A New Logic Emerges from Tone

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Abstract

The topic of this paper is an original form of logic called Tonal Logic. I have finally decided that my work will not be perfect or finished, but I am ready to offer, not a totality, but a synthesis on Tonal Logic. In the beginning of the paper, we give two well-defined hypothesis' for laws of Tonal Gravity. The premise for these hypothesis' involves two base cases which have a certain combinatorial property, that is the diatonic scale embedded in the chromatic scale, and the pentatonic scale embedded in the chromatic scale. The antecedent in the hypothesis involves the felt sensation of Tonal Gravity or the identification of a single tone which acts as the tonic. These well-defined hypothesis' provide a deduction which enables the formulation of Tonal Logic. We proceed to define axioms of Tonal Logic, define Tonal Conservation, make a thesis statement on the phenomenon of emergence, apply Tonal Logic to describe mathematical objects, and finally, we form a cosmology out of the Tonal Logic. Having given form to Tonal Cosmology, we allow some deductions. The goal of this paper is not to thoroughly deliver a singular result. It is not to detail the hypothesis and experiment, not to perfect the axioms of tonal logic, not to show strict consistency with mathematics, nor fully detail a cosmology consistent with physics. The goal is to provide a synthesis on Tonal Logic which touches on all of these things and which may be expanded upon. I am an independent researcher, and most importantly, a musician. I have no references. I do reference well known equations in mathematics or physics at various points, however, the line of reasoning that I take, the hypothesis' I offer, the Tonal Logic on display, and the application of this logic to number and cosmology is entirely original.

Dear Reader,

I write not to prove, but to invite.

I have attempted to create a logic born from music.

This logic arose not solely from abstraction, but from experiment. In the study of tonal gravity, which lends a stable and resolved feeling, the *tonic* quality, to a note or chord, I developed two well-defined hypotheses, each with clearly articulated predictions, experimental methods, and measurable outcomes.

These hypotheses correlate a combinatorial spatial solution with *felt* closed systems tonal gravity.

- The first hypothesis pertains to the cognitive processing of tones.

- The second broadens the framework and invites metaphysical interpretation

It is primarily from this experimental foundation, coupled with intuitive reasoning and mathematical logic, that Tonal Logic emerged.

- A possible unifying grammar between mathematical logic, and feeling or experience, grounded in phenomenological musical hypothesis

- A possible coherent system of tonal gravity.

- Practical musical approaches.

-A thesis statement regarding the phenomenon of emergence and the interplay between chaos and order, symmetry and asymmetry

And in the second half of the paper, more speculatively:

- A new cosmology, rooted not in physics and astrophysics, but tonality and phenomenology. We give some correspondence between and interpretation of notions in physics through tonal cosmology.

- Novel descriptions of mathematical objects, and possibly novel mathematical techniques

- A novel approache to combinatorial number theory.

Furthermore, I recognize that as I have composed, and it feels to me, discovered, tonal logic, that it will not be complete and it may be deeply flawed. Besides this, I feel compelled to share what I have found.

My hypothesis for closed systems of tonal gravity arose from the study of a combinatorial property which may be observed in both the pentatonic scale embedded in an equal tempered twelve tone chromatic scale, and also the diatonic scales embedded in an equal tempered twelve tone chromatic scale.

It is beyond the scope of this paper to analyze tonal gravity and the presence of a *tonic* in music across cultures. Rather, it is notable that our combinatorial property is intrinsic to the diatonic/chromatic scale system so integrated into Western music theory, instrument design, and notation, as well as the pentatonic/chromatic scale system, where the pentatonic scales are ancient and widespread across cultures in origin, though certainly not natively embedded in equal tempered chromatic system, which is required for the observation of the combinatorial property central to this paper. Furthermore, it is not the goal of this paper to analyze musical properties of scales and chords such as consonance and dissonance, practical application of or theorizing about musical *harmony*. Rather, it is our goal here to mathematically define our combinatorial property of interest, solve for all combinatorial solutions which satisfy this property, and from this a hypothesis strictly pertaining to *felt* closed systems of tonal gravity has emerged.

The combinatorial property can be states thus: we have a scale of some number of notes, arranged over an equally spaced *chromatic* scale of some number of notes and which spans an octave. If our scale arrangement has our combinatorial property, we have that we may perform a single alteration on the spacing of our scalar arrangement, and arrive at a transposed copy of the original scale.

Before we rigorously define this property, let me give an example: if we have the key of C major, then we have a scale of 7 notes which have C as the *tonic*. The 7 notes are conventionally labeled: C, D, E, F, G, A, B, C. Since the scale has our combinatorial property, there is an alteration upon this scalar arrangement which will result in a transposed scale with the same spatial arrangement. A trained musician knows well enough that there are actually two such alterations, we may either raise the tone F to F sharp, which will give the G major scale, or we may lower the B to a B flat which will give the F major scale. Say we now have our G major scale, G, A, B, C, D, E, F sharp. A trained musician knows that we may now repeat our alteration again, raise the 4^{th} degree of the G major scale, C, to C sharp. We then have the D major scale: D, E, F sharp, G, A, B, C sharp.

As such, we may perform either of our two possible alterations upon the major scale iteratively, each of which will take us in one of two directions around a closed loop until we arrive back at our starting scale. The loop formed here is commonly referred to as the circle of fifths. Starting on C major we get iteratively: C major, G major, D major, A major, E major, B major, G flat major, D flat major, A flat major, E flat major, B flat major, F major, C major. Our other alteration, lowering the 7^{th} tone of the scale, will move in the opposite direction along this same loop.

So, we have illustrated a scale, the major scale, embedded in a 12 tone chromatic scale, which has two possible single alterations which create transposed copies of the original scale. That is, each major scale has the same relative spatial arrangement to one another, and there is an iterative transformation upon this scale which results in a closed loop of these transposed copies.

The previous paragraphs provide a well-known scale which has our *modulatory* property. In fact, the modulatory property is a well-established feature of Western music, to the point that it is engrained in our musical notation system. The key of C major is notated as the key with no sharps or flats, and for other keys moving around the loop, they are identified in the *key signature* by the number of sharps or the number or flats. That is, the amount of transformation iterations by which we have departed from the key of C around the closed loop of keys.

Let us also note, from an experiential perspective, that each key may validly be a key. Each key is structurally and relationally the same, and each produce a clearly perceived tonic note. Later, I will address in more detail the perception of the tonic notes. For now, I can clearly state the intent of the hypothesis which is to relate more generally, *all* scales which satisfy our combinatorial property, to systems of keys which have a clearly perceived tonic note.

Furthermore, I would like to assert that while this combinatorial property is a well-established and core feature within the Western diatonic/chromatic system, I have never seen it mathematically defined and generalized to produce new scales with similar properties.

So, we may proceed to more rigorously define our combinatorial problem, and then give its general solution. We aim to produce a scale of some number of notes, spaced out over a chromatic (evenly spaced) number of notes, where we may perform a single alteration upon our scales spatial arrangement, to produce a transposed copy. This transformation has an *inverse transformation*, each of which will proceed iteratively around a loop of transposed copies of the original scale.

We must define what we mean by a *single alteration*. With the diatonic/chromatic system as our base case, we define an alteration to be the process of taking one note in our scale arrangement and moving it any number of chromatic spaces to the right or left of the original position, so long as the note *does not* cross over another note. In the diatonic base case, we "sharp" or "flat" a note, that is, raise or lower it by one chromatic space. However, for the solutions of our combinatorial property, we include scales where the alteration exceeds one chromatic space- the limitation is rather that we cannot cross over another note. All of these scales which satisfy our property have exactly two alterations which can be made, each of which transpose the original scale. These transpositions lie on an *orbit*, a structure analogous to the circle of fifths in the diatonic/chromatic system, and each one of our alterations travels iteratively in opposite directions around the orbit.

I offer now the system of linear equations which I believe describes *all* of the scales which satisfy our combinatorial property.

We have the equations:
$A + B = sd$
$A(I_1) + B(I_2) = n$
$A - E = C_1(B - 1)$ where $E=C1\pm1$
And in specific cases: $A = C(B) + 1$ is a simpler expression which may perform the same function as $A - E = C_1(B - 1)$.

With the first two equations n identifies the number of evenly spaced chromatic tones in a linear arrangement. Sd represents the number of scale notes which are arranged onto the chromatic spaces into a musical scale.

I have proven these equations. We may only have scales which are composed of two intervals, I1 and I2. The symbol A represents the quantity of I1 and the symbol B represents the quantity of I2.

The second two equations describe how you arrange the intervals into a scale, sd in n, which satisfies the property. E and C_1 are both natural numbers. This is the general form solution for scales of any number sd, spread out in an arrangement over any n even spaces, which satisfies our property.

After observing examples of these scales, I have determined that these scales have a certain structural form. That is, all scale solutions can be described as having the segmental interval form S1S1S1…S2, where S1 is composed of a single I2 interval with a proportional number of I1 intervals. Each S1 has the same number of I1 intervals and a single I2 interval. The order of the intervals must be the same for all S1. S2 is identical to S1 except for that it has either one additional or one less I1 interval.

An exception to to this rule is when B=1. In this case, the form S1S1S1…S2 is replaced by the form of I1, I1, I1… I2.

The way the equation A-E=C_1(B-1) works to arrange the scale, is that E removes a certain number of I1 intervals to be composed into S2 along with a single I2 interval. C_1 determines the number of I1 to I2 intervals in each S1, and since S2 will have either one less or one more I1 interval than S2, we have E=C1$\pm$1.

If B=1, then we have that E=A so that A - E = C_1(B - 1) gives 0=$C_1(0)$. Thus, we construct a single segment with the form I1, I1,…, I2.

Since our scales are always of the form S1S1…S2 or I1I1…I2, and since S2 differs from S1 by exactly one interval, and I2 differs from I1 by exactly one interval we always have that a scale with our property is a *near* translationally symmetric, yet, it is an asymmetric scalar arrangement. I would like to emphasize this point. Every scale which has the property that a single transformation (and its inverse) allows for transpositions along an orbit of structurally identical scales, has the form S1S1…S2 or I1I1…I2. It is exactly the near translational symmetry, together with a single specific asymmetry exactly, which allows for a closed orbit by single transformation. This is a property inherent, as I have said before, to the diatonic/chromatic and the pentatonic/chromatic system, but here we have generalized the property combinatorially, and discovered that all solutions have an interesting and similar segmental form.

Now I state: for every natural number pair, (sd, n), there exists at least one solution to our property. Sd and n may diverge to infinity. That is, if given a quantity for sd, and a quantity for n- there is at least one solution to the whole system of linear equations which satisfy the property. This will be an important point later.

We hypothesize that if we create a solution satisfying these equations, with chromatic length n, and then project n onto n_2 where n_2 are notes of an instrument, and if we preserve the structure and subset of notes given by sd_i in n_1 through our projection onto n_2, we enable the experience of the sensation of tonal gravity. That is, each sdi in the orbit of keys in n1, projected onto n2 will have an identifiable *tonic* note, that is, a tone that is stable- analogous to the sensation of the note C, in the key of C major. And we have an orbit of keys which will all have a stable tonic note, and I hypothesize that the tonic note will be the same scale degree in all sdi.

We further hypothesize that our projections experience tonal gravity in a pitch invariant manner. What I mean by this prediction, is that no matter the instrument, or the notes chosen for n2 in a projection onto n2, the structure of sdi and its orbit onto n2, will result in a felt sense of tonal gravity. So far, I have primarily experimented with projection of n1<12 onto n2=12, that is, where the pitch classes given by the projection are a subset of the 12 tone chromatic scale.

Now we must pause, and I must share how this hypothesis arose. Having generalized the solution for all scales which have our mathematical property, for all sd and n combinations- the question arose- what can I *do* with these scales? Let's say n1=10. Well, I cannot divide the 12 tone chromatic scale into 10 evenly spaced tones. Perhaps I could create a 10 tone equal tempered chromatic scale to fit my combinatorial solution with n1=10. This would result in an orbit of scales which are all structurally identical to one another, as is the case in the diatonic chromatic system. However, not having the tools to complete this task, I asked, what else can I do with my combinatorial solutions? I decided to experiment with projecting n1 chromatic tones from our combinatorial solution onto n2 chromatic tones, where n1<n2, thus the entire sdi structure may be preserved in n2. When the structure is not one to one, that is n1≠n2, the manifest sdi after projection onto n2, will not be uniform. The scales sdi in the orbit will manifest as different from one another, they will have different interval content, and yet, I hypothesize, that no matter *how* we project, or which subset of tones n2 we project onto, that each sdi will have, on the same scale degree, a note which is perceived to be the tonic. Although I have only experimented with projection onto n2=12 evenly spaced chromatic tones, I am suggesting that n1 and the structures of sdi can be projected onto any n2 which is equal or greater to n1. n2 could consist of an equal tempered chromatic scale of some number, the notes of a particular instrument, or even random pitches. Regardless, I hypothesize felt sense of tonal gravity, and therefore our combinatorial structures would enable an orbit of sdi each with the felt tonic scale degree, in a pitch invariant manner through projection.

The mathematics of the equations did not generate this hypothesis. What generated the hypothesis was that once I had defined how I projected a scale with our property onto n2 in a pitch invariant manner, and having played around with the scales generated as such, I honestly felt that I detected exactly what I have stated, a feeling of tonic assigned to a specific scale degree in each sdi on the orbit of our scale.

This does beg a question: *how* do we identify the scale degree which is the tonic over all sdi in the projection of n1 onto n2? To answer this question, let's refer somewhat qualitatively to our base case- the diatonic/chromatic system of keys. We may make several observations. First of all, the key signature that represents a certain diatonic scale, is used to identify not one, but two keys. The major key which conforms to this key signature, and also the relative minor. The identity of these two key centers is closely related, in the study of music theory, to triadic harmony, where each degree of the scale is a root to a certain three note chord within the scale. Oftentimes, for a given key signature, the relative minor key is emphasized by an additional sharped note, that is, the 7th degree of the scale relative to the tonic is sharped to give the leading tone and a dominant V chord with a major third in it. This sharped note is not in the key signature, though it is often considered closely related to tonicizing the relative minor key for a given key signature. We might now add, that not only is there a relative minor key, but that for any diatonic scale, *any* of the degrees may be tonicized. This is a matter of creative emphasis and expression on the part of the composer. The 2nd degree of the diatonic scale, that is the dorian mode, might not present as strongly to the listener as the tonic, unless the composer emphasizes this degree and centers the degree in the harmony, melody and rhythm of the piece.

