Waves, Velocity Addition and Doppler Effect in Light of EPR's Completeness Condition

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Abstract

It is a standard practice to derive velocity addition rules for point particles from Galilean and Lorentz transformations in point (classical) mechanics, and to apply such rules to wave velocities for explaining Doppler effect. However, in such standard practice, it is never shown whether the equation for wave propagation actually transforms in a way such that the velocity addition rules get manifested through the equation itself. We address this gap in the literature as follows. We claim that the *velocity addition for waves*, being the one and only mean to explain the empirically verified Doppler effect, should be considered as an element of physical reality in accord with EPR's completeness condition of a physical theory. Therefore, the 'equation for wave propagation' should manifest such velocity addition so as to be considered as a part of the respective physical theory of waves. We show that such manifestation is possible if and only if wave propagation is modeled with *first order partial differential equations*. From a historical point of view, this work settles the Doppler-Petzval debate which originated from Petzval's demand for an explanation of Doppler effect in terms of differential equations. From the foundational perspective, this work sets the stage for a renewed focus on the mathematical modeling of wave phenomena, especially in the context of various Doppler effects.

Keywords: Wave equation; Velocity addition; Coordinate transformation; EPR Completeness Condition; Doppler effect

1 Introduction

In any general discussion concerning modern physics, the term 'EPR', standing for Einstein-Podolsky-Rosen [1], comes up exclusively in association with quantum mechanics [2–4]. However, Einstein's personal views, as expressed in a separate solo article [5], concerns some general ideologies about the completeness of theories of physics in relation to human experience of the external world, which are not necessarily restricted to quantum physics. Indeed, if we focus on the completeness condition of a physical theory, as was stated by EPR [1], and which we refer to as EPR's Completeness Condition (ECC) henceforth, reads as follows:

"*every element of the physical reality must have a counterpart in the physical theory*",

where 'physical reality' takes the form of 'experiment and measurement' and 'physical theory' is what we express about 'physical reality' through language, generally both in verbal and computational form. Indeed, it has been demonstrated in refs. [6–9] that ECC actually seeks a truthful correspondence between human sense-experience, which is expressed through verbal language (e.g. English), and the respective mathematical

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equations, along with the logical conditions, on which a physical theory is based. It appears that, through ECC, an operational and a logico-linguistic refinement of the foundations is sought with an increasing degree of precision [10, 11]. Importantly, ECC is a reflection of Brouwer's views on human inquiries in science that complements Hilbert's views, on the same, as manifested in his sixth problem regarding axiomatization of physics [6–9]. While Hilbert's sixth problem concerns outer inquiry with the language of physics (i.e. what we can do with the standard axioms and hypotheses of physics), ECC directs the path of investigation inward and brings forth the significance of Brouwer's perspective of inner inquiry (i.e. how we can refine the standard axioms and hypotheses of physics to bring them closer to our experience) [6]. This, in the process, reveals the interplay among logic, intuition and operation. Significance of such a process of inquiry is revealed from some recent findings viz. formal unprovability of first Maxwell's equation due to the refinement of the Coulomb's law [12], undecidability of the charge gap as realized in the context of the oil drop experiment [6], undecidability of the logic of causality in the 'proof' of Poynting's theorem [9], the non-singular structure of gravitational two body interaction [13], etc.

It is with the mindset of refining the theory of waves in connection with human experience, in this work, we revisit the 'equation for wave propagation' in light of ECC to report a noteworthy consequence that can be debriefed as follows. Dependence of wavelength of a wave on the relative motions of source, detector and medium, generically known today as the Doppler effect, is one of the most widely used experimentally verified phenomena and, therefore, such dependence must be considered as an element of physical reality. We may mention that the role of the medium is explicit in the phenomenological formulae for Doppler shifts in frequency [37, 38, 68], except for the case of light propagation in vacuum [69]. Since, the only known way to explain Doppler effect is the use of velocity addition rules derived from Galilean and Lorentz transformations, therefore, the frame dependence of propagation velocity of waves (except light in vacuum) can be considered as an element of physical reality. Then, according to ECC, velocity addition rules must be manifested in the equations which are meant to represent 'wave propagation' and constitute a physical theory of waves. We observe and demonstrate that the presently accepted *equation for wave propagation* **(EWP)**, written in terms of a second order partial differential equation (PDE), does not satisfy such a criterion. Rather, EWP needs to be written in terms of a first order PDE so as to manifest the velocity addition rules and, therefore, to constitute a physical theory of waves in light of ECC. From a historical point of view, our work settles the Doppler-Petzval debate that is founded on Petzval's search for an explanation of Doppler effect based on differential equations. From the foundational perspective, our work raises a serious question about theoretical modeling of waves by considering ECC as a guiding principle to construct physical theories.

The structure of this work can be debriefed as follows. In section (2), we enlist the requisite properties which an EWP must satisfy in light of ECC. In section (3), we analyze the relevant PDEs under Galilean transformations so as to identify the EWPs in light of ECC. We note that our analyses settles the Doppler-Petzval debate by providing a basis for an explanation of the Doppler effect in terms of differential equations. In section (4), we analyze the relevant PDEs under Lorentz transformations so as to identify the EWPs in light of ECC. In section (5), we conclude with some remarks regarding our analyses.

