4-1 -from cosmology observations:

The energy density of vacuum as given by General Relativity is as follows[5] :

$$U_0 = \frac{\Lambda . c^4}{8\pi G} \approx 10^{-9} Joule.m^{-3} \quad (80)$$

With :

$\Lambda = 1{,}088\ 10^{-52}m^{-2}$ : cosmological constant ;

$c = 3\ \ 10^8 m.s^{-1}$ : light celerity in vacuum ;

$G = 6{,}67\ \ 10^{-11}\ SI\ units$ : gravitationnel constant ;

The total energy density of the vacuum is then according to the model of the black body theory:

$$U_0 = \int_0^\infty \frac{8\pi\nu^2}{c^3}.Ud\nu = \int_0^\infty \frac{4\pi h}{c^3}.\frac{\nu^3}{\exp\left(\frac{\nu}{\nu_0}\right)-1}d\nu = \frac{4\pi^5 h}{15.c^3}.\nu_0^4 \quad (81)$$

Equating (80) & (81) we get:

$$\nu_0 = \left[\frac{15\,\Lambda.c^7}{32.\pi^6.G.h}\right]^{\frac{1}{4}} \approx 0.7\;10^{12}Hz \quad (82)$$

This is an experimental proof that the electron is a 4-space dimension corpuscle. It means also that extension in space is energy.

**4-2-from chemistry:**

Oscillations is also rotations. Let's remarque that there is also an universal rotator with a moment of inertia of:

$$J_0 = \frac{h}{4\pi^2\nu_0} \approx 0.24\;10^{-39}\;gram.cm^2 \quad (83)$$

Many gases when excited with infrared radiation have approximatively a moment of inertia around this value and also peaks of radiation absorption which are interpreted that they are quantized rotational states and not transition between states (table01)[5] :

| Gas | Molecular mass $[atomic\ unit\ u]$ | Moment of inertia $[x10^{-39}\ gram.cm^2]$ | Rotational frequency $[x10^{12}Hz]$ |
|---|---|---|---|
| $H_{2\ para}$ | 2 | 0.221 | 0.8 |
| $H_{2\ ortho}$ | 2 | 0.096 | 1.8 |
| $HCl$ | 36.5 | 0.264 | 0.6 |
| $HBr$ | 81 | 0.327 | 0.5 |
| $HF$ | 20 | 0.137 | 1.2 |
| $D_2$ | 4 | 0.382 | 0.4 |

Table01: values taken from the book "the quantum theory" page169 & page 180 by Fritz Reiche-1920

This means that the frequency $\nu_0$ have a deep relation ship with atomic scale and we get a response why the rotational frequencies of molecules are about this value. No one have a response why gases have rotational frequencies around $10^{12}Hz$ and this means also that the 4th space dimension or other extra space dimension is about the same dimension of the atom or less. The frequency $\nu_0$ is a break down frequency between two scale levels .

**4-3-from photoelectric effect:**

Let's take the experiment of the photo-electric effect. We have a photo cell illuminated by light with a frequency $\nu$ . The photo-electrons are accelerated or decelerated by a tension $V$ applied on its ends and the correspondent current is measured.

Einstein equation for the photo-electrons:

$eV = h\nu - W_s$ (85)

$W_s = h\nu_s$ : the work done by light when $V = 0\ Volt$

$\nu_s$ : starting frequency for the photo-electric phenomena of the cell.

The uncertainty of the measuring of the potential $V$ is:

$e\Delta V = h\Delta\nu - \Delta W_s$ (86)

If we suppose that the frequency of the incident light is highly precise than in absolute manner we have:

$\Delta V \sim \frac{\Delta W_s}{e}$ (87)

With: $\Delta W_s = 2\alpha^4 mc^2$ & $m$: $the\ mass\ of\ the\ electron$

So from equation (87) we deduce that the curve $I = f(V)$ never coincide with the $V - axis$ at exactly when the current $I = 0$ even thought we eliminate any contact potential for the cell.

Grosso modo it is easy to estimate : $\Delta V \approx 3\ milli - Volt.$

**5-The verdict of thermodynamics:**

In 1900 ,to resolve the problem of black body radiation Planck had conceive a black body as a cave at an equilibrium temperature $T$ with oscillators on its walls exchanging energy with the electromagnetic radiation . Planck determine the entropy of an oscillator with the assumption that the statistical entropy is the same thermodynamic entropy. By referring to F.Kurlbaum measurements and Wien displacement law he determine the values of two universal constants $h$ & $k_B$ : Planck action element and Boltzmann constant.