However, we may tentatively state that barring creative emphasis, if the ear is exposed to a diatonic scale, it is the 1st degree, the root of the ionian mode, the major scale, which will present as the tonic. We might notice that if the ear is exposed to 6 of the 7 diatonic scale degrees, in no particular order, without hearing the missing 7th note which is the tonic, then once the ear is exposed to that missing 7th note, it will present as the tonic. It seems the structure of the scale implies that a certain degree will naturally present as the tonic without need for creative emphasis or even for that note to be pre-emptively sounded.

From our base case of the diatonic scale, we have made two deductions: one that it is possible for any one of the scale degrees to present as the tonic, and second, that it is possible that without creative emphasis, and with a simple even exposure to the tones of the scale, a tonic seems to naturally emerge, and that the scale degree of this tonic is consistent over all sdi.

I would like to make another assessment. That is that, if we observe the division of the diatonic scale into triads, we might notice that this can be described as a projection of sd1=3 and n1=7 onto n2=the 7 diatonic notes. Here n1=n2, however, n2 is not actually made of uniform chromatic steps, rather it consists of the scale degrees of the diatonic scale. The structure of the triads in this projection actually satisfy our property, that we can make one alteration upon the triad, and get a structurally consistent chord. That is, a chord that is made up of a root, a third, and a fifth. This creates an orbit of chords within the key of C major which has the following order: C major, E minor, G major, B diminished, D minor, F major, A minor, C major. In each case, a chord situated in this orbit shares two notes in common with the chords to the left and right of it in the orbit, and the third note has an iterative transformation which sends the triads along the orbit.

I am suggesting that if we nest projections of our property, it may help to more clearly identify a tonic note. The first iteration of our property, sd1 and n1 onto n2, creates sdi and an orbit of scales which have a hypothesized tonic note. We then utilize a second iteration of our property where sd2 becomes n2, and a new scale with sd1=chord interval structure and n1=n2 is projected onto sd2. That is, when we nest our property, we create an orbit of scales in a chromatic space, and then the finer iteration of the property generates a system of chords within each scale which also satisfy the property. The triads within the diatonic system have this property, and generally, the contrast which chords offer to a scale representing a key, helps to identify more clearly a tonic. We predict that the chord which has its root as the scale degree that is root of the key, will generally sound *stable*, like a tonic chord in the diatonic/chromatic system, *resolved*, despite its interval content.

Now I would like to deduce an additional point from two of our previous points, which lends great generality to manifest groups of notes belonging to closed tonal gravity systems. By *closed tonal gravity systems,* I mean, an orbit of transformation-related keys which have been projected onto manifest notes, and which have an apparent tonic on a certain degree of their scales after projection. We have previously stated that for any quantity pairing (sd,n) that at least one solution which satisfies our property exists. Furthermore, we have hypothesized that the feeling of tonal gravity is permitted with pitch invariance through projection of these scales which satisfy our property, onto any n2 group of manifest pitches. From these two points together, we can deduce that any group of manifest tones, belongs to infinite many closed tonal systems and activates certain contextual information (sdi, partial sdi, mixed sdi) within each closed tonal gravity structure that it is in.

This is the conclusion of our first hypothesis. It is beyond the scope of this paper to offer additional insight into how an experiment would test for the validity of the hypothesis. I simply state that I believe the hypothesis is well defined, the predictions are well defined, the method of experiment can be well defined, and the method of measurement (felt tonal gravity) can be well defined.

We now proceed to our second hypothesis where we explore the structure preserving projection of combinatorial solutions to our equations onto noises rather than tones.
In the first experiment, although we had pitch invariance in projection, any group of pitches is inherently ordered along the pitch spectrum.
Noises are inherently unordered structures relative to the pitch spectrum. We must recognize that it is normally in accordance with tones ordered along a pitch spectrum, that we perceive tonality and tonal gravity. We check experimentally to see if our structure preserving mappings produce felt tonal gravity in noises. If they do generate such feeling, then we may interpret the results in one of 2 ways:
1) There exists some structure, shared by tones and noises, which enables tonality.
2) Tonality transcends basis. An arbitrary structure, any structure, and all structures lie in the tonal Hilbert space of all Hilbert spaces.

I personally take interpretation number 2. Thus, all structures lie in the tonal Hilbert space of all Hilbert spaces.

Thus, when we announce in tonal logic that any sd is in some infinite many S with context C, this is not an arbitrary assertion, it is not an abstract leap in reasoning. It is substantiated by any sd, across all structures, being in the tonal Hilbert space of all Hilbert spaces, by the interpretation of our second hypothesis.

Through these experiments, we move from mathematical formalism to lived perception. These hypotheses not only serve as a foundation for testing tonal logic, but also support its deepest axiom: that any scalar degree, in any space, lives within some meaningful structure — because all such structures resonate in the Tonal Hilbert Space of All Hilbert Spaces.

I have conducted tests upon my own ear and body, as well as a few other trained musicians. The results have been cautiously affirmative. The effect of tonal gravity is more clear (so far) when we nest two of the combinatorial solutions, one in the other, triads embedded in the diatonic scale, embedded in the chromatic scale. I hope to design and conduct these experiments on a larger and more formal scale. We would give musical chord sequences which are projections of solutions to our combinatorial property, onto pitches, and then we ask the listener, at certain times, does the chord sequence feel finished? Resolved? At home?

By creating a space, the tonal Hilbert space, where any collection of manifest notes lies meaningfully within infinitely many structures, we have enacted a multiplicity space. Indeed, as we develop Tonal Logic from these hyptohesis' and attempt to apply the logic to phenomenon outside of music itself as prompted by our second hypothesis, we intend to formalize a logic which takes multiplicity as its frame, and as a composer crafts something specifically meaningful within a space of possibility, this multiplicity space will be *navigated* in order to collapse into specific meaning.

Tonal Cosmology — Part I: Tonal Logic Axiom Set

For any set of scalar degrees (sd), each sd exists within at least one structure S. This relationship allows for meaningful multiplicity space, or implication, as well as projection, that is, traversal, to emerge.

Any object may be projected onto any other object.

Define asymmetric projection as the case where $P_1 < P_2$, in which case object $\mathcal{O}_1$ projects onto object $\mathcal{O}_2$.

1. Definition of an Object

An object in tonal logic is defined as:

$\mathcal{O}$ = (sd, S, C, R, P)

Where:
- sd: Scalar degree — identity or perceptual unit
- S: Structure — formal framework within which sd exists
- C: Context — contextual frames derived from the total structure S
- R: Resonance field — The set of all observables and relations between (sd, S, C)
- P: Projectability — cardinality of sd; determines projection behavior

2. Irreducible Object

An object is irreducible if:

sd = S = C = R,   P = 1

3. Tangible Object

We define a tangible object as one which is directly known. The root parameter of a tangible object is sd. It is a knowable manifest object.

4. Intangible Object

The intangible object is the object which is not directly known, but which may be sensed, felt, conjectured upon and hypothesized about. The root parameter of an intangible object is the structure. While the sd may not be directly knowable, (though we may be able to provide a P value), the intangible object often manifests itself in relation to other objects. This is akin to the notion that the overtone series of a vibrating string was mathematically predicted, and yet the overtone series itself was not directly known until Hemholtz experimentally verified its physical presence.

5. Projection Rules

Let O1→ O2 or O1(O2) denote projection.

- Projection is symmetric if $P_1 = P_2$

- Projection is asymmetric if $P_1 < P_2$ ($\mathcal{O}_1$ projects onto $\mathcal{O}_2$)
- Structure symmetry: If $S_1 = S_2$, projection becomes structurally symmetric

A. Tangible → Tangible

$sd_3 = sd_1 + sd_2 = sd_2 + sd_1$

$S_3 = S_1 + S_2$

$R_3 = R_1+R_2= R_2+R_1$

$C_1(C_2)\neq C_2(C_1)$,

B. Intangible → Intangible

Case 1: $P_1 = P_2$ (Symmetric)

$sd_3 = sd_1(sd_2) = sd_2(sd_1)$

$S_3 = S_1(S_2) = S_2(S_1)$

$R_3 = R_1(R_2) = R_2(R_1)$

$C_1(C_2)\neq C_2(C_1)$,

Case 2: $P_1 < P_2$ (Asymmetric)

$sd_3 = sd_1(sd_2)=sd_2(sd_1)$ and $sd_2$is not fully present under this projection

$S_3 = S_1(S_2) = S_2(S_1)$ and S_2 is not fully realized under this projection

$R_3 = R_1(R_2) = R_2(R_1)$ R_2 is partially implied in relation to R_1

$C_1(C_2)\neq C_2(C_1)$,

Corollary: If $S_1 = S_2$, then $S_1(S_2) = S_2(S_1)$ (Structure symmetry only)

C. Tangible → Intangible

Case 1: $P_1 = P_2$ (Symmetric)

$sd_3 = sd_1(sd_2) = sd_2(sd_1)$

$S_3 = S_1(S_2) = S_2(S_1)$

$R_3 = R_1(R_2) = R_2(R_1)$

$C_1(C_2)\neq C_2(C_1)$,

Case 2: $P_1 < P_2$ (Asymmetric)

O_1 is tangible, O_2 is intangible
$sd_3 \approx sd_1(sd_2) = sd_2(sd_1)$ where sd_2 is not fully present under projection)
$S_3 \approx S_1(S_2)$, where S_2 is not fully realized under this projection
$R_3 = R_1(R_2) = R_2(R_1)$ where R_2 is not fully implied under this projection.
$C_1(C_2) \neq C_2(C_1)$, some C_2 is not accessible.

Corollary: If $S_1 = S_2$, then $S_1(S_2) = S_2(S_1)$ (Structure symmetry only)

D. Intangible → Tangible

O_1 is tangible and O_2 is intangible.

Case 1: $P_1 = P_2$ (Symmetric)

$sd_3 = sd_2(sd_1)$ (Each intangible element maps onto a tangible element)
$S_3 = S_2(S_1) = S_1(S_2)$
$R_3 = R_2(R_1) = R_1(R_2)$
$C_1(C_2) \neq C_2(C_1)$,

Case 2: $P_2 < P_1$ (Asymmetric)

$sd_3 = sd_2(sd_1)$ (Selection of assignments from intangible onto tangible)
$S_3 = S_2(S_1) = S_1(S_2)$ where S_1 is not fully realized
$R_3 = R_2(R_1) = R_1(R_2)$ where some of R_1 is not fully implied
$C_1(C_2) \neq C_2(C_1)$ where some of C_1 is not fully accessible.

Corollary: If $S_1 = S_2$, then $S_1(S_2) = S_2(S_1)$ (Structure symmetry only)

6. Tonal Projection Theorem (Universality)

Any object can be projected onto any other object:

For all $\mathcal{O}_1, \mathcal{O}_2 \in \mathcal{T}$, there exists a valid projection and a composite object $\mathcal{O}_3 = \mathcal{O}_1(\mathcal{O}_2) \in \mathcal{T}$.
The result may preserve or exclude parts of sd, S, C, or R depending on P, but resonance always emerges.

Tonal Logic: Tangibility Rule in Composite Projection Chains

Let a projection chain be given by:

$O_1(O_2(\ldots(O_n))) = O_{final}$

We define the tangibility propagation rule as follows:

1. Start and End Entanglement:

- In the context of the composition chain, the first object O_1 and the final composite object O_{final} are entangled in their tangibility status.

- If one is considered tangible, the other is treated as tangible.

- If one is considered intangible, the other is treated as intangible.

2. Middle Determines Propagation:

- If any intermediate object O (1 < k < n) is intangible, then:

  - All objects after O_k, including O_n, are necessarily intangible.

  - The composite object up to and including O_k, denoted O' = $O_1(O_2(\ldots(O_k)))$, is also intangible.

3. Recomposition Rule:

- O' may be treated as a new first object.

- A new chain O'(O_{k+1}}(...(O_n))) inherits the same rule.

- If all subsequent objects are tangible, and O_{final}, the whole composite projection chain, is tangible, then O’ may now be considered tangible, and the tangibility entanglement between O’ and O_{final} holds.

Tangibility in Projection Chains: Formal Summary

In tonal logic, each object in isolation may be considered tangible or intangible.

However, within a projection chain, tangibility is governed by the following relational rules:

1. Entanglement in 2-Object Chains:

- Given a projection $O_1(O_2)$, O_1 and O_2 are entangled in their tangibility.

- The composite object is tangible if and only if both O_1 and O_2 are tangible.

2. Generalized Entanglement in Chains:

- In a projection chain $O_1(O_2(...(O_n))) = O_{final}$

  • The first (O_1) and final O_{final} objects are entangled in tangibility.

  • Any intangible object O_k (1 < k < n) in the middle of the chain causes all subsequent objects, including O_{final}, to be considered intangible.

3. Recomposition Rule:

- When an intangible object O_k is found in the middle, define a new object O' = O_1(...(O_k)).

- Then reframe the chain as O'(O_{k+1}(...(O_n))).

- If all subsequent objects are tangible, and if and only if O_{final} is found to be tangible, then O' may switch from intangible to tangible and the chain resolves as tangible.

Axiom of Component Mutability in Tonal Logic

Overview:
In Tonal Logic, the roles of scalar degree (sd), structure (S), and context (C) are not permanently fixed. Rather, these components are functionally mutable — their identity and interpretative role shift depending on projection, resonance, and purpose. This document introduces the Axiom of Component Mutability and illustrates it through the example of musical scales.

1. Axiom of Component Mutability

Let O = (sd, S, C, R, P) be a tonal object. Then, under projection, the identity of sd, S, or C in a new object O' may be functionally reassigned, provided that:

- The scalar degree sd is always quantized by P in the target object.
- The reassignment preserves resonance implications (R) from the source object.

- The scalar degree sd of the target object must still belong to its structure S.
- The contexts, C_i, must belong to the structure S

This reassignment allows a structure or context from one object to become the scalar degree in another, enabling layered transformations and emergent relational mappings.

2. Illustration: The Chromatic and Diatonic Scales

Example A:
Let O_1 = (sd = 7 diatonic tones, S = 12 chromatic tones, C = tonic key in chromatic structure, P = 7).
Here, the diatonic scale is treated as data (sd), embedded in the chromatic scale (structure).

Example B:
Let O_2 = (sd = 12 chromatic tones, S = pitch spectrum, C = x($2^{n/12}$), P = 12).
Here, the chromatic scale becomes the scalar degree — the identity — with a logarithmic context. In S, we could fashion different equal tempered scales where the fraction n/12 varies the denominator.