2 Properties of EWP in light of ECC

Generally we explain Doppler effect by adopting the velocity addition rules, derived from Galilean and Lorentz transformations for point particles (hence, point/classical mechanics [14]), and applying them to the relation between velocity and wavelength of waves. The practice started with Doppler [15–22], continued through the works of Russell [23], Ballot [24], Rayleigh [25], Voigt [26, 27] and Mach [28–30][1] and continues to foster basic research [32–40]. Today, Doppler effect and its various other consequences have become useful tools of the scientists for analyzing the characteristics of waves due to the effect of motion [16–22] and have resulted in formulas such as those of Doppler-Fizeau effect [41, 42], reverse/inverse Doppler effect [43–63], Vavilov-Cherenkov effect [64] leading to various experimental applications in optics, acoustics and hydrodynamics till date. Such empirical success of Doppler effect seems to satisfy our curious minds enough not to question whether such phenomenological application of velocity addition rules can be explained in terms of EWP in wave mechanics. Indeed a similar concern was raised by Petzval who asked for the

[1] It is worth mentioning that it was Mach who changed the course of scientific queries regarding effects of motion on waves at the most elementary level by bringing in the concepts of Mach cone, Mach number, etc. – for example see ref. [31].

underlying differential equations for the frame dependence of velocity and frequency of waves [17,21,22,65]. Our intent is to search for an answer to this question.

Let us consider that S frame is the preferred frame of rest (e.g. air, water, optical medium, etc.) in which the EWP is constructed. The source is stationary, or at rest, with respect to (w.r.t.) this medium and situated at the origin O of S. The observer/detector is situated at the origin O' of S' frame, which moves with a constant velocity of magnitude v, with respect to the source. We expect that EWP must have the following two properties in light of ECC.

P1: EWP must maintain/retain its form, or remain invariant, under coordinate transformation.

[*Significance:* **P1** ensures that there is a counterpart in the physical theory corresponding to the physical reality that the *same* wave propagation is observed in both S and S' frames.]

P2: EWP must manifest the velocity transformation.

[*Significance:* **P2** ensures that there is a counterpart in the physical theory corresponding to the physical reality that the *same* wave propagation, with different velocities, is observed from S and S' frames.]

3 Analysis of propagation under Galilean transformation

In this section we revisit the velocity transformation of a point particle based on Galilean coordinate transformation, along with suitable diagrammatic representations, and then proceed towards a similar analysis to examine the frame dependence of wave velocity with an aim to identify the EWP.

3.1 Revisiting particle propagation

Let us consider two frames S and S', with origins O and O' respectively, such that the latter is moving w.r.t. the former, along x direction with constant velocity v. The Galilean transformation of coordinates are well known (e.g. see ref. [14]) and given by:

$$x' = x - \sigma v t \quad : \sigma = \pm 1, \tag{1}$$
$$y' = y, \qquad z' = z, \tag{2}$$
$$t' = t. \tag{3}$$

Here, the values of σ have the following significance:

1. "$\sigma = +1$" signifies "S' (or O') is moving along increasing/positive x direction and away from S (or O)". (Figure: 1a)
2. "$\sigma = -1$" signifies "S'(or O') is moving along decreasing/negative x direction and towards S (or O')". (Figure: 1b)

We may clarify a few points as follows which will help when we shall pass on to the scenario of wave propagation. We consider here that the particle is thrown by a source located at O and it is observed by an observer/detector located at O'. We note that in case of $\sigma = -1$, i.e. when O' moves towards O, there must be an initial separation between O and O', say a. Therefore, we must have $x' = x + vt - a$ so that the motion of O' ($x' = 0$) is given by $x = a - vt$ in S frame. Thus, in general we must write $x' = x - \sigma v t + \frac{1}{2}(\sigma - 1)a : \ a > 0$, in place of (1). However, this apparently does not affect the partial derivatives that we are going to consider for the construction of equation for wave propagation. Hence, we continue to ignore this subtlety about the effect of initial configuration on Galilean transformation.

For the motion of a point particle, the velocity transformation can be realized in a straightforward manner as follows:

$$\frac{dx'}{dt'} = \frac{dx}{dt} - \sigma v \quad \Leftrightarrow \quad \eta c' = \eta c - \sigma v \ : \eta c' = \frac{dx'}{dt'}, \ \eta c = \frac{dx}{dt}, \tag{4}$$

where ηc and $\eta c'$ are the velocities of the particle w.r.t. S and S' respectively. Here, "$\eta = +1$" signifies "forward propagating particle"(left to right) and "$\eta = -1$" signifies "backward propagating particle"(right to left). Thus, the velocity transformation can be written as follows:

$$c' = c - \frac{\sigma}{\eta} v. \tag{5}$$

■ When $\eta = +1$ (i.e. forward propagation) and the observer at O' is situated on the right side of O during the course of the phenomenon,

1. for $\sigma = +1$, we have $c' = c - v$. (Figure: 2a)
2. for $\sigma = -1$, we have $c' = c + v$. (Figure: 2b)

(a) $\sigma = 1$: S' moving in the positive x direction.

(b) $\sigma = -1$: S' moving in the negative x direction.

Figure 1: Relative motions of S' w.r.t. S.

■ When $\eta = -1$ (backward propagation) and the observer at O' is situated on the left side of O during the course of the phenomenon,

1. for $\sigma = +1$, we have $-c' = -c - v$. (Figure: 3a)
2. for $\sigma = -1$, we have $-c' = -c + v$. (Figure: 3b)

(a) Both the particle and S' are forward propagating. So the net velocity of the particle as observed from S' is $c - v$.

(b) The particle is forward propagating but S' is backward propagating. So the net velocity of the particle as observed from S' is $c + v$.

