New concept of H removes paradoxes of Infinity

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28th of June, 2025

1 Abstract

An alternative framework to zero and infinity can be presented through the new concepts of 0^m and H^m. These bold definitions aim to mathematically represent infinitesimal and infinite values, preserving our intuition while reducing many seemingly unnecessary contradictions encountered across several mathematical domains while dealing with infinity. I define 0^m as some hierarchy of infinitesimally small values, keeping the concept somewhat similar to finite numbers and using approximations (which have proved very useful in other areas of mathematics) as opposed to letting zero continue to represent absolute nothingness (and doing the same for H^m to replace the absurdity of infinity). Some applications include potential improvements to geometric series and supremum definitions, and clarifications to quadratic equation and inverse matrices - etc. Feedback on its implications and my communication are highly welcomed.

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2 Theory and Definitions

2.1 An adjustment of zero

Our current understanding of zero can be described decimally as ‘infinitely many zero digits, followed by a decimal point, followed by infinitely many zero digits’ or ‘...0.0...’. However, it could be that eventually there is a non-zero digit, and that we just don’t notice or care because it doesn’t affect our operations. Since the new and old definitions act similarly in nearly every way, I will continue to use the digit ‘0’ – but note that this now describes the new definition (which I will elaborate on later).

2.2 A new term for infinity

Just like with zero, it may be that the important part of the current definition of infinity is that it is more extreme than whatever number you could possibly be dealing with. This is a key part of my proposed understanding of infinity - from now on known as H to avoid confusion - and I will explain the difference between the current and new definition later.

2.3 A new type of zero

Let 01 be notation for a new type of number, which is similar to 0 in that it is impossibly small in comparison to any finite number. It is described by saying there are ‘infinite’ zero digits beyond the decimal before the first non-zero digit, and so it does largely behave like 0.

$$0^1 = ...0.0...010...$$

Which can be simplified, using new notation, as

$$0^1 = 0.[0]1$$

Where the [0] notates ’roughly’ H zero digits*.

Now, it’s clear that

$$3 \times 0^1 = 0.[0]3$$

And that

$$0^1 + 0^1 = 0.[0]2$$

And so

$$3 \times 0^1 \neq 0^1$$

And yet

$$3 \times 0 = 0$$

So they are not the same concept (despite the confusing notation)

And in fact

$$(0^1)^2 = 0^2 = 0.[0]0[0]1 \neq 0^1$$

And from this, we can see that [0] is a different concept to [0]0[0]. And yet both are similar to our previous understanding of zero in that they are impossibly small compared to finite numbers.

2.4 A new type of infinity

From our definition of 0^1 we can see that

$$\frac{1}{0^1} = \frac{1}{0.[0]1} = 1[0] =: H^1$$

It has always been puzzling that zero was treated as a number but infinity never was, even though zero was just as freaky – but now we can clearly treat H like a number too.

***Note:** I said [0] means roughly H digits as to simplify matters, but we must instead say roughly $\log_{10} H^1$ digits (which I'll show is allowed later through continuity of H^m). And so 0^2 has around twice the number of digits or $\log_{10} H^2$.

We can write H^m or 0^m where m is some finite number.

It's clear how H^m has some of the same properties as H, similar to the shared properties between 0^m and 0.

And now we can better define 0.

2.5 Fleshing out the old zero

The key point of 0^m is that even though it does have some infinitesimal value, no matter how it interacts with any finite number, it keeps its same extreme nature. For example

$$5 \times 0^1 + 3 \approx 3 \approx 3 + 0^3$$

We will now define the new definition of 0 by saying that: for any value $n \times 0^m$ (where n is a finite number), it's '0' is every 0^p such that p is a greater integer than m. And for an expression where each value can be expressed as some $n \times 0^m$, 0 is every 0^p such that p is a greater integer than the maximum value of m.

In this sense, if the expression was

$$3H^1$$

then because this is equal to

$$3 \times 0^{-1}$$

where m equals

$$-1$$

Then because 0^p is a value of 0 for any integer $p > m$, 0 can have the value

$$0^{m+1} = 0^0 = 1$$

Or actually any finite multiple of this. So all finite values are valid solutions for 0 as they does not affect the expression of $3 \times H^1$ no matter how finitely much it is added to it.

And so, as a side note, one is also able to write $3H^1 - H^1 - 2H^1 = 0^{0+}$ (where 0+ means any integer greater than or equal to 0).