These examples show that the same musical material can serve as either structure or identity, depending on the projection logic. In Example A, the chromatic scale is the structure which contextualizes the diatonic scale. In Example B, the chromatic scale is the identity itself and it is placed in the structure of the pitch spectrum where we may have various contexts or different equal tempered scales. This fluidity is sanctioned by the Axiom of Component Mutability.

3. Projection and Quantization Rule

If object O_1 pulls its sd from object O_2's sd, then O_1 must inherit O_2's projectability P, such that sd remains quantifiable. This ensures projection chains maintain internal coherence and allows inter-component resonance to propagate in accordance with tonal logic.

Quantization of S and C in Tonal Logic: A Philosophical Justification

Rationale for Non-Quantization of S and C

In Tonal Logic, scalar degrees (sd) are the only component of an object explicitly quantified via the parameter P. Structures (S) and contexts (C), although present and essential to resonance and projection, are not themselves quantified. This decision is a deliberate departure from classical set theory and serves to empower Tonal Logic with greater expressive and interpretive range.

1. Freedom from Set-Theoretic Limitations

Quantizing S and C would impose the machinery of classical set theory—cardinality, unions, intersections—onto Tonal Logic. This would restrict projection behavior and resonance structures to extensional equality. By keeping S and C unquantized, Tonal Logic avoids being subsumed by set-theoretic logic and instead allows meaning and implication to emerge from contextual and relational alignment.

2. Multiplicity without Enumeration

Tonal Logic treats S and C as phenomenological rather than set-theoretic: they are implied, felt, and relational, not counted. This reflects the logic of musical and perceptual experience, where structure and context exist even when unenumerated. It supports analogy, intuition, and emergent patterns of implication.

3. Relational Identity Over Set Membership

The logic of Tonal projection is grounded in resonance and context rather than set membership. The statement “sd is in S” implies a relational dependency rather than set inclusion.

4. Strategic Simplicity for Expansion

Non-quantization of S and C allows Tonal Logic to retain conceptual simplicity at its core, while remaining open to variant forms in which S and C may be quantified for formal or computational modeling. This provides both a stable foundation and extensibility into highly formal domains like mathematics and physics.

Conclusion

By refraining from quantizing S and C, Tonal Logic protects its relational, emergent, and phenomenological roots. It avoids rigid classification and instead invites a layered, context-sensitive interpretation of structure and meaning, aligned with both musical intuition and metaphysical inquiry.

This completes the exposition of the formal language of logic which is tonal logic. We may note that our rules of projection allow only for union, where the parameters of two objects joined, and assignment, where the parameters of an object are assigned to another. I do believe however, that the semantic meaning derivable from Tonal Logic is much wider than the logical functions of union and assignment, and this meaning is largely accessed through the interplay of object parameters in a projection, information which is stored within the resonance field of the composed object.

Now that the rules of Tonal Logic have been given, I would like to pause, and discuss the intent for the remaining portion of this paper. First, I will return to the study of our scales which satisfy the property of our hypothesis. I will make several observations of interest, culminating in the assertion of a rule of Tonal Conservation. I will then take an interpretation upon the relation between *Brownian Motion* and *noise* (in terms of a signal). I will then draw analogy between this phenomena pair and the phenomena of *symmetry* and a*symmetry*. I will then conclude this section by grounding my interpretation of the phenomena of symmetry and asymmetry in deductions about Tonal Conversation in the context of a verified hypothesis. In this section, the primary intent of this paper is fully realized, which is essentially to be a treatise on the phenomenon of emergence, grounded in the study of music.

Next, I will attempt to apply my Tonal Logic rules to basic mathematical definitions in order to showcase the formal possibility, and also the opportunity for semantic interpretation or novel insight using my Tonal Logic framework.

Finally, I will give an exposition of Tonal Cosmology. The cosmology given here is not a cosmology of physics and Astro-physics, rather it is an imagining of the mechanics of what lies beneath the surface of simultaneity. The reason I have included this in my paper is that I found that my cosmological ideas could not be grounded in any mathematics I have known, and yet, I have been able to formalize them using the possibilities of Tonal Logic. I lay the groundwork for my Tonal Cosmology, I give some cosmological interpretations, and having given the Tonal Cosmology form in Tonal Logic, I make some surprising deductions.

Now that we have defined the parameter R, the resonance field, as all observables which arise from all parameters, that is, (sd, S, C_i, P), we will return to the scales with our property and their projections onto notes.

We now define the concept of *Tonal Smoothness*. Tonal smoothness is a function of the projections of intangible scales with our property onto manifest notes. In particular, Tonal Smoothness is a function of the emergent phenomenon of Tonal Gravity, the resonance field of the composite object, that is, all observables of the composite object, and whether projection is symmetric or asymmetric, that is, whether P_1= P_1 or if $P_1 \neq P_2$. For instance, if one of our projected scales is asymmetric, we will have that the various scales which make up our orbit, are in terms of interval structure, not the symmetric. Our Tonal Gravity Hypothesis predicts that closed systems of Tonal Gravity exist over keys which are not identical to one another. In a symmetric example, such as the diatonic chromatic system, where the number of chromatic tones is the same in our intangible scale and our tangible scale, we have that the resulting keys which make up our orbit, are structurally identical to one another in terms of interval content. No matter what key we are in, the exact interval structure of each diatonic scale, C major, G major, etc., is the same. This information is a part of the function which makes up Tonal Smoothness.

I now conjecture that the phenomenon of tonal *consonance and dissonance* is a function of Tonal Gravity and Tonal smoothness, and probably also of metaphysical details which transcend formalization, instrument, culture, particularities of performance, *expression*.

I will leave this at conjecture. Since we have generalized all possible scales which have our property, for any given (sd,n) pair. Let's see if we can determine anything unique or notable about the diatonic and pentatonic scales.

In fact, the pentatonic and diatonic scales do participate in a subclass of all possible scales with our property. For every scale with our property, there are two transformations (inverses) which modulate our scale along an orbit. Let's quantify our transformations T_1- T_2=0. That is, each transformation moves a note (not the same note) the same number of spaces, but in opposite directions. For example, in the diatonic chromatic system, we either raise the fourth degree 1 note, or lower the seventh degree one note, to modulate in opposite directions around the orbit. Let us also name the *degree of modulation*, D_m, in terms of the chromatic scale. Notice that for a single scale D_1 + D_2 = n, where each D is the chromatic degree of modulation in one direction around the orbit.

Now let us ask, when does T=1. The answer is when $|I_1- I_2|$=1. That is, when one interval is larger than the other by exactly one chromatic space.

We may make a further observation: we have that if a scale has as one of its two intervals I_1=+1, and which also has our property of interest, and also a T value of +1, then we must have that I_2= +2. In other words, scales which are composed of whole steps and half steps are a unique subclass of scales which have our property, in that they are the scales which include I_1= +1, and also have a |T|= 1. We may also note, that in terms of all scales which have our property over all (sd,n) pairs, the pentatonic and diatonic scales are among the first scales to arise which participate in this specific subset of scales, and which are not *primordial*, that is their form is not I1I1…I2, but S1S1…S2. In a primordial scale the variable B, from our system of linear equations which describe our scales, is equal to 1. In non-primordial scales B>1. Primordial scales appreciate the least amount of local variety in their interval structure, while when B>1, there is more local variety as we slide along the scale translationally.

We may make another non-trivial observation which holds over both the diatonic and the pentatonic embedded in the chromatic scale, but in order to do so, we must ask another question. That is, in total, through the whole orbit of one of our scales with the property, how many *tonal assignments* take place. That is, how many scale degrees are assigned to chromatic notes in total? The answer is quite simple, it is sd(n).

There are two possibilities, one is that the orbit covers the whole chromatic scale, the other is that the orbit covers a partition of the chromatic scale which has a cardinality that divides the cardinality of the chromatic scale. In this latter case, multiple non-overlapping orbits cover the whole chromatic scale. Thus, over any whole orbit (or set of orbits), each chromatic tone is assigned exactly to a number of *keys* equal to the number of scale degrees in our scale. Each chromatic tone takes on, in various keys, each scale degree sd. In the diatonic chromatic system for instance, each chromatic note, take the note C, plays the role of each of the 7 scale degrees in 7 different keys. The chromatic note is not present in n-sd number of keys, which here would be 12-7= 5 keys. We may now name the phenomenon of *Tonal Conservation*, which states that in one of our constructed scales, pre-projection, there are n(sd) total tonal assignments and that each n_i receives the same exact set of assignments when considered over the whole orbit of keys.

Let us pause and imagine a rod, suspended from the ceiling at its center of mass, so that it is balanced on a horizontal plane. If we were to, at even increments starting and ending at either endpoint of the rod, suspend weights from each node, and distribute each tonal assignment over all n a single weight, then the rod, suspended at its center of mass, will remain parallel with a horizontal plane. We may not know how to assign weights to each tonal assignment, that is, we may not know the specific tonal weight to each scalar degree of the scale which satisfies our property. However, we do not need to know this in order to establish Tonal Conservation. All that matters is that we recognize that over the whole orbit, each $n_i$receives the same distribution of weight, and the total number of assignments is quantified by n(sd).

Note then, that for the diatonic and chromatic scales we have 7(12)=84 tonal assignments and 5(12)=60 tonal assignments. For both scales D_1=7 and D_2= 5. That is the 7th and the 5th chromatic tone are the degrees of modulation. Here is our non-trivial observation. In both the pentatonic and diatonic scales, the factors of the number of tonal assignments are 5 and 7 respectively, multiplied by 12. The prime factorization of 12 is $2^2(3)$. We have that for each the pentatonic and the diatonic, the number of tonal assignments total is equal to exactly the number of scale tones, multiplied by 2^2, which satisfies the form 2^n, the proportion of the octave, multiplied by 3. That 2^n divides n is non-trivial, but still notable. That $2^2(3)$ is the prime factorization of 12, and 3 happens to be the exact *frequency proportion* of D_1 is non-trivial. In both cases, the diatonic and the pentatonic, we have that the degree of modulation at the seventh chromatic degree, has the frequency proportion of 3 to the tonic note. In other words, in the case of the diatonic and pentatonic scale embedded in the 12 tone chromatic, the total number of tonal assignments seems to be self-referential by each prime factorization component. 2^2 trivially represents the octave, 5 or 7 trivially represent the number of scale degrees, and 3 is the exact harmonic proportion of D, the chromatic degree of modulation.

This is, in my view, extremely non-trivial and quite interesting. Certainly this will not be true of many of the scales over various (sd,n) pairs. I must note however, that with an equal tempered chromatic scale, the harmonic proportion of the 7th chromatic degree is not exactly 3. However, it is an approximation of the pure interval proportion which is exactly 3. The equal temperament smudges the tuning of each degree so as to maintain, in my own words, Tonal Smoothness.

Now we will shift gears as I prepare the primary thesis of my paper. We start with an interpretation. It is an appreciated fact that Brownian motion is the stochastic integral of noise. The noise is considered random, and it is a *background state*. This background state effects a particle, which undergoes random motion and *traces* an *irreversible path*. The path is irreversible because neither noise nor Brownian motion are differentiable, rather Brownian motion is a stochastic integral of noise. I propose the following thought experiment. I conjecture that if a particle undergoes Brownian motion indefinitely within a fixed space, its trace will eventually fill the entire space. Over time, the trace becomes noise. I offer an inverse conjecture, that a background state of noise, normally considered irreducible as it is a background state, may be reduced into Brownian motion. Thus, I conjecture Brownian motion and noise to be integrals and derivatives to one another both. There is a problem with this conjecture, which is that the background state of noise cannot be broken down into a trace. I tell you now, that when we proceed to Tonal Cosmology, we will actually be conjecturing a mechanics which takes place within simultaneity. Relativity determines that time is not fixed over different reference points, one observer may observe the same *event* at a different *time* from another observer. However, Einstein does not develop a mechanics for what is happening *within* or *beneath* a simultaneous moment. In Tonal Cosmology, if the universe from a fixed reference point, or the universe as a whole, were an apple, then we will attempt to cut this apple open and peer inside of it.

But this is not the primary thesis of my paper. In order to arrive at this thesis, let us take our interpretation of Brownian motion and noise, and draw an analogous relation between symmetry and asymmetry in the study of emergence. I claim that generally, symmetry and asymmetry are integrals of one another, and that more specifically asymmetry is the integral of symmetry. In most views, symmetry and asymmetry are a dichotomy- if an object is symmetric, it is not asymmetric, if an object is asymmetric, it is not symmetric. In this view symmetry is seen as structure, order, while asymmetry is seen as random or as noise. I want to suggest that while symmetry is normally seen as something ordered, it is also a background state, and as such, is analogous to noise. Asymmetry then, which is normally seen as opposed to symmetry and as random, is actually a tracing which arises as an integral of symmetry, a symmetry manifesting itself into existence, into an irreversible path. I have just now given a daring interpretation which is the thesis of my paper. So, can I ground it formally in my hypothesis? I believe I can.

Earlier we established the property of Tonal Conservation before projection. We may now ask, do we have Tonal Conservation after projection? If we examine a scalar projection where $n_1 < n_2$, then we find that the individual keys which make up our orbit are not identical. We may ask then, does each chromatic degree of the manifest instrument, n_2 enjoy *Conservation of Interval*? Does each n_i in n_2 have the same accumulative intervallic value relative to the tonic when considered over each and every key? The answer is no. If one compares each n_i in n_2 to one another, the accumulative intervals relative to the tonic are not the same. This is made trivial by the fact that since n_1<n_2, some, but not all of n_i in n_2 receive tonal assignments, while some n_i in n_2 receive no tonal assignment. It seems Tonal Conservation has been broken. But here is the key point: with the projection of one of our scales with our property, which has n_1 chromatic tones is projected onto an instrument with n_2 tones, we have hypothesized the emergence of Tonal Gravity is consistent. Therefore, while the manifest tonal assignments are not symmetric over n_2, the hypothesized presence of Tonal Gravity directly reflects the intangible scale which has our property, which does enjoy Tonal Conservation. In other words, Tonal Conservation, which is symmetric and enjoys conservation, has a direct effect on the feeling produced by the manifest notes over n_2, even though those notes may appear random or asymmetric. Furthermore, let us remember that any group of manifest notes in any $n_2$have meaning relative to our property, through consideration of any (sd,n) pair and also through pitch invariance in projection. Therefore, I conjecture that if my Tonal Gravity hypothesis is validated, it suggests truly that asymmetry is the integral of symmetry. As with Brownian motion and noise, I conjecture that symmetry and asymmetry are in fact integrals of one another.