Figure 2: Relative motions of S' w.r.t. S with particle with forward propagating particle with S' being on the right side of S for the entire motion

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■ When $\eta = -1$ (backward propagation) and the observer at O' is situated on the left side of O during the course of the phenomenon,

1. for $\sigma = +1$, we have $-c' = -c - v$. (Figure: 3a)
2. for $\sigma = -1$, we have $-c' = -c + v$. (Figure: 3b)

(a) The particle is backward propagating but S' is forward propagating. So the net velocity of the particle as observed from S' is $(-c-v)$.

(b) Both the particle and S' are backward propagating. So the net velocity of the particle as observed from S' is $-c+v$.

Figure 3: Relative motions of S' w.r.t. S with particle with backward propagating particle with S' being on the left side of S for the entire motion

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3.2 Analysis with 1st order PDE

We note that the partial derivatives transform as follows:

$$\frac{\partial}{\partial x} \quad \Leftrightarrow \quad \frac{\partial}{\partial x'}, \tag{6}$$
$$\frac{\partial}{\partial t} \quad \Leftrightarrow \quad -\sigma v\frac{\partial}{\partial x'} + \frac{\partial}{\partial t'}. \tag{7}$$

We write the propagating function as follows:

$$f_\eta(x-\eta ct) : \eta = \pm 1, \tag{8}$$

where

- "$\eta = +1$" signifies the "forward propagation/left to right" (along increasing x direction (Figure: 2)),
- "$\eta = -1$" signifies the "backward propagation/right to left" (along decreasing x direction (Figure: 3)).

Then, we can construct the following equation in S frame:

$$\frac{\partial f_\eta}{\partial x} = -\frac{1}{\eta c}\frac{\partial f_\eta}{\partial t}, \tag{9}$$

from which we can read off equations for "forward propagation" and "backward propagation" for $\eta = +1$ and $\eta = -1$ respectively, i.e., we write the following correspondence.

$$\text{Forward propagation } (\eta = +1): \qquad \frac{\partial f_+}{\partial x} = -\frac{1}{c}\frac{\partial f_+}{\partial t}, \tag{10}$$
$$\text{Backward propagation } (\eta = -1): \qquad \frac{\partial f_-}{\partial x} = +\frac{1}{c}\frac{\partial f_-}{\partial t}. \tag{11}$$

The above two equations can represent EWPs on satisfaction of both **P1** and **P2**, which is indeed the case as we see in what follows. Using (6) and (7) we can write the eq.(9) transformed to S' frame as follows:

$$\frac{\partial f_\eta}{\partial x'} = -\frac{1}{\eta c'}\frac{\partial f_\eta}{\partial t'} \;:\; c' = c - \frac{\sigma}{\eta}v. \tag{12}$$

Also, we may note that the phase of the wave must remain invariant under transformation between S and S' frames, which leads to the fact that the wavelength remains invariant as well, as follows:

$$x - \eta ct = x' - \eta c't' \quad \equiv \quad h\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)(x - \eta ct) = 0 \quad \equiv \quad \lambda = \lambda' \ \text{ for arbitrary } x, t \text{ satisfying } \ x \neq \eta ct, \tag{13}$$

where λ and λ' are the wavelengths of the wave in S and S' frames respectively. We may now calculate the relation between the frequencies of the wave, $\nu = c/\lambda$ and $\nu' = c'/\lambda'$, in S and S' frames respectively, to be given by the following relation:

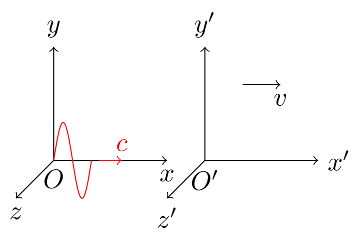
$$\nu' = \nu\left(1 - \frac{\sigma}{\eta}\frac{v}{c}\right). \tag{14}$$

So, from (12) and (14), we have the following cases for different values of η and σ.

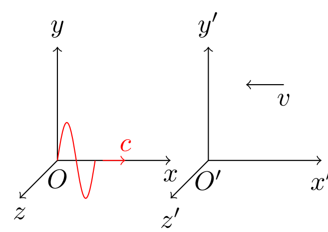
■ When $\eta = +1$ (forward propagation/left to right) and the observer at O' is situated on the right side of O during the course of the phenomenon (otherwise a forward propagating wave, originating from the source at O, won't reach the observer/detector/receiver at O'),

1. for $\sigma = +1$ (i.e. observer moving away from source in the increasing x direction, Figure: 4a), we have
$$\frac{\partial f_+}{\partial x'} = -\frac{1}{(c-v)}\frac{\partial f_+}{\partial t'} \quad \text{and} \quad \nu' = \nu\left(1 - \frac{v}{c}\right), \tag{15}$$

2. for $\sigma = -1$ (i.e. observer moving towards source in the decreasing x direction, Figure: 4b), we have
$$\frac{\partial f_+}{\partial x'} = -\frac{1}{(c+v)}\frac{\partial f_+}{\partial t'} \quad \text{and} \quad \nu' = \nu\left(1 + \frac{v}{c}\right). \tag{16}$$



(a) Both the wave and S' are forward propagating. So the net velocity of the wave as observed from S' is $c - v$.



(b) The wave is forward propagating but S' is backward propagating. So the net velocity of the wave as observed from S' is $c + v$.