And we should also write

$$5 \times 0^1 + = 3 = 3 + 0^3$$

instead of using '≈' which we reserve for a different purpose.

Naturally, the same argument goes for H and what it means to be the new infinity.

2.6 Putting the new old zero into action

This definition for 0 gives a much more intuitive argument as to why $\frac{0}{0}$ is undefined.

$$\frac{0}{0} :\approx \frac{0^2}{0^1} = 0^1 \approx 0$$

Is valid. As is

$$\frac{0}{0} :\approx \frac{2 \times 0^1}{0^1} = 2$$

And

$$\frac{0}{0} :\approx \frac{0^1}{0^2} = H^1 \approx H$$

And it means we never have to deal with the concept of actual genuine complete nothingness, or have to keep coming up with larger and larger infinities.

Note: $:\approx$ means I am temporarily setting 0 to be 0^m for some integer m and it defines this value of m at which we can swap back out every 0^{m+} for 0 again – which will be useful in the next section.

My H and 0 also maintains the fact that and shows how in virtually every context, $\frac{1}{0}$ seems to be $+\infty$ or $-\infty$.

$$\frac{1}{0} :\approx \frac{1}{0^1} = H^1 \approx H$$

Is valid, but so is

$$\frac{1}{0} :\approx \frac{1}{-0^1} = -H^1 \approx -H$$

I will now showcase the use of H^1 and 0^1 in slightly higher level mathematics through some examples.

3 Examples

3.1 Example with geometric sequence partial sums

This is possibly the best simple example.
The partial sums of a geometric sequence are

$$\sum_{k=0}^{n} r^k = \frac{1 - r^{n+1}}{1 - r}$$

As long as $r \neq 1$.
But it seems silly that the formula doesn't work for $r = 1$ so let's fix that.

Since

$$0 :\approx 0^1$$

We have

$$r = 1 = 1 + 0 \approx 1 + 0^1$$

So

$$\frac{1 - r^{n+1}}{1 - r} \approx \frac{1 - (1 + 0^1)^{n+1}}{1 - (1 + 0^1)}$$

$$= \frac{1 - (1^{n+1} + (n+1)0^1 + m0^2 + ...)}{1 - (1 + 0^1)}$$

$$= \frac{1 - (1 + (n+1)0^1 + m0^2 + ...)}{1 - (1 + 0^1)}$$

$$= \frac{-((n+1)0^1 + m0^2 + ...)}{-0^1}$$

$$= (n+1) + m0^1 + ...$$

$$\approx n + 1$$

Which is what we'd expect the partial sum to equate to.

3.2 Example with epsilon

The definition of the supremum is:

Let l be the least upper bound of a set $S \subset R$. Then for every $\varepsilon > 0$ there exists $s \in S$ such that $s > S - \varepsilon$.

I believe it is quite possible that since we're just dealing with classically real numbers in the set, we can just cut to chase and say instead:

Let l be the least upper bound of a set $S \subset R$. Since we're using the set R, let $0 = 0^{1+}$ and so there exists $s \in S$ such that $s \approx S - 0^1$, since clearly 0^1 is less than or equal to the magnitude of any non-zero real number, and this definition lets $s = S$ or $s = S - m0^1$ with m=some finite natural number.

3.3 Example with continuity

Is $f(x) = x$ continuous at x=H? We need to establish this if we want to say that $log_{10}H$ exists like we did earlier.

A function $f : M \to M$ with $M = R$ is continuous at $c \in M$ if for any $\varepsilon > 0$ there exists $\delta > 0$ such that

$$x \in M \text{ and } |x - c| < \delta \implies |f(x) - f(c)| < \varepsilon$$

This section is a little less certain but I believe it can be simplified by saying A function $f : M \to M$ with $M =$ the set of the new reals $= \{0, 0^1, 2 \times 0^1, ..., H^1 0^1 = 1, 2, ..., H^1, H\}$ is continuous at $c \in M$ if there exists $m \in Z$ such that

$$x \in M \text{ and } |x - c| \approx 0^m \implies |f(x) - f(c)| \approx 0^1$$

By this definition, $f : M \to M$ with $f(x) = x$ is continuous over the whole of M as it should be.

Also, $f : M \to M$ with $f(x) = \frac{1}{x}$ is continuous at $x = c \in M, x \neq 0$ as it should be.