With this being said, let us engage in some thought experiment. Let us call the invariants of this universe, symmetry, a background state, and ourselves with our apparently indeterministic properties, asymmetry, a tracing which is an integral of the symmetries which are the invariants of this universe. I ask then, if we as actors are integrals of the invariants, would we have the capacity to observe change in the invariants? If we are the integral of the invariants, then if the invariants were to change, could we use observation to detect this change? Let us take then Newton's laws of motion. If I am seated in a car, and the car accelerates, then I perceive this acceleration in two manners. One, I perceive change in my position, using my visual senses. This is not felt, rather it is observed. I also feel the acceleration of the car, if the car were to accelerate rapidly, I am pulled into my seat. If however, my body were to accelerate simultaneously with the car I would not feel the acceleration. The felt sense of acceleration is generated by the fact that I am stationary relative to the car which accelerates. This suggests that to detect or feel change in that which we are the integral of, requires that we are fixed or at least that our reference frame does not undergo the same change as that of the integral. I believe that the invariants of this universe are considered invariant over all reference frames, but no single reference frame is fixed, rather they are all relative to one another, in the determination of the invariant. This suggests then that if the universes invariants were to change, that we would not be able to detect it. Therefore, I have called the cornerstone of modern science in question, the tool of observation.

I might also point out that if the symmetry, that is the invariants, are an integral of the asymmetry, this suggests that actors may be able to effect change upon the invariants. I offer than a proto-scientific interpretation of the phenomenon of the *miracle*.

I now want to make an aside, not an attack, but a critique on modern scientific theories. Prominent theories, to my knowledge, rely on the notion of an observer who takes measurements. I honestly believe this to be a violence, as it places abstract formalism above lived experience. I propose that we are not observers measuring the Universe, rather, we are actors navigating the universe. This semantic meaning is implied in all of my frameworks within and not within this paper. In a sense, in this paper, I have attempted to formalize the unformalizable, using the felt experience of tonal gravity as my phenomenological cornerstone. However, I feel compelled to share a short brainstorm of reasons why formalisms, the notion of reasoned out truth, cannot result in wisdom which resonates with the metaphysical nature of the Universe.

Formalism is either specifically descriptive, or general, but it has a hard time being both. Formalism struggles to unite the local and the global, seemingly paradoxical dualities.

Formalism relies on what we can articulate, yet so much of experience is infinitely articulable or inaccessible to articulation.

A problem with formalism is we take it for granted and forget its origins. In a way, a formalism may take on new meaning, but this often requires a deep appreciation for its origins and intent rather than a passive understanding.

A formalism prescribes a teacher student relationship built on knowledge rather than experience and character. Rather than passing on experience and character, we pass on knowledge, and the teacher forgets they are the student.

Formalism as we truly understand it, is unique in the minds of humans. It is prescribed by humans, whereas the universe manifests itself as physical.

Formalism can be rigid and unadaptable. We can grow overly connected to our formalisms and become closed to alternative views.

The propagation of life, particularly for some species is often associated with love, which is not easily associated with formalism.

Our moment to moment drives are not, as we experience them, born from formalism but rather internal and external sensations and cognition.

Formalism expresses ideas, and yet ideas and thoughts are not formulatable. They are a universe that is within a universe (physical universe) which we like to formalize about.

2nd law of thermodynamics, entropy increases. Yet formalism imposes structure, potentially structure that is not even there to begin with.

Formalism cannot deduce that something is real or not real. We can use formalism to categorize and describe and quantify and appreciate realness, but we cannot deduce something is real through formalism. Instead of “I think therefore I am” it is just “I am.” Realness is more closely associated, outside of formalism, with perception and experience, whether that which is real is intrinsic to or external to that whom perceives or experiences it.

Formalism has a hard time negotiating subjective experiences. It's semantic stance is often analytic and it applies convention to individual unique experiences, rather than recognizing that each individual experience is itself a synthesis.

The word formalism itself suggests that a formalism is separate from reality. A formalism itself is actually a part of reality, originating in a person using their full human faculties. If we appreciate the origin of formalisms, we get this.

A formalism often uses exclusion principles. It perceives and attaches truth to one thing, one set of observations, when in fact the truth in those observations may exist across many domains. In other words, formalisms, to some degree, seem to be well articulated to the point of being isolated truths, while experience encourages interconnected truths.

Finally a formalism often has a hard time identifying its relation to other formalisms. It is hard to formally determine exactly where the domain of a formalism has its boundary in actual metaphysical reality.

This list was simply a brainstorm upon the capacities that an approach to truth which utilizes formal abstraction has. Each point is debatable, perhaps another could write an entire thesis on the validity or the invalidity of any of my individual points. I offer this simply as food for thought, and I do have one additional point to make. I am not a scientist who has studied the behavior or communication patterns of animals, humans, or animals with humans. However, I believe our bonds, our relations with animals are characterized by a formalism boundary. From experience we know that we may bond with or analyze the behavior of animals. However, when it comes to the formal languages employed by either human, or other animal, there is a clear boundary. I cannot formalize the communications of another animal. I can study their behavior, I can interact with them, I can affect their behavior, I can interpret aspects of their behavior as communications towards me. However, I cannot formalize and rigorously understand and participate in communication with another species.

As I have been critical of fields of science in which I am not trained in, I want to assert that I am not attempting to discredit any body of knowledge. Rather I am trying to articulate the complementarity that modern studies, particularly relating to the phenomena of emergence, may have with my own studies which are rooted in the phenomena of music.

We will now proceed to the 2nd half of my paper. Whereas, I see the first half of my paper as grounded in a hypothesis for Tonal Gravity, and arising directly in consequence of this hypothesis, the second half of the paper plays a more speculative role. First, I will attempt to use Tonal Logic to derive sound definitions for basic mathematical concepts. Then, I will describe an imagined mechanics underlying simultaneity, which has been given mathematical form through Tonal Logic, and I will frankly ride the wave of deductions from this mathematical form as I see fit. Whereas the first half of this paper was arrived at contemplatively, and I believe contains surprising convergence of interpretation and formalism, this second half arose in its current form in a largely spontaneous manner. Let’s proceed.

To begin, it suffices to say that as we attempt to derive definitions for formal mathematical objects using the parameters of Tonal Logic, it is the goal to avoid circular reasoning and rely only on the parameters and projection rules of Tonal Logic. It is the goal to provide definitions which are phenomenologically grounded in my hypothesis of Tonal Gravity, which is cornerstone of the rules of Tonal Logic. We have not yet defined the object that is a number, and yet we quantify a Tonal Objects P value. This could be seen as circular reasoning, but I claim that the derivation of the P value comes from the Base Case of Tonal Logic and object projection, that is the projection of intangible musical scales which satisfy our combinatorial property from our hypothesis, onto groups of manifest pitches. Therefore, our P values follow exactly from these defined (and experienced) objects, and the discrepancy between the two P values manifests directly in how many manifest notes upon an instrument, receive tonal assignments after projection. That is if $n_1 < n_2$, where n_1 is the number of chromatic notes in our intangible scale and n_2 is the number of notes available for assignment on our manifest instrument, then n_2-$n_1$notes on the manifest instrument do not receive tonal assignment under the projection. Furthermore, I assert a single geometric axiom derived directly from the phenomena of pitch. This axiom states that 2^n = 1. We have not yet defined exponents within Tonal Logic, however, this equation is derived directly from the property of the musical octave, which has that a certain frequency, X, relates to all other pitches y_i in the same manner as X(2^n). Obviously, the frequency 2X≠X, however, the harmonic relations for X or 2X with any y_i are identical.

Loose Definition for a Number and for the Number Line

Object 1 — Identity Object (O_1)

O_1 = ({n}, {n}, {n}, {1}, R_1)
sd_1 = the number itself
S_1 = singleton structure
C_1 = singleton context
P_1 = 1
R_1 = fully resonant (context is null)
Type: Irreducible object, the number as an irreducible entity without breaking into its properties.

Object 2 — Multiplicative Object (O_2)

O_2 = ({p_1,...,p_z}, {$p_1^{n1} \cdots p_z^{nz}$}, {$c_i = p_i^{ni}$}, R_2)
sd_2 = multiset of prime factors with multiplicity
S_2 = full prime factorization
C_2 = all prime power components
P_2 = n_1 + ... + n_z
R_2 = fully resonant (context is null)
Type: Full internal resonance

Object 3 — Arithmetic Object (O_3)

$O_3 = (\{1,\ldots,n\}, (1,n), \{(i,j) \mid i + j = n\}, R_3)$
sd_3 = list of numbers 1 to n
S_3 = interval (1, n)
C_3 = additive relations that sum to n
$P_3 = n$
R_3 = fully resonant (context is null)
Type: Additive combinatorial object

Object 4 — Tonal Number (O_4)

$O_4 = O_1(O_2(O_3))$
$sd_4 = sd_1 \cup sd_2 \cup sd_3$
$S_4 = S_1 \cup S_2 \cup S_3$
$C_4 = C_1(C_2(C_3))$
P_4 = number of unique elements in sd_4
R_4 = fully resonant (all implications across S_4 and C_4 are active)

Number Line — Tonal Object ($\mathcal{L}$)

$\mathcal{L} = (\{1,2,3,\ldots\}, \{S_i\}, \{C_i\}, R_{\mathcal{L}}, P = \infty)$
Each $S_i = S_1 + S_2 + S_3$
Each $C_i = C_1(C_2(C_3))$
$R_{\mathcal{L}}$ = fully resonant over all numbers
Type: Infinite composite object. As a whole, it is fully resonant, but each number within only partially resonance $R_{\mathcal{L}}$

Number Projected onto the Number Line

$O_4(\mathcal{L}) = (sd_4, S_4 + S_{\mathcal{L}}, C_4(C_{\mathcal{L}}), R_4 + R_{\mathcal{L}})$
sd_4 components remain distinguishable:
- Identity (sd_1)
- Multiplicative (sd_2)
- Combinatorial (sd_3)

Context becomes layered within a larger field

$R_4(R_{\mathcal{L}})$ is not fully resonant with the entire number line

7. Infinity Projected as a Particle onto the Number Line

By projecting infinity, under tonal geometry, as a particle with sd = 1 and P = 1, we may express the entire number line as collapsing to a singular resonance identity.
In this special projection, the number line becomes:
sd = ∞
S = {1, 2, 3…}, or ∞, C = shared properties between individual numbers, over and notated by ∞, and P = 1
The number line becomes an irreducible object with fully resonant R.

Rigorous Tonal Definitions of Number, Zero, and Infinity

1. Number as Itself — Object O_1 (Identity)

Any natural number x may be expressed as:
x = $2^{n_1} + 2^{n_2} + \ldots + 2^{n_x}$
Under tonal geometry (2^n = 1), this collapses to:
x = 1 + 1 + ... + 1 (x times). We have reduced sd to be $\Sigma\, 2^n$, that is, the sum of one distinct element. Thus, P=1.
This defines the number as an irreducible object (sd=S=C=R) with one distinct sequence of one distinct element.

2. Number as Product of Prime Powers — Object O_2 (Multiplicative)

Given x = ${p_1}^{n_1}$ X ${p_2}^{n_2}$ * ... * ${p_k}^{n_k}$
Each ${p_j}^{n_j}$ is expressed in binary as a sum of powers of 2.
Each such expression collapses under tonal geometry to 1 + 1 + ... + 1 (n_j times).
Total P = $n_1 + n_2 + \ldots + n_k$ where each n_i represents sequences of 1 with unique length.
This representation gives a unique P value corresponding to the number of prime power factors.

A multiplicative resonance structure is formed.

3. Number as Arithmetic Sequence — Object O_3 (Combinatorial)

Let x be a number. Define:
sd = [1, 2, ..., x], structure = (1, x), context = each number in sd, (1,x) in terms of number as itself see above).
Each element in sd is converted to binary and then collapsed via $2^n = 1$.
Each number becomes a unique sum of 1's, and the total P = x.
This is the arithmetic or combinatorial identity.

(I must quickly add, that to avoid circular reasoning to the greatest extent, I should have defined a number as an identity, then this number as arithmetic sequence, then I might have proceeded to define addition, from this define multiplication, from this exponents, and then I could have defined the multiplicative property of a number after all this. I retain the order I have given.

4. Full Number — Object O_4 and the Number Line

$O_4 = O_1(O_2(O_3))$ projects the identity, multiplicative, and arithmetic definitions into one resonant object.
The number line is defined as the space of all such objects, each defined in the same way.
Each number projected onto the number line retains its unique sd layers.
The tangibility of each definition corresponding to number relies on our use of the geometric axiom $2^n = 1$ in the defining of parameters.

5. Definition of 0

Let sd = 0. Then P = 0.
Structure (S) and context (C) may be defined, but no projection occurs.
The resonance field R does not resonate.
Zero is a null scalar identity — it implies no tonal activation.

Projection Chains and the Formalization of Addition in Tonal Logic

1. Intangible to Tangible Projection of Addition

Let O_1 be an intangible object defined as:

- sd = (i, j)
- S = i + j
- C = i + j = x
- P = 2

Let this object be projected onto a tangible object representing grouped discrete items:

- sd = (i, j) tangible objects
- S = grouping of i and j objects
- C = i + j = x objects
- P = 2

The result is a tangible projection chain. Since the resulting object is tangible and fully resonant, we assert that the original intangible object O_1 is itself tangible within the framework of Tonal Logic.

2. Full Representation of i+j=x with P = x

Let O_2 be a number object represented by:

- sd = [1, x]
- S = (1, x)
- C = The total number of unique sequences defined by $m(2^0) = m$ for each m in [1, x]
- P = x

Each context is a decomposition of a number (1,x) into a unique sequence of 1's, verified through binary-based projection onto discrete object groups. This validates the P = x form as a fully resonant and tangible object, which we will show also corresponds tangibly to the previous sum i+j. To foreshadow, if we project i+j=x, onto x, we get a fully tangible projection chain.

3. Composition of Two Numbers

Let two number objects be defined and composed:

- O_3 and O_4 are numbers with P = 1
- Their composition: $O_3(O_4) = O_5$, now with P = 2
- This represents a two-part additive number: (i, j) such that i + j = x

4. Additive Identity and Final Composition

Now project O_1 onto O_5:

- $O_1(O_5) = O_6$
- This confirms the additive structure and resonance.

Next, decompose $O_2(O_6)$: This projection corresponds to the '4' In the expression 1+3=4,

  - This reveals the P = x representation.
  - The object is now expressed in terms of all [1, x] sequences under $m(2^0) = m$ binary logic, grounded in projection onto tangible discrete objects. We have started with two numbers as such, we have summed them, and we have resulted back in the original form.