In 1906 Planck show that his oscillators with constant energy are described by an ellipse in the phase space domain $(p, q)$ moment & position and the area between two successive ellipses is equal to the action element $h$, which corresponds to the smallest existing volume of phase space. This volume corresponds to an elementary probability domain . The total volume of a cell bordered by an ellipse is proportional to the probability to find the oscillator in it. The result is justified by the hypothesis that all complexions are equally probable. The oscillators corresponding to this cell domain are indistinguishable.

The energy quantity $\varepsilon$ corresponding to $h$ verifies:

$h = \int_E^{E+\varepsilon} dqdp$ (88)

The ellipse of the phase space of equation $E = \frac{1}{2}Kq^2 + \frac{p^2}{2L}$has a surface $S(E) = 2\pi E\sqrt{\frac{L}{K}}$ so :

$h = S(E + \varepsilon) - S(E) = 2\pi\varepsilon\sqrt{\frac{L}{K}} = 2\pi\varepsilon.\frac{1}{\omega}$

With $K$ & $L$ are respectively the stiffness and the mass of the oscillator.

Than:

$\varepsilon = h\nu$ (89)

The magnitude of power $W$ corresponding for this same quantity of energy $\varepsilon$ verify :

$W = \int_{E}^{E+\varepsilon} \frac{K}{L} dq dp = \frac{K}{L}\frac{2\pi}{\omega}\varepsilon = 2\pi\omega\varepsilon$ (90)

Because $K = L\omega^2$ .

This is possible (to have the dimension of a power) if $\varepsilon$ verify equation (89) or also there is a constant $\alpha_0$ having the dimension of a power such that :

$\varepsilon = \alpha_0 \zeta$ (91)

With $\zeta$ : a characteristic time of the oscillator.

Constants $h$ & $\alpha_0$ are declared universal constants .

From equation (70) it is very easy to identify the constant $\alpha_0$ as:

$\alpha_0 = ac^2 = h\nu_0^2$ (92)

The constant "$a$" has the dimension of a mechanical impedance, in other words space-time cannot be conceived as completely empty: it is a superfluid of coefficient of friction "$a$" and having a negative pressure to cancel the viscosity effect. For negative pressure it is like the same idea in General Relativity : Planck oscillator can not be 1-space dimension oscillator as in classic mechanics but a 4-space dimension oscillator like in General Relativity.

In its ground state an oscillator absorbs a certain amount of energy and returns it to space-time in a perpetual fashion.

The electromagnetic field can be considered as an oscillator in its fundamental state and it fills all space-time. Thus the vacuum can be defined as being the fundamental state of the electromagnetic field: all the points of space-time are oscillators in their fundamental states. The vacuum energy predicted by General Relativity comes from this conception of the electromagnetic field in its ground state.

Of course we can obtain equation (92) in an ad hoc manner without referring to the Revisited Restraint Relativity (§01). Also we can continue in an ad hoc manner and we pose for equation(91) that $d\zeta = dt$ when the energy of the corpuscle is varying otherwise it is equal to zero. But we should always maintain in our mind that Planck oscillator is a classic oscillator with 4-space dimensions: where is the 4th dimension?: it is at first a mean for calculation, second the corpuscle (or the electron) is not a point material: it have extension and it can have extension in the extension and so on. The good question to response will be what is the structure of spacetime?.

It is evident that also for the same oscillator we have:

$\frac{h}{2} = \int_{E}^{E+\varepsilon} dqdp$ (93)

Which signify that we will have:

$\varepsilon = \frac{h}{2}\nu$ (94)

The energy (94) is the energy of an oscillator at its fundamental state i.e. does not vibrate or at near zero absolute temperature.

Let's exam the uncertainty on the position of the oscillator by the root mean square error i.e. the square root of the its mean quadratic fluctuations [6]:

$$\overline{\Delta q} = \sqrt{\overline{q^2} - \bar{q}^2}$$

We can pose that : $\bar{q} = 0$

So:

$\overline{\Delta q} = \sqrt{\overline{q^2}} = \sqrt{A^2\overline{\cos^2(\omega t)}} = \sqrt{\frac{1}{2}A^2}$ (95)

With: $A$: amplitude of vibration equivalent to the classic relation $\varepsilon = \frac{1}{2}LA^2\omega^2 = \frac{1}{2}KA^2$ than:

$\overline{\Delta q} = \sqrt{\frac{\varepsilon}{K}}$ (96)

With the same manner we have:

$\overline{\Delta p} = \sqrt{\overline{p^2}} = \sqrt{L^2A^2\omega^2 sin^2(\omega t)} = \sqrt{\frac{1}{2}L^2A^2\omega^2} = \sqrt{L\varepsilon}$ (97)

So we have:

$\overline{\Delta q}\overline{\Delta p} = \varepsilon\sqrt{\frac{L}{K}} = \frac{\varepsilon}{\omega}$ (98)

According to the uncertainty principle we have $\Delta q\Delta p \approx \hbar$ and the value of the lower limit of the product of the mean quadratic errors is $\frac{\hbar}{2}$ as we will have:

$\Delta q\Delta p \geq \frac{\hbar}{2}$ (99)

If we take in (98) as $\Delta q\Delta p = \frac{\hbar}{2}$ and comparing with (97) than:

$\varepsilon = \frac{\hbar}{2}\omega = \frac{h}{2}\nu$ (100)

Which mean that Planck model for black body radiation can be also useful as a model for vacuum energy density with a mean energy of the oscillator at low frequencies as equal to about $\frac{1}{2}h\nu_0$ where $\nu_0$ is an universal constant. The mean energy of the oscillator will be defined by analogy to Planck model of black body radiation as:

$$U = \frac{\sum_{n=0}^{n=\infty}\frac{n}{2}h\nu.\exp(\frac{-\frac{n}{2}h\nu}{\frac{1}{2}h\nu_0})}{\sum_{n=0}^{n=\infty}\exp(\frac{-\frac{n}{2}h\nu}{\frac{1}{2}h\nu_0})} = \frac{\frac{1}{2}h\nu}{\exp\left(\frac{\nu}{\nu_0}\right)-1} \approx \frac{1}{2}h\nu_0 \; if \; \nu \ll \nu_0 \qquad (101)$$

It is clear that our Universe can have many space dimensions: every corpuscle can have a 4th space dimension, the 4th space dimension can be considered in itself another Universe with 4-space dimensions and so on. To detect an extra space dimension it depends of our instruments accuracy, the energy used and also the representation of the corpuscle (point, wave, string). We have always those relations:

$$p = \hbar k = al \qquad (102)$$

Planck oscillators are classic oscillators. The mean energy of an oscillator in a cavity at an equilibrium temperature $T$ is according to Planck [7]:

$$E_{med} = \frac{h\nu}{\exp\left(\frac{h\nu}{k_B T}\right)-1} \qquad (103)$$

Every oscillator can absorb or radiate energy as per quanta multiple integer of $h\nu$ . The same thing for the oscillator about the power radiated or absorbed as per quanta multiple integer of $h\nu^2$ and the mean power of an oscillator is:

$$W_{med} = \frac{h\nu^2}{\exp\left(\frac{h\nu}{k_B T}\right)-1} \qquad (104)$$

We have for example for the energy of an oscillator as:

$$E = \int h\nu^2 dt = \int h\nu^2 \frac{d\nu}{\nu^2} = h\nu \qquad (105)$$

But we can't write it as $E = \int h\nu^2 dt = \int h\nu^2 . \frac{h}{ac^2} d\nu$ because $\vec{f} = m\vec{\gamma} = \frac{d\vec{p}}{dt}$ is valid for classic mechanics and it is invariant by Galileo transformations of space-time (Lorentz transformations of space-time with $c \to \infty$ ) & non valid for restraint relativity because it is not invariant by Lorentz transformations of space-time.

With Planck we deal with classic oscillator and if the quantum of energy $h\nu$ is OK for his model of black body than the quantum of power $h\nu^2$ is also OK for his model of black body even thought the speed of the oscillator can reach any value. We remain with Planck context even thought it appears that equation (102) is always valid for low speed corpuscles or high speed corpuscles. It is like that the Planck model is an extension to be valid for high speed corpuscles as an average model and experimentally strongly confirmed by thermodynamics.

The 4-vector force $\boldsymbol{f}$ which is invariant by Lorentz transformations is [8]:

$\boldsymbol{f} = \frac{d\boldsymbol{p}}{d\tau}$ (106)

And its contravariant components are:

$[f^i] = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}(\frac{\vec{f}.\vec{v}}{c}, \vec{f})$ (107)

With $\vec{f}$ is the ordinary 3-force and $\vec{v}$ the ordinary 3-speed measured in the frame $S$.