Figure 4: Relative motions of S' w.r.t. S with particle with forward propagating wave with S' being on the right side of S for the entire motion

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■ When $\eta = -1$ (backward propagation/right to left) and the observer at O' is situated on the left hand side of O during the course of the phenomenon (otherwise a backward propagating wave, originating from the source at O, won't reach the observer/detector/receiver at O'),

1. for $\sigma = +1$ (i.e. observer moving towards the source in the increasing x direction, Figure: 5a), we have
$$\frac{\partial f_-}{\partial x'} = \frac{1}{(c+v)}\frac{\partial f_-}{\partial t'} \quad \text{and} \quad \nu' = \nu\left(1 + \frac{v}{c}\right), \tag{17}$$

2. for $\sigma = -1$ (i.e. observer moving away from the source in the decreasing x direction, Figure: 5b), we have
$$\frac{\partial f_-}{\partial x'} = \frac{1}{(c-v)}\frac{\partial f_-}{\partial t'} \quad \text{and} \quad \nu' = \nu\left(1 - \frac{v}{c}\right). \tag{18}$$

In view of the above analysis, we may conclude that the 1st order PDEs encoded in (12), satisfy the properties **P1** and **P2** and, therefore, can be considered as EWPs. This mathematically justifies Doppler's intuition of considering frame dependent velocity of waves to explain frequency shifts.

(a) The wave is backward propagating but S' is forward propagating. So the net velocity of the wave as observed from S' is $-c - v$.

(b) Both the wave and S' are backward propagating. So the net velocity of the wave as observed from S' is $-c + v$.

Figure 5: Relative motions of S' w.r.t. S with particle with backward propagating wave with S' being on the left side of S for the entire motion

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3.3 Settling the Doppler-Petzval debate

We believe, in view of our analysis, it is worth devoting a section concerning the historical encounters between Doppler [21] and Petzval [65] which we may call the Doppler-Petzval debate [17, 21, 22, 65]. A very recent concise account on this can be found in ref. [17], from which we use some quotes in the following remarks. As has been noted in ref. [17], Doppler's empirical analysis of frequency shift of light was countered with fierce criticism from Petzval on the ground that Doppler's analysis lacked any mathematical basis as "*all natural phenomena were the manifestations of underlying differential equations*". Instead of providing a reply based on differential equations, Doppler's defense relied on the question "*whether an observed phenomenon must be deemed nonexistent if it cannot be derived from differential equations.*" and on the confidence due to the empirical verification of Doppler effect by Russell [23] and Ballot [24] (also see the references cited in ref. [17]). Doppler's reply was not considered to be satisfactory enough by most of his contemporaries, especially given the socio-scientific authority of Petzval at that time, which resulted in very disappointing consequences for Doppler. We believe that the analysis, which we have presented above, could have been the appropriate reply by Doppler to "Petzval's attack" [17], resulting in an avoidance of the unfortunate consequences. Now, it is interesting to note that, since the present analysis appears for the first time in the literature of science (up to the best of our knowledge), it indicates that the Doppler-Petzval debate has remained unsettled and ignored since its conception in the 19th century. Therefore, we may claim that now the Doppler-Petzval debate is settled as per our analyses.

3.4 Hearing sound backwards (Rayleigh)

Now, let us put some focus on the following particular fact. On p.154 of ref. [25], Rayleigh explained hearing sound backwards as follows: "*Since the alteration of pitch is constant, a musical performance would still be heard in tune, although in the second case, when c and v are nearly equal, the fall in pitch would be so great as to destroy all musical character. If we could suppose v to be greater than c, a sound produced after the motion had begun would never reach the observer, but sounds previously excited would be gradually overtaken and heard in the reverse of the natural order. If $v = 2c$, the observer would hear a musical piece in correct time and tune, but* <u>*backwards*</u>." ["*a*" has been replaced by "*c*" if compared to the original text.] This explanation is now mathematically realizable from eq.(15) and eq.(18) as follows.

■ From (15), for $v > c$ we have

$$\frac{\partial f_+}{\partial x'} = \frac{1}{(v-c)} \frac{\partial f_+}{\partial t'} \quad \text{and} \quad \nu' = -\nu \left(\frac{v}{c} - 1 \right) \tag{19}$$

and for $v = 2c$ we have

$$\frac{\partial f_+}{\partial x'} = \frac{1}{c} \frac{\partial f_+}{\partial t'} \quad \text{and} \quad \nu' = -\nu. \tag{20}$$

Eq.(19) means the forward propagating wave appears to the observer at O', who is situated on the right hand side of O and moving away from O (towards positive x direction), to be propagating backward with velocity of magnitude $(v-c)$ and frequency $\nu(v/c-1)$, where the negative sign of the frequency signifies that the sound is heard backward or in reverse order. Eq.(20) signifies the same except it is a particular case where the frequency is exactly same as the original in magnitude but the negative sign indicates the backward or reverse order of the sound heard by the observer.

■ From (18), for $v > c$ we have

$$\frac{\partial f_-}{\partial x'} = -\frac{1}{(v-c)}\frac{\partial f_-}{\partial t'} \quad \text{and} \quad \nu' = -\nu\left(\frac{v}{c}-1\right) \tag{21}$$

and for $v = 2c$ we have

$$\frac{\partial f_-}{\partial x'} = -\frac{1}{c}\frac{\partial f_-}{\partial t'} \quad \text{and} \quad \nu' = -\nu. \tag{22}$$

Eq.(21) means the backward propagating wave appears to the observer at O', who is situated on the left hand side of O and moving away from O (towards negative x direction), to be propagating forward with velocity of magnitude $(v-c)$ and frequency $\nu(v/c-1)$, where the negative sign of the frequency signifies that the sound is heard backward or reverse order. Eq.(22) signifies the same, except that it is a particular case where the frequency is exactly same as the original in magnitude but the negative sign indicates the backward or reverse order of the sound heard by the observer.