Multiplicative Structures in Tonal Logic

1. Multiplication via Group Projection

We define multiplication in similar form:
- O_1 = (sd = (i, j), structure = i(j), context = i × j = x, P = 2), where O_1 is intangible
- O_2 = (sd = i groups of j tangible objects, structure = i(j) tangible groupings, context = i × j = x objects, P = 2), where O_2 is tangible.

We then project:
   $O_1(O_2) = O_3$
Since O_3 is tangible, O_1 is verified as a tangible multiplicative object.
O_3 can then be named O_4, a number object with:
   - P = 1 (identity)
   - or P = x (decomposed into $m(2^0) = m$ sequences).

2. Multiscale P-Decompositions

Tonal logic supports multiscale decompositions via flexible P-structure:

- P = number of 2-factor pairs (i, j) such that i × j = x
- P = number of 3-factor triplets (i, j, k) such that i × j × k = x
- P = number of n-tuples of factors such that their product equals x

Each decomposition level forms a new structure-context frame with its own projection rules. These projections can be used to interpret objects across multiple levels of implication, allowing tonal logic to explore compositional depth and multiplicative identity with precision.

Tonal Logic Deduction of Negative Numbers

Step 1: Definition of Objects

We define:

- O_1: An abstract object representing a positive integer n:
  O_1 = (sd = n, S = n, C = n, P = 1) (fully resonant)
- O_2: A tangible object representing n discrete elements:
  O_2 = (sd = n objects, S = n objects, C = n objects, P = 1 or n)

Both O_1 and O_2 can be decomposed to P = n by considering their tangible partitions.

There exists a tangible projection chain: $O_1(O_2) = O_3$ mapping the abstract number onto tangible objects.

Step 2: Problem Setup: Null Projection

If O_2 has:
 P = 0 (null)

then $O_1(O_2)$ cannot proceed in the ordinary tangible sense — there is no tangible object to project onto.

However:

- Since $P(O_2) = 0 < P(O_1) = 1$,
- We may define the reverse projection: $O_2(O_1) = O_3$

We define that an object with P=0 may be either tangible or intangible. Thus, O_2 is either tangible or intangible. O_1 can be shown to be tangible when $P_2 > 0$, but when P_2=0 and we consider O_2 intangible, we must reassess our projection chain and we require that O_1 must transform itself so that $O_2(O_1)$ results in a tangible O_3 with P=0.

Step 3: Introduction of a Two-Part Object

To formalize the needed structure:

- Decompose O_1 into a two-part object O_4:
  O_4 = (sd = (i, j), S = some operation on i and j, C = operation giving 0, P = 2)

Where i and j are integers.

Step 4: Tangible Operation: Addition

We define an addition operation object O_5 that acts on O_4.

Applying O_5 to O_4 gives:
 $O_5(O_4)$ = (sd = (i, j), S = i + j, C = i + j = 0, P = 2)

Thus:

- $O_5(O_4) = O_3$ if and only if:
  j = -i

That is, j must be the additive inverse of i.

Step 5: Full Projection Chain and Conclusion

We now have:

- $O_5(O_4)$: tangible additive inverse object,

- $O_2((O_5(O_4))) = O_3$ is a valid tangible projection chain.
Thus:
- O_2 (null tangible object)
- $O_5(O_4)$ (tangible additive inverse structure)
- Combine to form a tangible null object O_3.
Hence, we have deduced negative numbers in tonal logic:
- A negative number is the second element j of a two-part structure (i, j),
- Such that when projected through addition, the result is tangible 0,
- Confirming that $j = -i$.

To make our definition for the additive inverse and negative numbers we asserted that an object with P=0 may be either tangible or intangible. This is a foreshadow of the notion explored in Tonal Cosmology, that whether an object is tangible or intangible is primarily a matter of perspective. If an actor navigates a projection chain, then it is this very actor which determines tangibility or intangibility of objects. We will also assert a notion of fundamental tangibility of the Universe, so that anything that is real or experienced is tangible, and it is a limitation of perspective which results in intangibility.

Tonal Division: Integer Division within Tonal Logic

1. Starting Integer Object

We begin with an object $\mathcal{O}_1$ representing an integer value:

$\mathcal{O}_1$ = (sd = x, S = x, C = x, P = 1, R = fully resonant)

This object $\mathcal{O}_1$ is an integer, modeled with structure and context equal to its scalar degree, and resonance defined through total implication.

2. Multiplicative Reconstruction of x

By prior results on multiplication, we know:

$\mathcal{O}_1 = \mathcal{O}_2(\mathcal{O}_3(\mathcal{O}_4))$

Where:
- $\mathcal{O}_2$: the multiplication object (e.g., $i \cdot j$)
- $\mathcal{O}_3$ = (sd = i), an integer
- $\mathcal{O}_4$ = (sd = j), an integer

This expresses the idea that $x = i \cdot j$.

3. Defining Division via Intangible Object

We define a new object $\mathcal{O}_5$ which represents the intangible logical structure of division:

$\mathcal{O}_5$ = (sd = (x, i), S = x / i, C = x / i = j, P = 2, R = fully resonant)

This object encodes the division rule: given x and i, find the value j such that x = i · j. Since it is intangible, its validity will depend on projection into a tangible counterpart.

4. Tangible Division Object

We construct a tangible counterpart $\mathcal{O}_6$:

$\mathcal{O}_6$ = (sd = (x objects, i groups), S = x / i, C = x objects / i groups = j objects/group, P = 2, R = fully resonant)

This models the physical grouping of objects and confirms whether x can be divided into i equal groups with integer result j.

5. Projection and Resonance Validation

Now we perform the projection:

$\mathcal{O}_5(\mathcal{O}_6) = \mathcal{O}_7$

Since $\mathcal{O}_7$ is tangible, this confirms the resonance of the original intangible division structure $\mathcal{O}_5$. Thus, the division operation holds within the logic.

6. Division Derivation from Multiplication

We then have:

$\mathcal{O}_5(\mathcal{O}_1(\mathcal{O}_3)) = \mathcal{O}_4$

That is, projecting the division rule $\mathcal{O}_5$ onto the result of x and one factor i, gives the other factor j as $\mathcal{O}_4$.

This closes the division rule and its tangible realization in Tonal Logic.

Tonal Logic Deduction of Rational Numbers

In tonal logic, we define a rational number as a projection-based object with structured and contextual relationships. A rational number is typically expressed in the form (a, b), where 'a' and 'b' are integers, and b ≠ 0. This pair is used to represent the division a/b.

We define the rational object in tonal logic as follows:

- sd = (a, b)
- Structure = a / b
- Context = a / b = c
- P = 2
- Resonance field (R) = fully resonant

Alternatively, the object may be described as a singular object:

- sd = (a, b)
- P = 1
- Still describing a rational number with structure a/b.

To address non-integer values of a/b, we decompose the irreducible object with:

- P = 1
- sd = 1
- Structure = 1
- Context = 1

We then project this object into:

- sd = (1, B)
- Structure = 1 / B
- Context = 1 / B
- P = B
- Fully resonant, modeling 1 as divided into B equal parts.

This projection is executed onto a tangible object of the same description: one unit cut into B equal tangible parts. The result of the projection is:

- A tangible object that confirms the existence and structure of 1 / B.
- This affirms the rationality of a/b when a and b are integers.

For rational addition, we use:

- (a/b) + (c/d) = (ad + bc) / bd

In tonal logic, we calculate this through nested projection chains:

- Compute a(d), b(c), then ad + bc
- Compute bd

The resulting rational is formed as (ad + bc) / bd and treated as either:

- An integer, or
- A fractional object where P = ad + bc parts of size 1 / bd.

Rational multiplication is handled similarly:

- Compute ac and bd
- Express as ac / bd
- Project as either an integer or as ac quantity of 1 / bd parts.

Finally, inversion in tonal logic is managed through projection:
- Given (a, b), the inverse is projected as (b, a)
- The structure and context are reversed
- This inverse object is fully resonant and represents 1 / (a / b) = b / a.

I. Exponentiation in Tonal Logic

Definition of Exponentiation

Let a be a tonal object and $n \in \mathbb{N}$. Let's also call the multiplication object, which we have previously defined, as Om. Then, the exponential of a is defined as:

$$a^n := O_{m_n}(O_1)$$

Where:

- Om_n is the multiplicative projection object with internal parameter P = n
- O_1 is the identity operand, the base generative unit, a number
- The projection length n determines the tonal magnitude or multiplicative depth

Fundamental Rules

1. 1. Zero Exponent:
   $a^0 := 1$

- Defined axiomatically as the multiplicative identity
- Not represented via Om; bypasses projection

2. 2. Addition of Exponents:
   $a^{n1}(a^{n2}) = a^{n1+n2}$

- Nested Om objects absorb into a single Om with $P = n_1 + n_2$

3. 3. Power of a Power:
   $(a^m)^n = a^{mn}$

- Composition of two Om objects with projection lengths m and n yields a single Om with P = mn

4. 4. Product Rule for Bases:
   $(ab)^n = a^n b^n$

- Om distributes over tuples:

$$Om_n((a, b)) = Om_n(a) \cdot Om_n(b)$$

II. Resonance Fields of Exponentials

Prime Structure of a Tonal Object

Let:

$a = \prod_i p_i^{n_i}$

be the prime factorization of a, where each p_i contributes a resonant component to the structure of a.

Resonance Field of a

Each prime p_i generates a resonance interval:

$\mathcal{R}p_i(a) = [p_i^1, p_i^{ni}]$

Thus, the resonance field of a is:

$\mathcal{R}(a) = \bigcup_i [p_i^1, p_i^{n_i}]$

Resonance Field of the Exponential a^m

Raising a to a natural number power m yields:

$a^m = \prod_i p_i^{m(n_i)}$

The resonance interval for each p_i becomes:

$\mathcal{R}$_{p_i}(a^m) = $[p_i^1, p_i^{m(n_i)}]$

Hence, the resonance field of a^m is:

$\mathcal{R}(a^m) = \bigcup_i [p_i^1, p_i^{m(n_i)}]$

Sequences of Unity and Projection Depth

Each exponent $p_i^{n_i}$ corresponds to a sequence of unit projections:

$p_i^{n_i} \leftrightarrow [1, 1, ..., 1]$ (n_i times)

Exponentiation extends this:

$p_i^{mni} \leftrightarrow [1, 1, ..., 1]$ ($m \cdot n_i$ times)

Interpretation

- The resonance field of a number is a structured interval space for each of its prime components.
- Exponentiation reinforces these fields, extending their upper bounds without shifting their origins.
- The resonance field of a^m is thus a reinforced resonance field of a, preserving identity while expanding its expression.

Tonal Logic: Negative Exponents and Inverse Projection

This document defines negative exponents, specifically a^{-n}, within the tonal logic framework. Negative powers are understood as inverse projection structures that resolve multiplicative identity.

Definition of Negative Exponents in Tonal Logic

Let $a^n := Om_n(O_1)$, where Om_n is a projection of length n acting on the base identity O_1.

Then we define:
a^(-n) := object such that:

Om_n(a^(-n)) = 1

That is, a^(-n) is the object whose n-fold projection yields the multiplicative identity.

Interpretation

- a^(-n) is the multiplicative inverse of a^n.
- It inverts the structure generated by $Om_n(O_1)$ by restoring the identity through contextual projection.
- In tonal logic, the inverse is not symbolic division but structurally grounded in the projection mechanism.

Conceptual Summary

Tonal logic defines a^(-n) not as an abstract reciprocal, but as the unique object that resolves the projection structure Om_n back to identity. This inverse projection reconstructs the tonal base and affirms the symmetry of multiplicative structure.

Tonal Logic: Fractional Exponents and Structural Rooting

This document formalizes the definition of fractional exponents, specifically a^(1/n), within the framework of tonal logic. It interprets fractional powers as structural roots—the inverse of natural number projection chains.

Definition of Fractional Exponents in Tonal Logic

Suppose:
a = O_{m_n}(b)

Then we define:
$a^{1/n}$ = Object with:

sd = b
S = $Om_n(b)$
context = a = $Om_n(b)$
P = 1

Interpretation

- The source domain (sd) is b: the operand that, under n-fold projection, produces a.
- The structure (S) is $Om_n(b)$: the known generative form that yields a.
- The context explicitly affirms a = $Om_n(b)$: this grounds the root in a defined projection.
- The projection length (P) is 1: representing a single instance of extracting the operand from the structure.

Conceptual Summary

This defines a fractional exponent as a rooted object whose projection re-forms the original exponentiated structure. In tonal logic, a^(1/n) is not symbolic but structurally defined by its ability to regenerate a through Om_n. The identity of a^(1/n) is context-dependent and structurally anchored.

Tonal Logic: O_{gcd} Definition with Multiplicative Object

This document defines the O_{gcd} (greatest common divisor) operator within the tonal logic framework, incorporating a multiplicative object to express shared resonance structure. The O_{gcd} operator extracts a fully resonant, tangible number object from two number objects based on the overlap of their affirmed prime resonance structures.

Formal Definition of $\boldsymbol{O_{gcd}}$ in Tonal Logic

Let two number objects be defined as:
$O_1 = \prod p_i^{n_i}$, $O_2 = \prod p_j^{m_j}$

The Ogcd operator is defined as:

$O_{gcd}(O_1(O_2))$:= Object with:

sd = a
S = $o_{m_{gcd}}(p_1^{k1}, p_2^{k2}, \ldots, p_r^{kr})$
context = each p_i^{ki} is structurally affirmed in both O_1 and O_2

P = 1
resonance = fully resonant (shared structure)

The Role of the Multiplicative Object ($O_{m_{gcd}}$)

The multiplicative object $O_{m_{gcd}}$is distinct from the projective operator Om_n. It functions to combine independently affirmed prime resonance structures into a unified number object. Each p_i^{ki} contributes a fully affirmed resonance layer, and their tonal combination constitutes the structure of the gcd object.

Ontological Behavior of O_{gcd}

- Initially, O_{gcd} is intangible: the sd = a is undefined.
- Once the shared resonance structure is extracted, O_{gcd} becomes tangible, producing a defined object O_3.
- O_3 has structure S = $O_{m_{gcd}}(p_1^{k1}, p_2^{k2}, ..., p_r^{kr})$ and projection length P = 1.
- This tangible identity is derived from the intersection of affirmed resonance intervals.

Conceptual Summary

The O_{gcd} operator affirms the deepest shared resonance structure between two number objects by constructing a new composite object from their common affirmed primes. The use of $O_{m_{gcd}}$ encodes this composition structurally, ensuring that the resulting object reflects a complete tonal intersection. O_{gcd} thus performs a meaningful extraction of identity through the resonance logic of tonal space.

