The Minimal Set of Primes in Base 12

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Abstract

The purpose of this note is to describe the construction of a mathematical gem, the minimal set M_P of primes in base 12, namely,

$$M_P = \{2,3,5,7,\mathrm{E},11,61,81,91,401,\mathrm{X}41,4441,\mathrm{X0X1},\mathrm{XXXX1},\mathrm{44XXX1},\mathrm{XXX0001},\mathrm{XX000001}\}.$$

The minimal set M_P illustrates how naturally base 12 encodes the primes and is one of the reasons why base 12 is the canonical base for mathematics, the universe, and everything.

1 Introduction

The author considers base 12 as the canonical base for mathematics, the universe, and everything. Here are some of the reasons.

A prime is a sum of squares if and only it is of the form $4n+1$, and is not a sum of squares if and only it is of the form $4n-1$ [8]. Any prime greater than three is of the form $6n \pm 1$. Since $\operatorname{lcm}(4,6) = 12$, base 12 is a natural base in which to encode the prime numbers. Consequently, any prime greater than three ends in one of the digits $\{1,5,7,\mathrm{E}\}$. Any prime ending in 1 or 5 is a sum of squares, and any prime ending in 7 or E is not a sum of squares. Furthermore, the group of units of $\mathbb{Z}_{12}$ is $\{1,5,7,\mathrm{E}\}$, the Klein four-group [2], and the multiplication table for $\mathbb{Z}_n$ contains 1's only on the diagonal if and only if n is a divisor of 24 [1], and the multiplication table for $\mathbb{Z}_n[x_1,\ldots,x_m]$ contains 1's only on the diagonal if and only if n is a divisor of 12 [3]. The spectral basis of $\mathbb{Z}_{12}$ is $\{9,4\}$, consisting only of powers. We have $F_{10} = 100|_{12}$, where F_{10} is the twelfth Fibonacci number. The canonical subdivision of the unit circle is with $6! = 5 \cdot 100|_{12}$ equally spaced points. Lastly, 12 is the first *sublime number* [4], that is, a number whose number of divisors and sum of divisors are both perfect.

2 Outline of the construction

Let $p = d_1 \cdots d_k|_{12}$ be a prime, and define the form $*d_1 * \cdots * d_k*$ to represent all primes whose digits in base 12 contain the digits $d_1,\ldots,d_k$ in any position, as long as order is preserved. For example, the form $*1*5*$ represents all primes that contain the digits 1 and 5 in any position, as long as order is preserved. Define d^* to represent zero or more successive occurrences of the digit d. If S is a set, then S^* is the set of all finite strings of the elements of S.

The minimal set is initially empty, $M = \{\}$. The first prime is 2, and we consider this 2 as representing all primes that contain the digit 2 in base 12. Equivalently, 2 represents all primes matching the form $*2*$. Consequently, M={2}. Recursively, we look for the first prime $p > \max M$, $p = d_1 \cdots d_k|_{12}$, such that no element of M matches the form $*d_1 * \cdots * d_k*$, and then add p to M. The minimal set is always finite [5] so eventually we run out of forms to match. For the construction of the minimal set of primes in base 10, and the motivation for this note, see [6].

3 One digit primes in the minimal set

The digits in base 12 are $\mathbb{Z}_{12} = \{0,1,2,3,4,5,6,7,8,9,X,E\}$. Since the one-digit primes in base 12 are $\{2,3,5,7,E\}$, the minimal set up to one digit is

$$M_1 = \{2,3,5,7,E\}. \tag{1}$$

4 Two digit primes in the minimal set

Recall that primes greater than three in base 12 end in the digits $\{1,5,7,E\}$. Thus, from now on, only primes ending in a 1 in base 12 are in the minimal set. Consider primes of the form $d1$, where d is a nonzero digit in the set $E_1 = \{0,1,4,6,8,9,X\} = \mathbb{Z}_{12} \setminus M_1$. Thus,

11	prime,
41	$= 7 \cdot 7$,
61	prime,
81	prime,
91	prime,
X1	$= E \cdot E$.

Consequently, the minimal set up to two digits is

$$M_2 = \{2,3,5,7,E,11,61,81,91\}. \tag{2}$$

The only digits to survive are $\{0,4,X\} = E_1 \setminus \{1,6,8,9\}$.

5 Three digit primes in the minimal set

The only forms to consider from now on are $4\{0,4,X\}^*1$ and $X\{0,4,X\}^*1$. Thus,

401	prime,	X01	$= E \cdot XE$,
441	$= 5^4$,	X41	prime,
4X1	$= 15 \cdot 35$,	XX1	$= 7 \cdot 167$.