3.4.1 Inverse/Reverse Doppler Effect

The study of inverse/reverse Doppler is a matter of importance both from theoretical and experimental point of view [43–63], which can now be realized directly in terms of PDEs from the above analysis just by considering the case $v > 2c$. To see this let us write $v = 2c + v_e$ where $v_e > 0$ and the subscript 'e' stands for 'excess over $2c$'.

■ For $v = 2c + v_e : v_e > 0$, eq.(19) yields the following:

$$\frac{\partial f_+}{\partial x'} = \frac{1}{(v_e+c)}\frac{\partial f_+}{\partial t'} \quad \text{and} \quad \nu' = -\nu\left(\frac{v_e}{c}+1\right). \tag{23}$$

Eq.(23) means the forward propagating wave appears to the observer at O', who is situated on the right hand side of O and moving away from O (towards positive x direction), to be propagating backward with velocity of magnitude (v_e+c) and frequency $\nu(\frac{v_e}{c}+1)$, where the negative sign of the frequency signifies that the sound is heard backward or in reverse order.

■ For $v = 2c + v_e : v_e > 0$, eq.(21) yields the following:

$$\frac{\partial f_-}{\partial x'} = -\frac{1}{(v_e+c)}\frac{\partial f_-}{\partial t'} \quad \text{and} \quad \nu' = -\nu\left(\frac{v_e}{c}+1\right). \tag{24}$$

Eq.(21) means the backward propagating wave appears to the observer at O', who is situated on the left hand side of O and moving away from O (towards negative x direction), to be propagating forward with velocity of magnitude (v_e+c) and frequency $\nu(\frac{v_e}{c}+1)$, where the negative sign of the frequency signifies that the sound is heard backward or in reverse order.

Therefore, in both the above cases, the observer is moving away from the source, but the frequency is increasing. This is called reverse/inverse Doppler effect, albeit restricted to the situation where the source and the observer are moving away from each other and the source is stationary w.r.t. the medium. An explanation of inverse Doppler effect, for mutually approaching source and observer, requires few other subtleties to be addressed, regarding which some remarks will follows shortly.

Now, there are two points which are worth noting here.

- For $v_e = Nc : N = 1, 2, 3, \cdots$, we have $\nu' = -\nu(N+1)$ i.e. integral multiple of the original frequency is heard by the observer.

- The observed wave is backward only. That is, the reverse/inverse Doppler effect can only be perceived with the wave propagating in the reverse direction compared to the original direction (w.r.t. the source).

So, we may arrive at the following conclusion. If the alternation of direction of the wave due to the relative velocity of the source and the observer is explained as a type of aberration, as has been done in ref. [66], then inverse Doppler effect is always associated with an aberration effect. It will be interesting to see the consequences of such analysis in case of cosmic microwave observations [67].

□ *Mutually approaching source and detector* Up to this point, throughout our analyses we have kept the source to be stationary w.r.t. the medium. However, in case of the mutual approach of the source and the observer, for explaining inverse Doppler effect, albeit within the Galilean framework, it is necessary for the source to be in motion w.r.t. the medium. Additionally the situation raises some basic concerns about coordinate transformations and EWP. We explain the issues as follows.

The phenomenological formula that we generally deal with in such a situation, as available nowadays in standard textbooks (e.g. see ref. [68]), is written as:

$$\begin{aligned} \nu' &= \frac{c+v_r}{c-v_s}\,\nu \\ &= \frac{1+\beta_r}{1-\beta_s}\,\nu \quad \text{such that} \quad \beta_r = \frac{v_r}{c}, \beta_s = \frac{v_s}{c}, \end{aligned} \tag{25}$$

where v_r is the observer velocity w.r.t. the medium towards the source and v_s is the source velocity w.r.t. the medium towards the observer. Now, it is clearly visible from (25) that there is a signature change only when $v_s > c$, whereas there is no effect of v_r. However, our common sense suggests that there should be a dependence on v_r as well (like the case of mutually moving away source and observer). So, the question arises whether (25) is actually an approximation like the following:

$$\begin{aligned} \nu' &= \frac{1}{(1-\beta_r)(1-\beta_s)}\nu \\ &\simeq \frac{1+\beta_r}{1-\beta_s}\nu \quad \text{for} \quad \beta_r \ll 1, \end{aligned} \tag{26}$$

where the dependence on v_r (β_r) is now manifest, through the *exact* equation, in a way that indicates a signature change for $v_r > c$ ($\beta_r > 1$), leading to an inverse Doppler effect, as well. If this is true, then a further question arises whether Galilean coordinate transformation is applicable to handle the situation or we need to think about something else e.g. see refs. [34] for a different perspective on waves and coordinate transformation. Furthermore, refs. [37, 38] are suggestive of the fact that the Doppler formula indeed gets very subtle when the motions of the source, the observer and the medium are involved. Therefore, the case of inverse Doppler effect needs separately devoted attention, especially regarding the coordinate transformation and the structure of the equation for wave propagation itself, which we plan to report elsewhere. In this work, we shall keep our discussion restricted to Galilean transformation and the structure of EWP, as our main motto is to identify the EWP that is consistent with the prevalent standard coordinate transformations.