If the speed of the corpuscle tends to zero than:

$[f^i] \approx (\frac{h\nu^2}{c}, \vec{0}) \rightarrow (\frac{h\nu_0^2}{c}, \vec{0})$ (108)

And the constant $\nu_0$ can be not equal to zero i.e. the mean speed of the corpuscle can be equal to zero but its mean square speed can be not equal to zero and this correspond to a perpetual sinusoidal motion which mean vacuum is full with an energy which drive the corpuscle around and equilibrium position with a force equal to "$a\vec{v}$" and a mechanical impedance of vacuum $a = \frac{h\nu_0^2}{c^2}$ . If we assign to the corpuscle a position in time as $\zeta$ it will have a moment along the axle time as $a\zeta c$ and an energy $H$ ( in terms of analytical mechanics with 4- space dimensions where $ct$ is the 4$^{th}$ space dimension) [8] as :

$\frac{\partial H}{\partial(a\zeta c)} = c$ so $H = a\zeta c^2$ (109)

The force which acting along the time axle ( in terms of analytical mechanics) is:

$-\frac{\partial H}{\partial(c\zeta)} = -ac$ (110)

Also we have :

$\frac{\partial H}{\partial(c\zeta)} = ac$ (111)

Which mean there is only conservative forces.

The position of the corpuscle at rest on the axle time is $\zeta_0$ as :

$\zeta_0 = \frac{m}{a}$ (112)

With $m$: mass of the corpuscle.

And at any time the relation ship between its inertial time $\zeta$ and its position in rest $\zeta_0$ is given by Lorentz transformations as :

$\zeta_0 = \zeta\sqrt{1 - \frac{\vec{v}^2}{c^2}}$ (113)

The moment of the corpuscle in the 3-oridnary space will be :

$\vec{p} = a\zeta\vec{v} = \frac{m}{\sqrt{1-\frac{\vec{v}^2}{c^2}}}\vec{v}$ (114)

With : $a\zeta = \frac{m}{\sqrt{1-\frac{\vec{v}^2}{c^2}}}$ : *the mass of the corpuscle in motion.*

And this moment is in accordance with analytical mechanics as:

$$\vec{p} = \frac{\partial \mathcal{L}}{\partial \vec{v}} \qquad (115)$$

With : $\mathcal{L} = \vec{p}.\vec{v} - H = mc^2\sqrt{1-\frac{\vec{v}^2}{c^2}}$ : Lagrange function of the corpuscle (or kinetic potential).

There is not any experience proofs that our Universe had only three space dimensions. It can have $D$ −space dimensions . The other space dimensions can be curled in little spheres about the Planck length and more.

For a black body in $D - space\ dimensions$ Universe the number of modes with frequencies between $\nu$ & $\nu + d\nu$ is [9]:

$$N(\nu)d\nu = (D-1)V\frac{2}{\Gamma(\frac{D}{2})}\left(\frac{\sqrt{\pi}}{c}\right)^D \nu^{D-1}d\nu \qquad (116)$$

With: $V$: volume of the black body

$\Gamma(x) = \int_0^\infty t^{x-1}\, e^{-t}dt$ : Gamma function.

$D > 1$: number of space dimensions of the Universe.

The density of power between $\nu$ & $\nu + d\nu$ is :

$$w_T(\nu)d\nu = 2\left(\frac{\sqrt{\pi}}{c}\right)^D \frac{D-1}{\Gamma(\frac{D}{2})}\frac{h\nu^{D+1}}{\exp\left(\frac{h\nu}{k_BT}\right)-1}d\nu \qquad (117)$$

The total density of power is obtained by integration of (117):

$$w_T = \int_0^\infty w_T(\nu)d\nu = \int_0^\infty 2\left(\frac{\sqrt{\pi}}{c}\right)^{D+1-1}\frac{D+1-2}{\Gamma(\frac{D+1-1}{2})}\frac{h\nu^{D+1}}{\exp\left(\frac{h\nu}{k_BT}\right)-1}d\nu$$

And using the identity $\Gamma(2x) = \frac{2^{2x-\frac{1}{2}}}{\sqrt{2\pi}}\Gamma(x)\Gamma(x+\frac{1}{2})$ we get:

$$w_T = \frac{c}{\sqrt{\pi}}\frac{\Gamma(\frac{D+1}{2})}{D}\frac{D-1}{\Gamma(\frac{D}{2})}a_{D+1}T^{D+2} \qquad (118)$$

With :

$$a_D = \left(\frac{2}{hc}\right)^D \left(\sqrt{\pi}\right)^{D-1}{k_B}^{D+1}D(D-1)\Gamma\left(\frac{D+1}{2}\right)\zeta(D+1) \quad (119)$$