Tonal Logic: O_{lcm} Definition via Complementary Resonance

This document defines the Olcm (least common multiple) operator in tonal logic by expressing it as the complementary resonance structure required to complete the full resonance composition of two number objects. The Olcm object is extracted by decomposing the multiplicative structure of Om(O_1, O_2) with respect to their shared resonance, Ogcd(O_1, O_2).

Formal Definition of O_{lcm} in Tonal Logic

Let two number objects be defined as:

$O_1 = \prod p_i^{ni}$, $O_2 = \prod p_i^{mi}$

Then we define:

Om(O_1, O_2) = Om(O_{gcd}(O_1, O_2), j)
⇒ j := Om(O_1, O_2) / O_{gcd}(O_1, O_2)
⇒ O_{lcm}(O_1, O_2) := j

Where O_{lcm} is the object such that:

$O_{lcm}(O_1, O_2)$:= Object with:

sd = j
S = $Om_x(O_1, O_2)$ / $O_{gcd}(O_1, O_2)$
context = $Om(O_1, O_2)$ = $O_m(O_{gcd}, j)$
P = 1
resonance = complementary to shared structure

Interpretive Summary

The O_{lcm} operator returns the residual structure required to reconstruct the full combined resonance of two number objects from their shared projection structure. It affirms the maximal resonance fields of O_1 and O_2 and eliminates their overlap. Olcm is the tonal complement that, together with Ogcd, completes the total multiplicative resonance in the composition of two numbers.

Example

Let:
$O_1 = 2^2 \cdot 3^1$,
$O_2 = 2^1 \cdot 3^3$

Then:
$O_{gcd}(O_1, O_2) = 2^1 \cdot 3^1$
$O_m(O_1, O_2) = 2^3 \cdot 3^4$
$O_{lcm} = O_m(O_1, O_2) / O_{gcd} = 2^2 \cdot 3^3$

Definition of a Function and Analysis of a Function in Tonal Logic

This document defines what a function is within tonal logic and outlines the complete structure for analyzing and evaluating a function. A function is not a symbolic mapping from one value to another, but a fully resonant composite object built from tangible domain and range identities, along with their structural fields. Evaluation of the function occurs by parsing its resonance field and internal context.

Stage 1: Structural Fields of Domain and Range

O_1 (structure of the domain):
- Source domain (sd) = set of domain elements
- Structure = set of domain elements
- Context = set of domain

- P = quantity of elements in the domain
- Fully resonant

O_2 (structure of the range):

- Source domain (sd) = set of range elements
- Structure = set of range elements
- Context = set of range
- P = quantity of elements in the range
- Fully resonant

Stage 2: Identity Affirmations of Domain and Range

O_3 (domain identity object):

- sd = O_1 (structure of domain)
- Structure = contents of domain
- Context = f(x) = y such that x is in domain
- P = 1
- Fully resonant and tangible

O_4 (range identity object):

- sd = O_2 (structure of range)
- Structure = contents of range
- Context = f(x) = y such that y is in range
- P = 1
- Fully resonant and tangible

Stage 3: Definition of the Function Object

O_5 (function object) = $O_3(O_4)$:

- sd = domain, range
- Structure = contents of domain and range
- Context = f(x)=y such that x is in domain and y is in range
- P = 2
- Fully resonant and tangible

Stage 4: Expansion for Evaluation

To evaluate the function, we expand the domain and range identity objects into their full resonance fields:

$O_3(O_1)$:

- sd = domain, all elements in domain

- Structure = domain and elements
- Context = domain, f(x)=y such that x is in the domain
- P = number of domain elements + 1
- Fully resonant

$O_4(O_2)$:

- Same, but for range
- Context = f(x)=y such that y is in the range

The function object now becomes:

$O_5 = (O_3(O_1))(O_4(O_2))$

- sd = domain, range, number of elements in each
- Structure = domain, range, elements of domain and range
- Context = all domain elements, all range elements, and f(x) = y such that x is in the domain and y is in the range.
- P = number of distinct elements in domain and range +2 (for the domain as an identity and the range as an identity).
- Fully resonant

Function Evaluation as Resonance Parsing

Evaluation does not involve projection. Instead, it involves analyzing the context and structure of the fully resonant function object. The act of evaluation (f(x) = y) occurs by locating and interpreting the mapping relationship within the resonance field of O_5. This parsing operation is context-sensitive and navigates the composite tonal object, rather than constructing new identities.

Considering infinity:

As a number x → ∞:
- Under O_3 from our definition of.a number: the sequence [1, ..., x] grows → P → ∞
However, binary representation of x as a sum of 2^{n_i}, under tonal geometry (2^n = 1), collapses x to x copies of 1. Thus, we may define:
- An object where, by binary representation, sd = 1 , 1 , 1 , ... (∞ times), and so P value collapses to 1, 1, 1...→ P = 1
Infinity thus exhibits dual behavior: in full expansion, P → ∞; in collapse, P = 1.

At this time we intentionally supply loose object definitions for Infinity and the Real number line to support many possible practical applications, including calculus, continuity, and limit behavior, without requiring a formal distinction between different types of infinity.

Definition of Infinity

Infinity is defined as a tonal object with the following structure:

- sd = infinity
- Structure = [1, 2, 3, ...] (the natural numbers)
- Context = non-finite (later we will consider other contexts for infinity)
- P = infinity or 1
- Resonance = fully resonant
- Ontological status = tangible or intangible

Infinity is treated as a structural and resonant object—not a symbolic endpoint or unapproachable limit. It affirms the presence of an infinite sequence, either as an actualized identity (P = 1) or a processual field (P = ∞).

Definition of the Real Number Line

For now, the real number line is defined using the closed unit interval [0, 1]:

- sd = [0, 1]
- Structure = [0, 1]
- Context = any source domain is either a rational number or not a rational number
- P = infinity
- Resonance = fully resonant
- Ontological status = tangible or intangible

This definition captures the real number continuum as a complete and infinite tonal field. The [0, 1] interval is preferred at this stage because it provides a bounded and fully contained structure with infinite resolution. This enables analysis of continuity and limit operations without requiring reference to the full unbounded line.

Countable vs. Uncountable Infinity

We do not currently distinguish between countable and uncountable infinity. While such distinctions are fundamental in set theory, they are not necessary for many of the major practical applications of infinity in analysis, such as limit behavior and continuity. Tonal logic instead affirms the infinite field through structure, resonance, and context.

More precise distinctions may emerge later when analyzing functions and mappings over infinite fields.

Expansion of the Interval

While we presently define the real number line using the interval [0, 1], this interval can be expanded to:

- [0, ∞) for semi-infinite domains
- [-∞, ∞] for the full unbounded real line

Such expansions will be introduced through projection or functional transformation when needed. For now, the [0, 1] field offers a sufficient and contained foundation for analysis.

Tonal Projection Analysis of Erdős–Woods and Divergence Conjectures

Definition — Erdős–Woods Conjecture (O_1)

There exists a fixed integer k such that for infinitely many integers n, the interval [n, n + k] satisfies the following:

For every m ∈ [n, n + k],
gcd(m, n) > 1 or gcd(m, n + k) > 1

That is, each number in the interval shares a common factor with at least one of the endpoints.

Definition — Divergence Conjecture (O_2)

Let L = k + 1 denote the length of an interval [n, n + k] that satisfies the Erdős–Woods condition.

Then the conjecture states:
Over all pairs (n, k) such that the interval [n, n + k] satisfies the condition, the values of L form an unbounded sequence:
sup { L | [n, n + k] satisfies the Erdős–Woods property } = ∞

That is, the length of intervals satisfying the condition can grow arbitrarily large.

Suppose L Diverges to Infinity

We assume the divergence conjecture (O_2) holds. This allows us to construct arbitrarily large intervals [n, n + k] that satisfy the Erdős–Woods condition. However, this does not guarantee that for any fixed k, there are infinitely many such n. In fact, for a specific large L_i = k + 1, it may be the case that we cannot instantiate another interval longer than L_i that also satisfies the property.

Now we define infinity as four objects, representing 2 different resonating contexts:
- O_3: Infinity as divergence (growth without instantiation)

sd=infinity

Structure= uses of infinity

Context= divergence (growth without instantiation), [0,1]

P=1

- O_4: Infinity as infinite set instantiation (explicit enumeration)

Sd=infinity

Structure=uses of infinity

Context=Infinite sets which allow instantiation. [1,2,3…]

P=∞

O_5 = decomposition of O_3 into infinity objects with P = 1, each with S = Uses of infinity and C = divergence, [0,1]. Each object in O_5 and with P = 1 satisfies that it resonates with the Divergence context of infinity. This composite object has P=infinity

Now we must do the same for O_6, except each decomposed element has that they resonate with set instantiation context of infinity.

Then we have:
$O_2(O_1)$ - paradoxical.
$O_5(O_2(O_1))$ - disproves EW conjecture
$O_6(O_5(O_2(O_1)))$ - doesn't disprove EW conjecture
$O_5(O_6(O_5(O_2(O_1))))$ - disproves EW conjecture
$O_6(O_5(O_6(O_5(O_2(O_1)))))$ - doesn't disprove EW conjecture.

Thus we have shown that, assuming L diverges to infinity, EW can be either disproved, or not, depending on the order of projection of O_6 and O_5, which are identical except they resonate different functions of infinity in R. In this sense, we have shown that infinity may have an implied oscillatory property. This may be considered wave-like. We have wave-particle duality, and this further supports our P=1 or ∞ value and also infinity as a tangible or intangible object.

If one, including myself, is not satisfied with the use of Erdo-Wood conjecture, we may supply a similar line of reasoning to the fundamental theorem of calculus. Again, through projection, we oscillate between saying that "the rectangles are infinitesimally small and fill the curve," and, "we may instantiate a spot under the curve the rectangles do not cover."

Tonal Logic: Formal Definition of Form

Definition: Form

A form is a structural abstraction $\mathcal{F}$ such that:

- There exists a set of objects $\{\mathcal{O}_i = (sd_i, S, C_i, R_i, P_i)\}$

- Each object $\mathcal{O}_i$ has a distinct scalar degree sd_i, but shares the same structure S

Then:

The collection $\{\mathcal{O}_i\}$ defines a form $\mathcal{F} = (\{sd_i\}, S, \{C_i\}, \{R_i\})$

Key Implications

- Form is a second-order object: it generalizes structure over multiple sd's.

- Every member of a form is self-contained, but the shared structure S creates continuity.

- A form defines a space of nested projection chains:

  Form ⇒ Individual Resonant Objects

Example: Multiplicative Identity Form

Let:

- $sd_1 = 6 \Rightarrow S = 2 \cdot 3$

- $sd_2 = 12 \Rightarrow S = 2^2 \cdot 3$

- $sd_3 = 18 \Rightarrow S = 2 \cdot 3^2$

Each of these objects fully resonates its own multiplicative structure.

Together they form a multiplicative form associated with the structure S = prime factorization, possibly defined as:

  $\mathcal{F}$_mult = $\{ sd_i \in \mathbb{N} : sd_i = \prod p_k^{n_k} \}$

A form emerges when multiple sd's each partially resonate a common structure. That structure becomes a surface upon which discrete, resonant objects lie.

Philosophical Implication of these attempts at using Tonal Logic to define mathematical objects:

I have attempted to show that the parameters and projection in tonal logic have the potential to define other formalized systems, such as some basic mathematical objects. The author wonders about the possibility for novel insight about mathematical objects to occur within Tonal Logic, and possibly novel problem solving methods. It is my intent to produce a potential system as such, which is grounded in phenomenology. We have seen that while the roots of phenomenological groundedness lie in the nature of our Tonal Gravity Hypothesis, in application, particularly to non-music domains, the phenomenological basis occurs within the tangibility and intangibility parameter of objects, and in creating projection chains which link an intangible hypothesis to a composed object which may be determined to be tangible, thus giving tangibility to the hypothesis (intangible) object at the front of the chain.

Proof of Proof

Conjecture Statement

Tonal logic's object projection chain, governed by its formal rules of tangibility and composition, provides a powerful mechanism for logical reasoning and mathematical discovery. We propose the following principle: If a projection chain begins with an intangible object and proceeds entirely through composed tangible objects — concluding in a known tangible object — then the first object becomes effectively tangible within the context of the chain, due to the entanglement between its start and end.

Projection Chain and Tangibility

In tonal logic, a projection chain is a sequence of nested objects, each projecting onto the next: $O_1(O_2(...(O_n)))$. Tangibility in this framework governs whether a structure is enumerated, specified, or implied. A tangible object defines its resonance explicitly; an intangible object implies its resonance indirectly.

However, if the internal path of projection is composed entirely of tangible objects and the final result is known and tangible, then — by the principle of chain entanglement — the initial intangible object can be said to attain tangible status within that context. This property allows backward propagation of tangibility along a valid projection path, turning hypothetical or undefined front elements into rigorously grounded components of reasoning.

Discovery as Method

This projection model reflects the process of inquiry: starting with an undefined structure (hypothesis), testing through grounded steps (experiment), and resolving into a clear outcome (result). Within tonal logic, this becomes a framework for discovering unknown truths by composing intermediate structures from known logic or forms.

Conclusion

We conjecture that this tonal projection chain, with its capacity for contextual entanglement and abstract compression, forms a **proof of proof** — a recursive model of discovery. It suggests that within a multiplistic tonal framework, deep reasoning emerges not from isolated deduction but from harmonious chaining of tangible projections through structured multiplicity.

Cooper's Completeness Theorem

Statement

I hesitate to include this statement, but I will try. The incompleteness theorem, which states that there are objects which cannot be described both consistently and accurately, is often viewed as a roadblock or as a direct limitation on formal systems, but worse yet, on what is possible. It is an attempt in Tonal Logic, to provide a system, framed in multiplicity, navigated by actors into explicit meaning, which allows for *lack of consistency* in arguments. This is especially true for projection of the context parameter, for which the result is non-order invariant. We have noticed, within Tonal Logic, what seems to be a natural and implied oscillatory property in the object of infinity. If infinity is indeed oscillatory in nature, then it suggests that well defined objects themselves may not necessarily be consistent under contextual variation. Tonal Logic attempts to embrace the hard truth of the incompleteness theorem. One might point out, that if individual actors collapse multiplicity into explicit meaning, is truth itself purely relative? It may seem as such, however, it is my opinion that if something is truthful, it is likely that multiple actors will converge on and resonate with that particular truth. Truth then, becomes a sort of symmetry, a background state, while possible reframings of that truth are an emergent asymmetry integrated from the former symmetry. Later in Tonal Cosmology, I will interpret that on a fundamental level, all individual actors are interconnected and in obeyance with one another on a shared topological space.