Consequently, the minimal set up to three digits is

$$M_3 = \{2,3,5,7,E,11,61,81,91,401,X41\}. \tag{3}$$

6 Four digit primes in the minimal set

We have the form 44^*X^*1, since 40 and X4 are excluded. We have the form $X\{0,X\}^*1$, since X4 is excluded. Thus,

4441	prime,	X001	$= \mathrm{E} \cdot \mathrm{XXE}$,
44X1	$= 7 \cdot 767$,	X0X1	prime,
4XX1	$= 31 \cdot 171$,	XX01	$= 81 \cdot 141$,
		XXX1	$= 6\mathrm{E} \cdot 16\mathrm{E}$.

Consequently, the minimal set up to four digits is

$$M_4 = \{2,\ 3,\ 5,\ 7,\ \mathrm{E},\ 11,\ 61,\ 81,\ 91,\ 401,\ \mathrm{X}41,\ 4441,\ \mathrm{X0X1}\}. \tag{4}$$

7 Five digit primes in the minimal set

We have the form $44\mathrm{X}^*1$, since 444 is excluded. We have the form XX^*0^*1, since X0X is excluded. Thus,

44XX1	$= 95 \cdot 575$,	X0001	$= 7 \cdot \mathrm{E} \cdot 1685$,
		XX001	$= 1\mathrm{E} \cdot 579\mathrm{E}$,
		XXX01	$= 17 \cdot 61 \cdot 117$,
		XXXX1	prime.

Consequently, the minimal set up to five digits is

$$M_5 = \{2,\ 3,\ 5,\ 7,\ \mathrm{E},\ 11,\ 61,\ 81,\ 91,\ 401,\ \mathrm{X}41,\ 4441,\ \mathrm{X0X1},\ \mathrm{XXXX1}\}. \tag{5}$$

8 Six digit primes in the minimal set

We have the form $44\mathrm{X}^*1$, with no more than three consecutive X's. We have the form XX^*0^*1, with also no more than three consecutive X's. Thus,

44XXX1	prime,	X00001	$= \mathrm{E} \cdot 3\mathrm{E} \cdot 2951$,
		XX0001	$= 17\mathrm{E} \cdot 663\mathrm{E}$,
		XXX001	$= 85 \cdot 13665$.

Consequently, the minimal set up to six digits is

$$M_6 = \{2,\ 3,\ 5,\ 7,\ \mathrm{E},\ 11,\ 61,\ 81,\ 91,\ 401,\ \mathrm{X}41,\ 4441,\ \mathrm{X0X1},\ \mathrm{XXXX1}, 44\mathrm{XXX1}\}. \tag{6}$$

Observe that the form $4\{0,4,\mathrm{X}\}^*1$ is now closed, since no more primes of the form $44\mathrm{X}^*1$ are possible.

9 Seven digit primes in the minimal set

The only possible remaining form is XX^*0^*1, with no more than three consecutive X's. Thus,

X000001	$= \mathrm{E} \cdot 4\mathrm{E} \cdot \mathrm{E}7 \cdot 237$,
XX00001	$= 15 \cdot 779215$,
XXX0001	prime.

Consequently, the minimal set up to seven digits is

$$M_7 = \{2,\ 3,\ 5,\ 7,\ \mathrm{E},\ 11,\ 61,\ 81,\ 91,\ 401,\ \mathrm{X}41,\ 4441,\ \mathrm{X0X1},\ \mathrm{XXXX1}, 44\mathrm{XXX1},\ \mathrm{XXX0001}\}. \tag{7}$$

10 Eight digit primes in the minimal set

We have the form XX^*0^*1, with no more than two consecutive X's. Thus,

$$\begin{array}{ll} \text{X0000001} & = \text{E}\cdot 1\text{E}\cdot 583731, \\ \text{XX000001} & \text{prime.} \end{array}$$

Consequently, the minimal set up to eight digits is

$$M_8 = \{2,\ 3,\ 5,\ 7,\ \text{E},\ 11,\ 61,\ 81,\ 91,\ 401,\ \text{X41},\ 4441,\ \text{X0X1},\ \text{XXXX1}, \text{44XXX1},\ \text{XXX0001}, \text{XX000001}\}. \quad (8)$$

The only remaining form is X0^*1, but any element in this form has sum of digits equal to E, and is necessarily divisible by E. Therefore, the form X$\{0,4,\text{X}\}^*$1 is closed, and we have found the full minimal set, that is, $M_P = M_8$ [7].