The radiancy or the rate of radiation of energy per unit area of the cavity surface of the black body or generalized Stefan-Boltzmann law is [9]:

$$R_T = \sigma_D T^{D+1} \qquad (120)$$

With :

$$\sigma_D = \left(\frac{2}{c}\right)^{D-1} \left(\sqrt{\pi}\right)^{D-2} \frac{k_B^{D+1}}{h^D} D(D-1)\Gamma(\frac{D}{2})\zeta(D+1) \qquad (121)$$

Using the identity $\Gamma(x+1) = x\Gamma(x)$ we get:

$$w_T = (D-1)\sigma_{D+1}T^{D+2} \qquad (122)$$

It is clear that for a Universe with $D - space\ dimensions$ there is $D - 1$ sides of propagation of energy to cover all the Universe (example: for $D = 3$ there is 2 sides of propagation of energy: front and rear). The density of power is not equal to the radiancy except for a $D = 2\ space\ dimensions$ of a Universe where it is equal to the radiancy of $D = 3\ space\ dimensions$ of a Universe because the electromagnetic field can exist at list in a two dimensional Universe.

**6-Quantum gravitation:**

The equations of gravitational field according to General Relativity are [10]:

$$R_{ik} - \frac{1}{2}Rg_{ik} = -\frac{8\pi G}{c^4}T_{ik} - \Lambda g_{ik} \qquad (123)$$

With $R_{ik}$ : curvature tensor;

$R$ : scalar (curvature of space-time)

$T_{ik}$ : momentum-energy tensor of matter

$g_{ik}$ : metric tensor with signature (+,-,-,-)

$i, k = 0,1,2,3$ : tensor indices

Equating again (80) & (81) we deduce that:

$$\Lambda = \frac{32\pi^6 hG}{15.c^7}.\nu_0^4 \qquad (124)$$

For a space with $D$ space-dimensions ($D \geq 4$) the half quantum of vacuum energy is:

$$\frac{1}{2}h\nu_0 = D.\frac{1}{4}\alpha^4 mc^2$$

So $$\nu_0 = \frac{D}{2h}\alpha^4 mc^2 \qquad (125)$$

Replace (125) in (124) we get :

$$\Lambda = \frac{2\pi^6\alpha^{16}m^4 cG}{15.h^3}D^4 \qquad (126)$$

With: $m = 9{,}1\ 10^{-31}Kg$ the mass of the electron.

The length $\bar{\lambda}_0 = \frac{c}{2\pi\nu_0} \approx \sqrt{l_{Pl}l_U} \approx 7\ 10^{-5}\ meter$ can be understood [11] as the mean of the smallest geometric length of the Universe Planck length $l_{Pl} = \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-35} meter$ and the largest geometric length of the Universe: the observable length of the Universe $l_U \sim 10^{26}\ meter$.

It is also understood as a cutting length (a sort of DE Broglie wavelength )to calculate vacuum energy density with $E = \frac{1}{2}\hbar\omega$ the fundamental state of an oscillator conform to that density obtained by cosmological observations [12].

We deduce directly that:

$$G = \frac{\hbar^3}{16\alpha^{16}m^4 c l_U^2} \qquad (127)$$

Replace (127) in (126) we get:

$$\Lambda = \frac{\pi^3 m^3 \alpha^{12} c^6}{960\ l_U^2} D^4 \qquad (128)$$

Replace $\alpha$ by $N\alpha$ in equation (127) and by $Z\alpha$ in equation (128) to get quantified constants $G$ & $\Lambda$ with $N$ & $Z$ positive integers ( to consider the first layer of Bohr atom to calculate exchanged vacuum energy for the electron).

Equations of the gravitational field becomes:

$$R_{ik} - \frac{1}{2}Rg_{ik} = -\frac{\pi\hbar^3}{2N^{16}\alpha^{16}m^4c^5 l_U^2}T_{ik} - \frac{\pi^3 Z^{12} m^3 \alpha^{12} c^6}{960\ l_U^2} D^4 g_{ik} \qquad (129)$$

For $N = Z = 1, D = 4$ & $l_U \approx 2.9\ 10^{26}\ meter$ we get Einstein equations.

**7-Conclusion :**

A corpuscle can be considered as a packet of strings and a packet of waves. The link between the two packets is deduced from atomic scale and cosmology and this can resolve the problem of vacuum energy density as it is the same density at any scale.

Einstein equations of the gravitational field are the same with quantified constants $G$ & $\Lambda$. If considering the radius of the observable Universe is a function of time than those constants will be functions of time.

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