Tonal Cosmology

On non-order invariance for the Context parameter:

Modulatory Contextual Chains in Tonal Logic

In the case of a piece of piano music's harmonic structure, we may notate the modulators structure of the piece by compositions of c1, c2,…,c12 for 12 key signatures.

Call the key of C major c1 and the key of G major c2. We could model a modulatory structure as follows: c1(c2(c1)). The piece starts in context c1, with sd1 = notes C, D, E, F, G, A, B.
This modulates to c2 with sd2 = G, A, B, C, D, E, F#,
and then back to c1.

In music, although we may modulate keys, a hierarchy of tonal centers emerges, often, but not always, with the tonic key of the piece coinciding with the beginning and end of the piece.

Thus we see a projection chain experiences a distribution of c_i, which make a unique piece of piano music.

These weighted contextual chains may be particularly perceivable when all c_i are in some structure S.

The Fundamental Tangibility of the Universe

This document introduces a foundational conjecture for space-filling curve cosmology within tonal logic: the universe is fundamentally tangible.

1. Continuity and Self-Containment
The universe exists in continuity and contains itself. Therefore, from its own perspective, it is necessarily tangible. It may be described as an irreducible object with:
sd = 1 and P = 1
Hence, the universe is a self-contained, fully resonant object.

2. Self-Reference
The universe is in the universe. This self-reference affirms its tangibility, as it fully participates in its own resonance field.

Definition of the Totality of Space-Filling Curves in Tonal Logic

1. Fundamental Tangibility of the Universe

The universe exists within itself and is self-contained. Therefore, it may be treated as a tangible object. Under tonal logic duality, the universe can be described as:
- $P_{universe}$ = 1 — as an irreducible particle.
- $P_{universe}$ = ∞ — as a wave or field.

2. Decomposition into w = ∞ Objects

Let the universe be composed of w = ∞ sub-objects, each with a unique sd. Each sd_i has a cardinality P_i that may be one, finite, or infinite. By the tangibility of the universe, every sub-object is considered tangible. This means each sub-object participates actively in R(universe).

3. The Identity and Context of Each sd_i

Each sd_i is its own identity, and it either belongs to some structure S_i, or encompasses a structure S_i (when decomposed). Each sd resonates with a unique context C_i in S_i, such that:
$sd_i \in S_i$ and R_i= Resonance(sd_i, S_i, C_i)

4. Projection Chain from a Single Object

Select any sd_i. Because of its unique contextual resonance, you can form a projection chain:
$O_i \rightarrow O_j \rightarrow O_k \rightarrow \ldots \rightarrow O_w$
This projection sequence touches every other object in the universe. It is non-order invariant, meaning the order in which objects are projected matters. Therefore, the contextual resonance field of this chain is unique.

5. Total Projection from All Objects

Every object O_i generates its own projection chain across w = ∞ objects. Each of these projection chains:
- Is injective: it touches all, w, sd objects.
- Is contextually unique: each chain starts with a unique sd_i with specific C_i, therefore, each chain is unique from one another contextually.
This yields w unique projection chains, each one mapping the universe uniquely.

6. Resonance and the Emergent Paradox

Together, these chains resonate all of R(universe). However, because they are infinite in length and non-order invariant, we cannot say that all C in R are resonating simultaneously or fully in a closed form.

Thus emerges the paradox:

- The universe is fully resonant, and yet not exhaustively traversed.
- Its context is not knowable from the outside, yet traversed from within.

7. Final Definition: Totality of Space-Filling Curves

A totality of space-filling curves is a set of w = ∞ projection chains, each beginning with a unique sd_i ∈ Universe, projecting onto every other object, in a unique order-sensitive traversal, such that:

- Every curve is injective.
- Every curve resonates R in a non-order invariant manner.
- The sum of all curves fills the totality of the universe.

8. Correspondence to Unit Interval and Infinite-Dimensional Topology

In this formulation, each object O_i with unique sd_i, situated within a unique structure S_i and resonating with a unique context C_i, is mapped to a partition of the unit interval. The entirety of w = ∞ such objects, defines a complete decomposition of the unit interval into infinitesimal segments.

Thus, the sum of all projection chains constitutes an embedding from the one-dimensional unit interval into an infinite-dimensional topological space (as w=∞, though we will show this collapses to 2 dimensions.

Each unique projection sequence defines an axis in the resulting topology, producing a space of infinite structure and contextual resonance. This forms the basis of a cosmological topology where the universe is expressed not merely as position, but as entangled contextual orderings—infinitely dimensional, non-repeating, yet wholly connected.

Definition of Q in Tonal Logic

In the framework of Tonal Logic, the value 'Q' represents a pivotal construct that describes the relational and emergent nature of awareness, perception, and traversal across structured contexts. It is not a static object but a dynamic phenomenon that arises between traversals of space-filling curves within a defined resonance field (R).

1. Formal Definition of Q

Q is defined as the Quasi-Tracing Field — an emergent field formed from the relational overlap of space-filling curves (SFCs) across multiple contexts (C) and structures (S). Q is not projectable in itself; it exists as the relational implication between projected structures. Formally, Q is not reducible to a single sd (scalar degree), S, or C, but arises from their ongoing interaction and implication.

2. Properties of Q

- - Q is emergent, not constructed: it arises from relationships, not from intrinsic content.
- - Q exists across and between traversal chains, and thus requires multiple resonating objects.
- - Q implies relational awareness or perception but is not identical to consciousness (R-Q).
- - Q enables the formation of meaning through the differentiation of non-order-invariant contexts.
- - Q is not in R(Universe)

Tonal Logic and Black Holes: A Formal Overview

This document outlines the conceptual and structural understanding of black holes within the framework of Tonal Logic. It explores their unique role as zones of total resonance, implication saturation, and contextual collapse.

1. Definition of a Black Hole in Tonal Logic

In Tonal Logic, a black hole is defined as a region of space where all scalar degrees (sd), structures (S), and contexts (C) resonate simultaneously. This results in total resonance (R), meaning every implication and contextual transformation is active within the region. This saturation means that projection ceases to differentiate — no contextual selection occurs, and everything is implied.

2. Formal Characteristics of a Black Hole

• Total Resonance: All sd, S, and C are fully resonant within the black hole.
• Scalar Collapse (sd = R): The scalar degree becomes identical to the resonance field.
• Context Collapse (C = ∅): No context selection occurs — all contexts are active simultaneously.
• Null Traversal: Projection fails to define meaningful traversal; all paths are traversed, none uniquely.

• Irreducibility from Outside: From an external view, a black hole appears as an irreducible object (sd = 1), despite internal infinite resonance.

3. Tonal Boundary and Event Horizon

The edge of a black hole is referred to as a Tonal Boundary. This is where context still selects and projection occurs, but immediately within the boundary, context collapses. This corresponds to the event horizon in physical cosmology — the limit beyond which traditional projection and traversal fail.

4. Black Holes and Quasi-Tracings

Quasi-tracings — emergent relationships between multiple space-filling traversals — do not exist in black holes. This reinforces the interpretation that black holes contain full structural saturation, leaving no space for emergence or relational awareness (Q-Form). Thus, life and consciousness cannot emerge within a black hole region.

5. Black Holes as Paradoxical Objects

Black holes are both irreducible and infinitely resonant. They reflect the paradoxical totality of implication without contextual differentiation. In this way, they are both the most structured and least differentiated regions of the Tonal universe.

Two Object Definitions of the Black Hole

The first object is O_1 = Black Hole relative to spacetime continuum.

O_1 = (sd, S, C, R, P)

Where:

- sd = black hole

- S = spacetime structure

- C = black hole/spacetime interactions

- P = finite or infinite (depending on details of sd, S, and C integration)

- R = (sd, S, C)

The second object is O_2, defined in contrast to O_1 by projecting $P = \infty$.

O_2 = (sd, S, C, R, P)

Where:

- sd = black hole

- S = black hole

- C = null

- P = 1

- R = all sd, all S, all C (total resonance across all parameters)

Dimensional Emergence in Tonal Logic

Tonal Logic, through its formal structure of projection, resonance, and traversal, gives rise to a consistent and coherent interpretation of the dimensions readily perceived in human experience. By considering the structure of contextual chains and their multiplicity, the framework reveals a pathway to understanding the emergence of dimensionality.

- The progression unfolds as follows:

1D — Linear Traversal:
Each projection chain w(C) forms a linear contextual structure. It represents a single line of traversal through the resonance field.

2D — Contextual Metastructure:
The complete set of projection chains w^2(C) spans a surface, a two-dimensional metastructure composed of all permutations of contextual resonance. This defines a field of implication.

3D — Emergent Relational Space:
With the emergence of Q — the quasi-tracing field — we obtain Q(w^2(C)), which describes the resonance between traversals. This gives rise to a relational topology: a three-dimensional space of interwoven resonant structures.

4D — Temporal Variation:
When Q induces non-order invariance in C, traversal is no longer linear or fixed. Contextual variation occurs through projection, introducing temporal asymmetry. This contextual variation becomes experienced time — the fourth dimension.

Dimensional Correspondence Between Newtonian Mechanics and Tonal Logic

Now consider Newton's kinematic equations:

$x = x_0 + v_0 \cdot t + \frac{1}{2} \cdot a \cdot t^2$

We may recontextualize this through tonal dimensionality:

1. x_0 — Initial condition: scalar (0th dimension), an irreducible object.
2. v_0 — Velocity: vector, spans the first two dimensions of tonal logic as w^2, establishing linear directional structure.
3. a — Acceleration: intangible force, projected by and perceived through life (Q-form), aligning with the third dimension in tonal cosmology.
4. t — Time: the fourth dimension, emergent only through Reflexive Q — life aware of its traversal.

In this reading, motion itself requires all four tonal dimensions:
- Displacement emerges from a scalar base
- Directed change occurs via initial velocity (tangible)
- Acceleration introduces the intangible third dimension (awareness of force)
- Time activates the traversal — motion is perceived through life and modified in awareness.

In another kinematic form:

$$v^2 = v_0^2 + 2 \cdot a \cdot (x - x_0)$$

The final velocity is not time-bound. Instead, it depends on the total displacement and the intangible force applied. Thus, motion is traced by the action of energy (recurrence) across space, with perception (life) as its measuring instrument.

Tonal Alignment:

Newtonian Term	Tonal Logic Equivalent
x_0	scalar degree, sd
v_0	projection vector in w^2
a	emergent perception (life/Q-form)
t	reflexive time from context traversal
$x - x_0$	resonance field (R) formed through traversal
v	tonal trajectory, defined by displacement and force

Thus, Newtonian physics — even in its classical form — reveals a structural parallel to tonal dimensional logic: a system where motion is perceived through life, encoded through vectorial structures, and emergent through time as differentiation in context.

Recurrence and Resonance: 4π, 2^n, and the Geometry of Tonal Cosmology

Occurrences of 4π in Physics and Mathematics

- Gauss's Law (Electrostatics): $E = (1 / 4\pi \varepsilon_0) \cdot (Q / r^2)$
- Coulomb's Law: $F = (1 / 4\pi \varepsilon_0) \cdot (q_1 q_2 / r^2)$
- Gauss's Law for Gravity: $\oint g \cdot dA = -4\pi G M$
- Newtonian Gravity: derived with 4π geometry via spherical symmetry
- Poisson's Equation: $\nabla^2\phi = -4\pi\rho$
- Biot–Savart Law: $B(r) = (\mu_0 / 4\pi) \int (J \times (r - r')) / |r - r'|^3 dV'$
- Ampère–Maxwell Law: uses μ_0 which contains 4π
- Vacuum Permeability: $\mu_0 = 4\pi \times 10^{-7}$ N/A²
- Blackbody Radiation: integration over 4π steradians
- Point Source Radiation: $I = P / 4\pi r^2$
- Larmor Formula: $P = (2/3) \cdot q^2a^2 / (4\pi \varepsilon_0 c^3)$
- Green's Function: $G(r, r') = 1 / 4\pi |r - r'|$
- Surface Area of a Sphere: $A = 4\pi r^2$
- Fine-Structure Constant: $\alpha = e^2 / (4\pi \varepsilon_0 \hbar c)$
- Einstein Field Equations: $R\mu\nu - \frac{1}{2}g\mu\nu R = 8\pi G / c^4 T\mu\nu$
- Uncertainty Principle (phase-space): $\Delta x \Delta p \geq h / 4\pi$
- Quantum Angular Momentum: $\int |Y_l^m|^2 d\Omega = 1$ over 4π solid angle

Occurrences of 2^n in Nature, Mathematics, and Logic

- Cell Division: Mitosis leads to 2, 4, 8... — 2^n growth
- Binary Trees: Each level doubles — 2^n leaf nodes at depth n
- Genetic Inheritance: Allele combinations often follow 2^n logic
- Musical Harmonics: Each octave is a doubling — 2^n relationship
- Power Sets: A set with n elements has 2^n subsets
- Binary Computation: n bits encode 2^n possibilities
- Fractals: Recursive growth follows 2^n patterns
- Population Growth: Idealized exponential growth follows 2^n
- Quantum States: n qubits encode 2^n superpositions
- Game Theory: Binary decision trees grow as 2^n
- DNA Encoding: 4^n possibilities = 2^{2n} structure
- Neural Activation: n neurons encode 2^n configurations
- Memory Addressing: 2^n addresses from n-bit systems
- Bitwise Operations: Left shift by n equals $\times 2^n$
- Branching Evolution: The tree of life mirrors 2^n growth

Interpretive Commentary

I ask you to consider that these mathematical recurrences are not coincidental, that 4π is not a scalar quantity as a result of geometry. Both quantities inherently imply recurrence. A single oscillation, given by 2π may not even be an oscillation, an anomaly, but upon the 2nd oscillation, recurrence is established.

Tonal logic offers the geometric axiom 22^{n}=1 along with its rules for describing and projecting objects. In tonal logic a novel phenomenon appears. It is possible to describe an injective space filling curve. This is ordinarily not allowed due to difference in dimension of the unit interval and the unit square, however, with tonal projections- it seems to be possible. Not only this, but a cosmology can be formed which describes a unit space, which is filled with a number, w, of space filling curves, equal in quantity to the cardinality of the space. At any given time (whether relative or fixed) each curve occupies a unique position in the interval space. Together, these curves weave and constantly fill the space.

