A Geometric Interpretation of Lorentz Length Contraction Using an Inscribed Triangle in a Circle

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Abstract

This paper focuses not on deriving the Lorentz length contraction from first principles, but on providing a geometric visualization of this well-known relativistic phenomenon. The aim is to offer an intuitive and accessible interpretation through a simple construction: a right triangle inscribed in a circle. In this model, the observed contraction of a moving rod's length as a function of velocity is illustrated geometrically. While the Lorentz contraction is formally derived via Lorentz transformations, the approach presented here serves as a complementary visual tool that may be particularly useful for educational purposes.

Keywords: special relativity, Lorentz length contraction, relativistic geometry, Pythagorean theorem, proper length, moving rod

1 Introduction

We aim to explore and clarify the fundamental connection between the relativistic formulas and their underlying geometric interpretations. In particular, this study investigates how certain kinematic expressions arising in the theory of special relativity—especially those related to length contraction—can be understood not only algebraically through Lorentz transformations but also via intuitive geometric constructions.

2 Historical Background

According to the theory of special relativity, an object moving at a significant fraction of the speed of light appears to be contracted in length along the direction of motion when observed from a stationary reference frame. More precisely, the measured length of a moving rod is shorter than its proper length, that is, the length measured in the rod's own rest frame [1]. This phenomenon, known as Lorentz contraction, is a direct consequence of the invariance of the speed of light and geometric structure of Minkowski spacetime [2].

By reinterpreting this contraction through a geometric lens, we aim to provide a more intuitive understanding of how velocity affects spatial measurements and show that the contraction formula can be derived from a purely geometric construction without initially invoking the coordinate transformations. Similar visual and conceptual approaches have been explored in the recent literature [3–6].

This concept originated from the FitzGerald–Lorentz contraction hypothesis [7], proposed in the late 19th century as a theoretical mechanism to explain the unexpected null result of the Michelson–Morley experiment [8]. According to this hypothesis, a moving body physically contracts along the direction of motion relative to the supposed luminiferous ether, thereby compensating for any differences in the speed of light that might otherwise be detected in different directions.

Although initially introduced as an ad hoc explanation within the framework of ether theory, this idea later gained a more rigorous foundation through the development of Einstein's theory of special relativity, where length contraction arises naturally as a consequence of the invariance of the speed of light and Lorentz transformations.

3 Geometric Interpretation

Let us consider a circle (Fig. 1), in which a right triangle is inscribed with its hypotenuse ℓ passing through the center of the circle. The hypotenuse ℓ is interpreted as the proper length of the rod, i.e., its length in the rest frame. One of the legs is equal to $\Delta\ell$, which is interpreted as the length of the rod measured in a reference frame in which it is moving with velocity v.

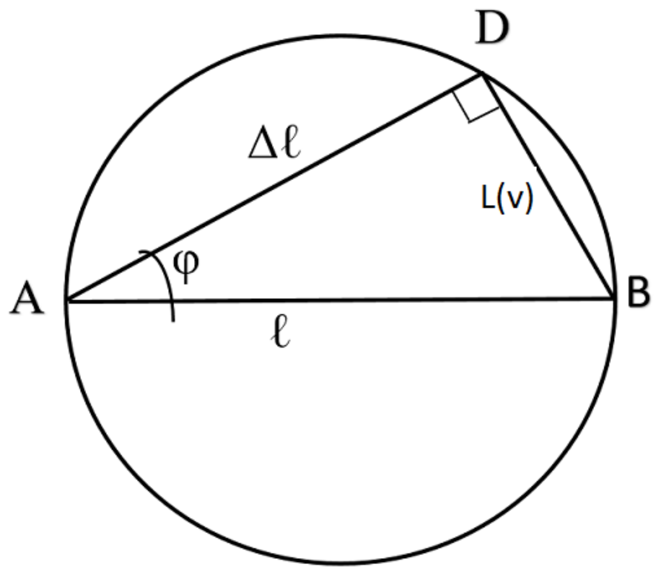

Figure 1. *Triangle representing rest length, contracted length, and angle φ.*

If the hypotenuse ℓ, representing the proper length, is used as a scaling reference in conjunction with the speed of light c, then it becomes possible to express the velocity v as a function of the angle φ in the given triangle.