3.5 Analysis with 2nd order PDE

Now, let us analyze the situation with second order PDE. From (9), we construct the following equation through the standard steps of calculations:

$$\frac{\partial^2 f_\eta}{\partial x^2} = \frac{1}{\eta^2 c^2}\frac{\partial^2 f_\eta}{\partial t^2}, \tag{27}$$

which is generally identified as EWP in the standard literature. However, this equation does not transform to an EWP in S' frame and rather leads to the following result:

$$\frac{\partial^2 f_\eta}{\partial x'^2} = \frac{1}{(\eta^2c^2 - \sigma^2 v^2)}\left(\frac{\partial^2 f_\eta}{\partial t'^2} + 2\sigma v \frac{\partial^2 f_\eta}{\partial x' \partial t'}\right). \tag{28}$$

The above equation can not be an EWP because of its failure to satisfy **P1** and **P2**. It is certainly true on mathematical ground that in S frame, the forward and backward propagating functions and their linear superposition satisfy the second order PDE. However, it fails to retain its form in S' frame and, therefore, incapable of providing the mathematical support to Doppler's intuition through a manifestation of velocity addition.

Digression: In Galilean transformation, the relation $t' = t$ signifies the fact that two synchronized clocks continue to remain synchronized irrespective of whether they are in relative motion or at rest w.r.t. to each other. Einstein explained in ref. [69] that this is not true if light travels with a constant velocity in the vacuum and the two clocks can be synchronized only by communication through light signals. However, recently the relation $t' = t$ has been criticized on certain grounds concerning acoustic Doppler effect and a new set of transformed Galilean transformations have been reported in refs. [34, 35]. While a detailed discussion regarding such criticism and the new transformations is neither our main interest nor within the scope of this article, nevertheless, such work intrigue and compel one to think about the fundamentals of coordinate transformations and the consequent relations with wave phenomena. We hope to elaborate more on such issues elsewhere.

4 Analysis of propagation under Lorentz transformation

In this section we do similar analyses with Lorentz transformations. We revisit the velocity transformation for particles and then analyze propagating functions to decide what we may call EWP.

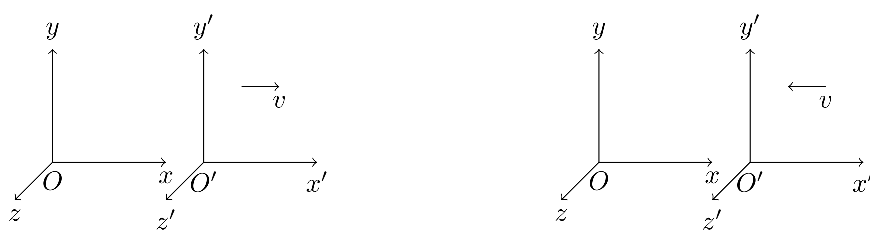
4.1 Revisiting particle propagation

Let us consider two frames S and S', with origins O and O' respectively, such that the latter is moving w.r.t. the former along x direction with constant velocity v. Then the Lorentz transformations that relate these two frames are given as follows [14]:

$$\begin{aligned} x' &= \gamma x - \sigma\gamma v t \quad : \sigma = \pm 1 && (29)\\ y' &= y, \qquad z' = z, && (30)\\ t' &= \gamma t - \sigma\gamma \frac{v}{c^2} x. && (31) \end{aligned}$$

σ can be either +1 or -1 with the following significance.

1. "$\sigma = +1$" signifies "S' (or O') is moving along increasing/positive x direction and away from S (or O)".
2. "$\sigma = -1$" signifies "S'(or O') is moving along decreasing/negative x direction and towards S (or O')".



(a) $\sigma = 1$: S' moving in the positive x direction.

(b) $\sigma = -1$: S' moving in the negative x direction.

Figure 6: Relative motions of S' w.r.t. S.

The velocity transformation for a point particle goes as follows.

$$\frac{dx'}{dt'} = \frac{\frac{dx}{dt} - \sigma v}{1 - \frac{\sigma v}{c^2}\frac{dx}{dt}} \implies \eta V' = \frac{\eta V - \sigma v}{1 - \frac{\sigma v}{c^2}\eta V} \quad : \eta V' = \frac{dx'}{dt'},\ \eta V = \frac{dx}{dt} \tag{32}$$

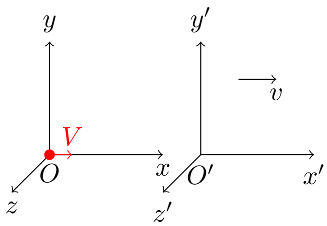
$\eta = +1$ implies "forward propagating particle" and $\eta = -1$ implies "backward propagating particle". Hence, the velocity transformation can be written as

$$V' = \frac{V - \kappa v}{1 - \kappa \frac{v}{c^2} V}, \tag{33}$$

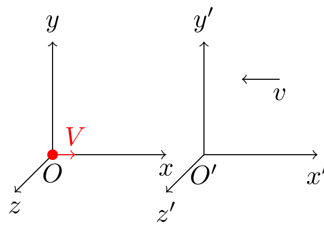
where $\kappa = \frac{\sigma}{\eta} = \sigma\eta$.