11 The universal prime in base 12

The smallest prime that satisfies all patterns in the minimal set is called the *universal prime* for the minimal set. The current candidate for universal prime in base 12 is

$$\text{1234456789X04XXX00E0001}. \quad (9)$$

12 Other minimal sets

The following base 12 minimal sets are computer generated and provided without proof.

12.1 Minimal set of primes of the form $4n+1$

The base 12 minimal set of primes of the form $4n+1$ is

$$\begin{aligned} M_{4n+1} = \{&5, 11, 31, 61, 81, 91, 221, 241, 271, \text{2X1}, \text{2E1}, 401, 421, 471, \text{4E1}, 701, 721, 771, \text{7X1}, \text{X41}, \text{E21}, \text{E71}, \\ &2001, 4441, 7441, \text{7E41}, \text{X0X1}, \text{X201}, \text{E001}, \text{E0E1}, \text{EE01}, \text{EE41}, \text{7EEE1}, \text{X07E1}, \text{X7EE1}, \text{XX7E1}, \\ &\text{XXXX1}, \text{XXEE1}, \text{E04X1}, \text{EXX01}, \text{EXXX1}, \text{EEEE1}, \text{44XXX1}, \text{XX00E1}, \text{XEXXE1}, \text{XEEXE1}, \text{XEEEX1}, \\ &\text{EXEXE1}, \text{EEXXE1}, \text{EEEXX1}\}. \end{aligned}$$

12.2 Minimal set of primes of the form $4n-1$

The base 12 minimal set of primes of the form $4n-1$ is

$$M_{4n-1} = \{3, 7, \text{E}\}.$$

12.3 Minimal set of primes of the form $6n+1$

The base 12 minimal set of primes of the form $6n+1$ is

$$\begin{aligned} M_{6n+1} = \{&7, 11, 31, 51, 61, 81, 91, 221, 241, \text{2X1}, \text{2E1}, 401, 421, \text{4E1}, \text{X41}, \text{E21}, 2001, 4441, \text{X0X1}, \text{X201}, \text{E001}, \\ &\text{E0E1}, \text{EE01}, \text{EE41}, \text{XXXX1}, \text{XXEE1}, \text{E04X1}, \text{EXX01}, \text{EXXX1}, \text{EEEE1}, \text{44XXX1}, \text{XX00E1}, \text{XEXXE1}, \\ &\text{XEEXE1}, \text{XEEEX1}, \text{EXEXE1}, \text{EEXXE1}, \text{EEEXX1}\}. \end{aligned}$$

12.4 Minimal set of primes of the form $6n-1$

The base 12 minimal set of primes of the form $6n-1$ is

$$M_{6n-1} = \{5, \text{E}\}.$$

12.5 Minimal set of palindromic primes

The base 12 minimal set of palindromic primes is

$$M_{\text{PAL}} = \{2, 3, 5, 7, \text{E}, 11\}.$$

12.6 Minimal set of composite numbers

The base 12 minimal set for composite numbers is

$$M_{COMP} = \{4, 6, 8, 9, \text{X}, 10, 12, 13, 20, 21, 22, 23, 2\text{E}, 30, 32, 33, 50, 52, 53, 55, 70, 71, 72, 73, 77, 7\text{E}, \text{E}0, \text{E}1, \text{E}2, \text{E}3, \text{EE}, 115, 151, 15\text{E}, 257, 275, 311, 317, 31\text{E}, 351, \text{E}57, \text{E}75, 1111, 1117, 111\text{E}, 5111\}.$$

The base 12 minimal sets $M_{POW2} = \{2, 4, 8\}$, $M_{POW3} = \{3, 9\}$, $M_{POW4} = \{4\}$, $M_{POW6} = \{6, 30, 900\}$, $M_{POW8} = \{8, 54\}$, and $M_{POW9} = \{9\}$ are easy to determine. The following minimal sets were computed up to the 512th power.

12.7 Minimal set of powers of 5

The base 12 minimal set for powers of 5 is

$$M_{POW5} = \{5, 21, 441, 9061, 16\text{X}081, 69919101\}.$$

12.8 Minimal set of powers of 7

The base 12 minimal set for powers of 7 is

$$M_{POW7} = \{7, 41, 58101, 3888921259681, 9\text{EE}96\text{X}306899\text{E}55\text{X}535881\}.$$

12.9 Minimal set of powers of X

The base 12 minimal set for powers of X is

$$M_{POW\text{X}} = \{\text{X}, 84, 6\text{E}4, 5954, 3423054, 1\text{E}30\text{E}91054\}.$$

12.10 Minimal set of powers of E

The base 12 minimal set for powers of E is

$$M_{POW\text{E}} = \{\text{E}, \text{X}1, 8581, 715261\}.$$

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