And this includes $c = 0^1$ by letting $0 = 0^{3+}$, and it includes $c = H$.

3.4 Example with limits tending to infinity

I haven't had the time to finish this section but I'd be shocked if it wasn't the case that another one of new 0 and H features is that it simplifies limits and makes all the facts proved about limits seem quite obvious.

3.5 Example with quadratic formula

It is thought that there is only one solution to the equation

$$bx + c = 0$$

But actually, if we allow ourselves to use the H system then we can set

$$bx + c = 0x^2 + bx + c = 0 :\approx 0^1x^2 + bx + c$$

So we can use the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4 \times 0^1 c}}{2 \times 0^1}$$

$$= \frac{-b \pm (b - \frac{2\times 0^1 c}{b} - m0^2 - ...)}{2 \times 0^1}$$

$$= \frac{-b \pm (b - \frac{2\times 0^1 c}{b} - m0^2 - ...)}{2 \times 0^1}$$

$$= \frac{-\frac{2\times 0^1 c}{b} - m0^2 - ...}{2 \times 0^1}, \frac{-2b + \frac{2\times 0^1 c}{b} + m0^2 + ...}{2 \times 0^1}$$

$$= -\frac{c}{b} - \frac{m0^1}{2} - ..., -bH^1 + \frac{c}{b} + \frac{m0^1}{2} + ...$$

Now let's check that these are in fact solutions:

$$0x^2 + bx + c :\approx 0^1x^2 + bx + c$$

$$= 0^1(-\frac{c}{b} - \frac{m0^1}{2} - ...)^2 + b(-\frac{c}{b} - \frac{m0^1}{2} - ...) + c$$

$$= 0^1(\frac{c^2}{b^2} + \frac{cm0^1}{b} + ...) + (-c - \frac{bm0^1}{2} - ...) + c$$

$$\approx -c + c = 0$$

This is correct though expected since it uses quite a standard value for x. But also

$$0x^2 + bx + c :\approx 0^1x^2 + bx + c$$

$$= 0^1(-bH^1 + \frac{c}{b} + \frac{m0^1}{2} + ...)^2 + b(-bH^1 + \frac{c}{b} + \frac{m0^1}{2} + ...) + c$$

$$= 0^1(b^2H^2 - 2cH^1 + \frac{c^2}{b^2} - \frac{bm}{2} + ...) - b^2H^1 + c + \frac{bm0^1}{2} + ... + c$$

$$= b^2H^1 - 2c + 0^1(\frac{c^2}{b^2} - \frac{bm}{2} + \frac{bm}{2}) + ... - b^2H^1 + c + ... + c$$

$$\approx b^2H^1 - 2c - b^2H^1 + c + c = 0$$

Which is what we wanted.
Therefore, a new solution is produced which involves H.

3.6 Example with inverse matrices

Supposedly, for a matrix, M, if $det(M) = 0$ then M^{-1} is undefined.
This is because if

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Then

$$M^{-1} = \frac{1}{det(M)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

But if we acknowledge that $det(M) = ad - bc :\approx 0^1$ then $\frac{1}{det(M)} \approx H^1$ can be reasoned with.

Now, for M^{-1} to be the inverse of M we need

$$M^{-1}M = \frac{1}{det(M)}M \approx H^1 \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$= H^1 \begin{bmatrix} ad - bc & bd - bd \\ cd - cd & ad - bc \end{bmatrix}$$

$$\approx H^1 \begin{bmatrix} 0^1 & n0^{2+} \\ n0^{2+} & 0^1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & n0^{1+} \\ n0^{1+} & 1 \end{bmatrix}$$

$$\approx \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

So M^{-1} is an inverse with

$$M^{-1} = \frac{1}{det(M)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \frac{1}{0} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \begin{bmatrix} dH & -bH \\ -cH & aH \end{bmatrix}$$

4 Conclusion

I sincerely hope this is interesting enough to hear some feedback upon - and that my poor writing still allows the reader understand the idea. To get the idea across concisely, I have of course not described the details rigorously - passing over negative and complex values for the most part, and altering notation, etc.

There is nothing more rigorous than the theory I have thus presented but I hope the examples of its use and neatness give it some credit.

And I am yet to explore where this fits in with mathematics other than in the examples described, after some feedback of course.

Please do take the time to understand this theory of H and to discuss it with me.

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