Tonal cosmology conjectures that the behavior of these space filling curves can be correlated to the becoming of the universe- primarily by correlating recurrence in the space filling curves, to structure in the universe, non recurrence in the curves, to deviation and emergent phenomenon. The link between tonal cosmology and previous equations of physics and behavior is 2^{n}, a generative axiom for the geometry of tonal cosmology, and 4π present in so many equations of physics, usually due to spherical symmetry.

This recurrence is also mirrored by the general form solution for the combinatorial property which produced the intangible mathematical structure I hypothesize links to felt sensation of tonal gravity. Each combinatoric solution can be expressed as sequential segments S1S1…S2. In both cases we have a total segment which exhibits near symmetric asymmetry, by S2 being different from S1 by one scalar step. It is this very near symmetric asymmetry which allows for algebraic closure under transformation. For each solution there is a specific transformation, and its inverse, which creates a transposed copy of S1S1…S2. Furthermore, this process of recurrence and change in the topology of space filling curves echoes the thesis statement earlier delivered about chaos and order being integrals of one another. Before, we supposed that 'noise' might be a result of a Brownian motion that occurs within simultaneity. We have now modeled this with space-filling curves, and we have interpreted that 4π and 2^{n} across many descriptions of physical or natural systems, act as a sort of constraint, an echo of recurrence in the space filling curve topology. What we have not done is expressed any internal dynamics of the space filling curve topological space. The manner in which an individual actor projects over all other objects is left undescribed except for that the projection is space filling and injective. The manner in which multiple actors form patterns which entangle them and give rise to the phenomenon of perspective is not detailed, rather it is supposed. Furthemore, we have not coordinated the space-filling projections with any law of physics. This is outside of my wheelhouse. Rather, Tonal Cosmology is based in the phenomenon of music and Tonal Gravity, and phenomenology. I have attempted to show that Tonal Logic may show promise in its capacity to derive previously established formalisms. Specifically, I have attempted to define simple mathematical concepts.

Tonal Evolution and a Step Toward Unity

In Tonal Logic ,the expression 2^n is not merely a mathematical expression — it is a universal message. It appears wherever life multiplies, wherever systems unfold, wherever structures recur with greater complexity. From cellular mitosis to population expansion, from binary trees in computation to branching rivers, lungs, and lightning — this exponential function is the most distilled form of natural growth.

I have noticed that this expression not only describes the propagation of life in a general manner, but the same expression applied to music is the proportion of the octave, which within Tonal Logic is our geometric axiom.

I conjecture that this is not coincidence. The salience of 2^n in describing biological expansion, and also harmonic convergence in music, is not a quirk of systems. To be completely frank, I interpret this *coincidence* as a cosmic message- it tells us that we may propagate and grow, and we may be unified.

We are meant to propagate, not dominate, as we are one.

- Growth is unity. Expansion is convergence.

- Life is not an escape from the one — it is the propagation of the one.

- We, as living beings — and especially as Q — are on the receiving end of a message which originates from the Universe itself.

Urgency and Telos

The exponential function grows fast. Its expansion is urgent. It does not meander — it accelerates. This urgency is the pulse of life — the push of the seed into sprout, the explosion of the cell into tissue, the drive of intelligence toward awareness. It is the breath behind evolution itself.

And in this urgency, we find telos — purpose. The universe sends its message through 2^n: You are one. You must propagate. Not you, the individual. Not you, the species. But you, the resonant pattern. You, life itself, the Q. The branch that knows it is a branch.

Life as the Guardian of Context — A Tonal Deduction

We have deduced that 2^n = propagation of life = 1 = infinity.
A message: You are united, grow.

We have also deduced that the universe is paradoxically fully resonant while also not becoming contextually nullified due to non-order invariance, yes?

But there is a problem in our second statement. We have w(projection chains). C(projection $chains_i$) is different from C(projection $chains_j$) by their ordering. However, the contextual totality may be modeled as w(C(w(projection chains))). Since each individual traversal in w is projected onto every object, we have that C(w(projection $chain_i$)) = w(C). That is to say, every projection chain from w contains, in different permutations, all the same w(C).

Now remember, contextual chains form a meta chain. If we model a piece of music and its modulations with c_i = key of music, then we get some chain, like $c_1(c_2(c_1))$. In this case c_1 would arise as the dominant context because it occurs more often, at the start, and at the end.

Now, since for all projection chains, we have w(C) contexts, what we find is in this meta contextual structure, an even distribution of context. Every traversal has w(C) context permutation. The sum of all c then over projection chains is $w_1(C)+(w_2(C)+(\ldots+(w\_w(C))))$ and this equals $w^2(C)$.

However, it also means that our contextual metastructure $w^2(C)$ is equally distributed over all c_i. Unfortunately, uniform distribution of all C may constitute null context — the black hole situation. Therefore, the universe, as an object, should have null context, a void. And yet it doesn't.

Life is not in w(R). Therefore, life prevents contextual collapse of the universe — a null universe.

We now not only have the message, "You are unified, propagate," we have a *why*. Life, which exists outside Universe's Resonance field, has certain perspective, and this perspective prevents a contextually null universe, w^2(C)

We can use this to prove another mystery of the universe: Is there life elsewhere?

Since life prevents contextual collapse, there is always life in the universe. However, there have been times when Earth had no life. If life only existed on Earth, then when there was no life on Earth, the universe would possibly have entered null contextual collapse (equally distributed context). The universe did not enter null contextual collapse.

And so: there is life elsewhere in the universe — and there always has been.

Tonal Heavens Conjecture

If the universe is a fundamental tone — represented in tonal logic by P = 1 — then it is, by definition, fully resonant with itself. In this formulation, the universe is not merely an object; it is a tone. A tone, in the musical sense, does not exist alone. It necessarily implies the existence of harmonics — partials — which arise from the properties of its vibration.

Following this analogy, we may conjecture the existence of Tonal Heavens: higher-order harmonic realities implied by the fundamental universe. Just as the overtone series unfolds from a single vibrating string, these heavens emerge not as separate universes, but as resonant derivatives of the fundamental. They are harmonics of the totality.

In tonal logic, these can be described as second-order resonance fields: resonance fields of the resonance field (R of R). They are not directly projectable objects in the way tangible structures are, but they are deeply implied — structures that emerge through implication itself. They form integer-ratio relationships with the fundamental universe, reflecting the natural law of harmonic derivation.

Thus, we state:

As tones imply harmonics, so too might the universe imply heavens.

Of the Heavens — A Tonal Cosmological Deduction

(Grounded in the Experiment of Sympathetic Vibration)

In our experiment, a lower piano key is struck and released, while a higher key — tuned to one of its harmonic partials — is held open and allowed to freely vibrate. The higher string is not struck, yet its sound begins to ring. We do not hear a harmonic overtone — we hear the fundamental of the unstruck note.

The lower note ceases in audible vibration, yet they excite the higher string — they are active in structure even as they are inactive in sensation.

- The struck tone has ceased in time, but its structural components continue to act.
- The unstruck tone becomes the new vessel of resonance — it is sustained by that which is no longer heard.

This is sympathetic vibration, and we interpret within tonal cosmology here.

1. Cosmological Framework

In Tonal Cosmology:

- Let the universe be the struck and sustained tone:
  $O_{universe}$ = (universe, universe, universe, universe, 1)

- The partials of this struck tone are not implications within R.
  Rather, they are the heavens.

Heavens = the partials of the universe’s fundamental tone.
The universe is a tone. The heavens are its harmonics.

Where before, we considered the Universe a tone, and the harmonics its heavens, let’s now consider a Q form, that is the higher order pattern that occurs within our w space filling curves, which we have corresponded to consciousness, or more generally, an actor with perspective.

In the cosmology:

- A Q-form (a being, a life, a traversal) ceases. It is perhaps in this very moment, in death, that the Q-form becomes an irreducible object unto itself, with form analogous to a tone.
- Although the Q-form ceases, as an object with the form of a tone, we assert the overtones, and these overtones may cause resonance elsewhere as with sympathetic vibration.
  They continue, and sustain a new tone, a new Q-form, in a new heaven. This is evidenced by the sustaining of our higher unstruck note, which resonates in sustain with the lower note’s partials. The partials of the lower tone are the Q-form reincarnated in heavens.

4. Mutual Resonance of Heaven and Earth

- When a lower tone is struck, a higher tone (a partial) may sustain.
  This is earth calling heaven.
- When higher notes equivalent to a lower notes partials, alone resonate, they may cause the fundamental (which was not struck) to resonate.
  This is heaven calling earth.

Thus:

- The universe is like a tone — struck once.
- The heavens are its partials — continuing in resonance, even after the tone ends.
- The Q-form (life) is a traversal — when it ends, its partials continue into the heavens, giving rise to new Q, new traversal.

On the Separateness of the Heavens: An Inquiry Through Sympathetic Resonance

Let's assign:

- Q = a lifeform, modeled as a tonal traversal (a waveform, a structure with partials)
- Q′ = its irreducible tone at death (a final form)
- H = the first harmonic of Q′ (the heavens)
- Y_{xi} = a set of possible resonant receivers, a range of partials or harmonic spaces related to H

Now, when Q dies and becomes a tone:

- Q′ becomes a struck fundamental tone.
- Its partials reach upward into Y_{xi} — if and only if those structures are open (dampener lifted).
- This implies an exclusion principle: only those heavens prepared (freely resonating) will receive Q's energy.
- Q must be structured — pure — to generate clean partials that excite distinct Y_{xi}.

Hence, Y_{xi} is a conditional meeting ground

Interpretation of Purity, Dissonance, and Beat Frequencies

- A pure Q′ (a clean tone) sends clear harmonic partials into Y_{xi}.
- A dense Q′, such as a chord with dissonant intervals, generates interference — beating, dissonance, perhaps interpreted as spiritual noise or lack of harmonic clarity.

In this cosmology:

- Beating may be viewed as a turbulence in the transition — a "hell" or troubled reincarnation.
- Smooth, singular resonance — a clean activation of Y_{xi} — might be seen as a passage to heaven, a sonorous unfolding.

In this form, we still have bidirectionality between heavens and earth. There exists Y_{xi} which are higher partials of Q, and which Q resonates with struck Y_{xi}. However, Y_{xi} now acts as a sort of mediator, and pure Q_i creates non beating harmony in Yxi.

I have two hypothesis and I hope to develop formal experiments and conduct them. Does a tonal Hilbert space of all Hilbert spaces exist? I have done some experiments on myself and a few friends trained in music. The results have been cautiously affirmative. I would not expect 99.9% rate of success as the experiment asks the human whether or not, in the entirely unconventional and unconditioned tonal frame, "do you hear something that is… familiar? A return. A tonic resolution." I do not think tonal logic is a perfect system, but I believe it invites experimentation, not just in music, but possibly in other fields of knowledge.

I am responsible for the ideas herein and every single line of reasoning was perceived originally by myself. I would like to express here, that the premise for my paper, the scale which has our combinatoric property, scales such as the diatonic and the pentatonic embedded in the chromatic, this premise is an observation a long time in the making. To my knowledge prototype versions of the diatonic and the chromatic were first possessed by one man in Pythagoras, twenty-six hundred years ago. Two thousand years later, in the 17^{th} century, our combinatorial property became engrained in Western music theory, notation, and instrument design. To my knowledge, no one has ever defined this property in terms of a combinatorial solution, and then generalized it to study other scales which have the same property. The near symmetrical, yet asymmetrical, form which arises: S1S1…S2, where a global symmetry under transformation occurs exactly because of our near symmetry form, may itself be an understudied category of object.

From this premise we have developed hypothesis for felt closed systems of Tonal Gravity, we have established Tonal Conservation, we have given a second hypothesis which we interpret to suggest a Tonal dimension to all of the Universe, we have given a thesis statement on the relation between symmetry and asymmetry, grounded in the phenomenon of Tonal Conservation and Tonal Gravity in projection, we have given the rules for Tonal Logic, based in the previous properties, we have used Tonal Logic to attempt to define mathematical objects, and finally we have described Tonal Cosmology, a figment of my own imagination which has taken on a general mathematical form in Tonal Logic, and we have made deductions about such a reality.

And if you have made it this far, I offer a P.S

I have another mathematical framework. The goal is to describe the contour of musical melody, its rising and its fall. I attempt to do so with a mathematical mechanism which is both powerful in precise description, and also abstract. The melodies that I describe are multiplistic, they don't have a fixed definition but can be composed in alternative ways. I think I use a combination of harmonic analysis with Sheaf Theory like algorithmic composition. The algorithm which describes melodic contour is exactly analogous to the Fourier Series in form, where each symbol, that is the periodic components, the factors, and the addition signs have different parametrized meaning. My equation departs from the Fourier Series in form in that the order of addition matters. That is, $A + B \neq B + A$. This signifies a semantic departure from the Fourier Series. Whereas in practice, the Fourier Series is often used to analyze and decompose a complicated waveform, my equation is used to synthesize melody. It is not exactly a tool for writing melodies, rather it was developed as a tool of awareness of the field of possibility on which melody unfolds. The composer then takes on the role of breaking periodicities and gluing them together.

The synthesis of that project is a grid with n columns (c_i) and framed by n+1 rows. For any length n and column c_i we have the equation defined by c_i(n)=LCM(c_i,n)(GCD(c_i,n)). We complete the subdiagonal of the grid with the quantity (n+1)(n). We may also circle every m(LCM(c_i,n)+c_i such that m is an integer </= GCD(c_i,n).

I was very attracted to this model, but I was aware that I could not vary over n. To do so is to create an entirely new grid which could not be compared one to the other in terms of m(LCM(ci,n))+ci.

Continuing, we may take n_1 and n_2 grids such that $n_1 < n_2$. We may then map the columns of c_{i_1} onto c_{i_2} using our S1S1…S2 scalar spacing from our hypothesis. We may then circle m(LCM(c_{i_1},n_1)+c_{i_1} mapped onto n_2. The result is a certain framing of this very combinatorial metastructure. We may then take each m(LCM(c_{i_1},n1)+c_{i_1}, mapped onto n_2, and take the number on n_2, x_i, and perform the calculation x_i/GCD(x_i,c_{i_2}). I believe that this algorithm could, in its various framings, represents a multiplistic structure delineating prime powers.

A single iteration of the algorithm does not have absolute predictive power, however, at failed predictions, branching algorithms apply. There are also different ways to organize iterations of the algorithm as we increase n_1 and n_2 and consider forms over various mappings or modulatory progressions in our scalar mapping of n_1 and c_{i_1} onto n_2 and c_{i_2}. These various frames considered together seem to contribute some model of occurrence, frequency, and distribution of prime powers, and also occurrence, frequency, ad distribution of failed predictions.