■ When $\eta = +1$ (i.e. forward propagation) and the observer at O' is situated at the right side of O during the course of phenomenon,

1. for $\sigma = +1$, we have $V' = \frac{V-v}{1-\frac{vV}{c^2}}$ (Figure: 7a),
2. for $\sigma = -1$, we have $V' = \frac{V+v}{1+\frac{vV}{c^2}}$ (Figure: 7b).



(a) Both the particle and S' are forward propagating. The net velocity of the particle as observed from S' is $\left(\frac{V-v}{1-\frac{vV}{c^2}}\right)$.

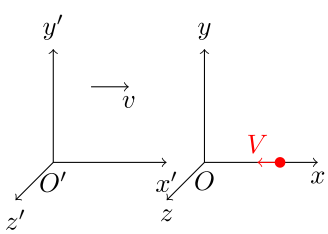


(b) The particle is forward propagating but S' is backward propagating. The net velocity of the particle as observed from S' is $\left(\frac{V+v}{1+\frac{vV}{c^2}}\right)$.

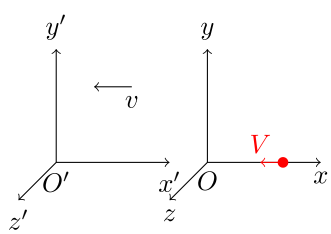
Figure 7: Relative motions of S' w.r.t. S with forward propagating particle with S' being on the right side of S for the entire motion.

■ When $\eta = -1$ (i.e. backward propagation), and the observer at O' is situated at the left side of O during the course of phenomenon,

1. for $\sigma = +1$, we have $V' = \frac{-V-v}{1+\frac{vV}{c^2}}$ (Figure: 8a).
2. for $\sigma = -1$, we have $V' = \frac{-V+v}{1-\frac{vV}{c^2}}$ (Figure: 8b).



(a) The particle is backward propagating but S' is forward propagating. So the net velocity of the particle as observed from S' is $\left(\frac{-V-v}{1+\frac{vV}{c^2}}\right)$.



(b) Both the particle and S' are backward propagating. So the net velocity of the particle as observed from S' is $\left(\frac{-V+v}{1-\frac{vV}{c^2}}\right)$.

Figure 8: Relative motions of S' w.r.t. S with backward propagating particle with S' being on the left side of S for the entire motion.

4.2 Analysis with 1st order PDE

The partial derivatives transform as follows.

$$\frac{\partial}{\partial x} \equiv \gamma \frac{\partial}{\partial x'} - \sigma\gamma \frac{v}{c^2} \frac{\partial}{\partial t'} \tag{34}$$

$$\frac{\partial}{\partial t} \equiv -\sigma\gamma v \frac{\partial}{\partial x'} + \gamma \frac{\partial}{\partial t'} \tag{35}$$

The propagating function can be written as follows.

$$f_\eta(x - \eta V t) : \eta = \pm 1. \tag{36}$$

So the propagation equation can be written as,

$$\frac{\partial f_\eta}{\partial x} = -\frac{1}{\eta V} \frac{\partial f_\eta}{\partial t}, \tag{37}$$

where $\eta = 1$ signifies forward propagation and $\eta = -1$ signifies backward propagation. The propagation velocity is V in S frame. We can write the following correspondence.

$$\text{Forward propagation equation } (\eta = +1): \quad \frac{\partial f_+}{\partial x} = -\frac{1}{V} \frac{\partial f_+}{\partial t}, \tag{38}$$

$$\text{Backward propagation equation } (\eta = -1): \quad \frac{\partial f_-}{\partial x} = +\frac{1}{V} \frac{\partial f_-}{\partial t}. \tag{39}$$

The same wave as viewed in S', using (34) and (35), can be written as,

$$\frac{\partial f_\eta}{\partial x'} = -\left(\frac{1 - \eta\sigma \frac{vV}{c^2}}{\eta V - \sigma v}\right) \frac{\partial f_\eta}{\partial t'}. \tag{40}$$

■ When $\eta = +1$ (i.e., forward propagation or left to right moving) and the observer at O' is situated on the right side of O for the entire course of phenomenon,

1. for $\sigma = +1$ (observer moving away from source in increasing x direction, Figure: 9a),

$$\frac{\partial f_+}{\partial x'} = -\left(\frac{1 - \frac{vV}{c^2}}{V - v}\right) \frac{\partial f_+}{\partial t'} \tag{41}$$

2. for $\sigma = -1$ (observer moving towards source in decreasing x direction, Figure: 9b),

$$\frac{\partial f_+}{\partial x'} = -\left(\frac{1 + \frac{vV}{c^2}}{V + v}\right) \frac{\partial f_+}{\partial t'} \tag{42}$$

■ When $\eta = -1$ (i..e, backward propgation or right to left motion) and the observer at O' is situated in the lef hand side of O during the course of phenomenon,

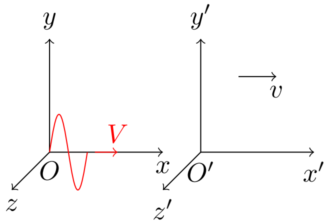
1. for $\sigma = +1$ (observer moving towards the source in the increasing x direction, Figure: 10a),

$$\frac{\partial f_-}{\partial x'} = -\left(\frac{1 + \frac{vV}{c^2}}{-V - v}\right) \frac{\partial f_-}{\partial t'} \tag{43}$$

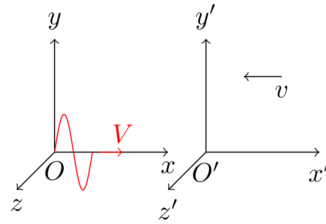
2. for $\sigma = -1$ (observer moving away from the source in the decreasing x direction, Figure: 10b),

$$\frac{\partial f_-}{\partial x'} = -\left(\frac{1 - \frac{vV}{c^2}}{-V + v}\right) \frac{\partial f_-}{\partial t'} \tag{44}$$

It can be seen that when we substitute $V = c$ in the above cases we get back equations for wave traveling with velocity c. Further, from the above simple analyses it becomes evident that L_1 and L_2 are mathematically in tandem with the first order PDEs. So, the above first order PDEs, which we have called propagation equations, can now be termed as EWPs.



(a) Both the wave and S' are forward propagating. So the net velocity of the wave as observed from S' is $\left(\frac{V-v}{1-\frac{vV}{c^2}}\right)$.



(b) The wave is forward propagating but S' is backward propagating. The net velocity of the wave as observed from S' is $\left(\frac{V+v}{1+\frac{vV}{c^2}}\right)$.

Figure 9: Relative motions of S' w.r.t. S with forward propagating wave with S' being on the right side of S for the entire motion

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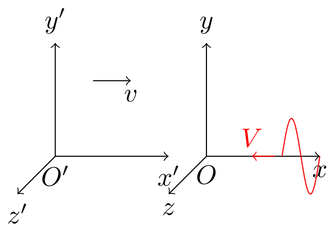
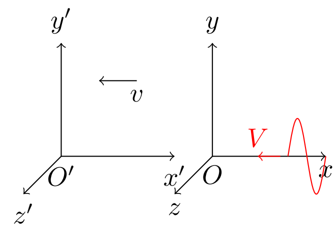


(a) The wave is backward propagating but S' is forward propagating. So the net velocity of the wave as observed from S' is $\left(\frac{-V-v}{1+\frac{vV}{c^2}}\right)$.



(b) Both the wave and S' are backward propagating. So the net velocity of the wave as observed from S' is $\left(\frac{-V+v}{1-\frac{vV}{c^2}}\right)$.

Figure 10: Relative motions of S' w.r.t. S with particle with backward propagating wave with S' being on the left side of S for the entire motion.

4.3 Analysis with 2nd order PDE

The question remains whether similar analyses can be done with second order PDEs. To begin with, from (37), the following second order PDE can be written.

$$\frac{\partial^2 f_\eta}{\partial x^2} - \frac{1}{V^2}\frac{\partial^2 f_\eta}{\partial t^2} = 0. \tag{45}$$

Using (34) and (35),

$$\frac{1-\frac{v^2}{V^2}}{1-\frac{v^2}{c^2}}\frac{\partial^2 f_\eta}{\partial x'^2} - \frac{2\sigma v(\frac{1}{c^2}-\frac{1}{V^2})}{1-\frac{v^2}{c^2}}\frac{\partial^2 f_\eta}{\partial x'\partial t'} + \frac{\gamma^2}{c^2}\left(\frac{v^2}{c^2}-\frac{c^2}{V^2}\right)\frac{\partial^2 f_\eta}{\partial t'^2} = 0 \tag{46}$$

The above equation can not be considered as an EWP because of its failure to **P1** and **P2**. It is certainly true on mathematical ground that in S frame, the forward and backward propagating functions and their linear superposition satisfy the second order PDE. However, it fails to retain its form in S' frame.

5 Conclusion and Outlook

Based on this work we may conclude that wave equations should be modeled with first order partial differential equations so that Doppler's effect, in particular velocity addition for waves, can be explained in terms of equations – a necessity imposed by the EPR's completeness condition (ECC) for a physical theory of waves. Considering this work as the first step of a new line of investigation concerning velocity transformation and the structure of wave equation, we have restricted our work to one spatial dimension and considered the source to be fixed with the medium wherever applicable. Certainly we plan to investigate the scenarios where the medium, the source and the detector all are in motion and also generalize such investigations for three spatial dimensions. Further, from a historical point of view, we may assert that the Doppler-Petzval debate, which has hitherto remained ignored and unresolved for nearly two centuries, is now settled. However, such a settlement is not complete as it comes with the following limitation.

We note that this work is rooted to the following claim. The experimentally observed Doppler shift in frequency is only explained through velocity addition and, therefore, must be manifested through the concerned equation for wave propagation in light of ECC. However, strictly speaking, the frequency shift is an element of physical reality and *not* the velocity addition because the shift in frequency is only observed experimentally and the velocity addition is just a theoretical mean to explain such observation. In case one explains Doppler effect without using the velocity addition rules, then the velocity addition need not be manifested in the equation for wave propagation in light of ECC. Indeed in refs. [34, 35] the Galilean transformation has been modified in such a way that the velocity of sound is independent of the frame of observation, but the modified transformation conforms with the Doppler effect. We plan to explore such a limitation of our work in the near future.

In spite of the above limitation, considering the standard accepted way of explaining Doppler effect conforming with the standard Galilean and Lorentz transformation, it is not hard to foresee the significance of the present work. We can, at least and without making an overstatement, ask the following question. Do Maxwell's equations lead to any first order differential equation which can be considered as the equation for light wave propagation? This question becomes important especially in the context of light propagation in a material medium which acts as the preferred rest frame e.g.

- an optical medium where various types of Doppler effect have been observed,
- cosmic observations where the concept of ether (i.e. a preferred rest frame) is reemerging from various point of views [70–75] and Doppler shift is the key to the inferences drawn from such observations.

We plan to explore such a line of investigation in the near future.

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