The other leg, denoted as L(v), is not equal to the velocity v itself, but is proportional to it and has the same physical dimension as length.
From the geometry of the triangle, it follows that: $\Delta\ell = \ell \cdot \cos(\varphi)$, where φ is the angle between the sides ℓ and $\Delta\ell$, geometrically determined by the configuration of the triangle inscribed in the circle.
On the other hand, in special relativity, according to the Lorentz transformations, the length of a moving object is related to its proper length by the expression:

$$\Delta\ell = \ell\sqrt{1-\frac{v^2}{c^2}}$$

By comparing the two expressions, it can be observed that the factor cos(φ) and $\sqrt{1-\frac{v^2}{c^2}}$ play the same role: both represent a proportionality coefficient that relates the observed length $\Delta\ell$ to the proper length ℓ.

Thus, the geometric model can help us visualize the concept of Lorentz length contraction.

4 Mathematical Derivation

As a consistency check, we examined how the cosine of angle φ is related to the magnitude of velocity v.

Physical meaning:

- ℓ — the proper length of the rod (in its rest frame).
- $\Delta\ell$ — the length of the same rod measured in a frame where it is moving with velocity v.
- c — the speed of light.
- v — the relative velocity between the rod's frame and the observer.

We attempt to establish a correspondence between the geometric and physical forms of the expression. According to the trigonometric identity:

$$cos(\varphi) = \sqrt{1 - sin^2(\varphi)}$$

From Figure 1, it follows that:

$$sin(\varphi) = \frac{\mathrm{L(v)}}{\ell}$$

and therefore:

$$\mathrm{L(v)} = \ell \cdot sin(\varphi)$$

If we now turn to a similar triangle inscribed in a circle (see Fig. 2), it can be established that angle is related to the ratio of the rod's velocity v to the speed of light c. Based on the geometry of the triangle, the angle φ can take values in the range of 0°–90°. The velocity v can take values within the range $0 \leq v \leq c$.
If the angle φ is the same in both triangles shown in Figures 1 and 2, the triangles are similar and fully consistent in their geometric structure.

Although the legs and hypotenuse in the triangles of Figures 1 and 2 differ in physical dimensions, they are proportionally equivalent when the angle φ is the same in both cases. By appropriately scaling the quantities v and c in Figure 2, they correspond proportionally to L(v) and ℓ in Figure 1.

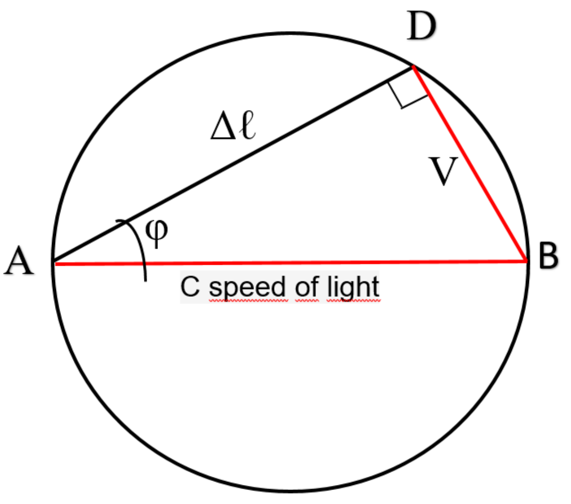

Figure 2. *Same triangle interpreted via velocity components.*

This allows us to express the relationship as follows: $sin(\varphi) = \frac{v}{c}$

and therefore: $L(v)=\ell \cdot \frac{v}{c}$

Substituting this expression into the Pythagorean theorem, we obtain the following relationship:

$$(\Delta\ell)^2 + L(v)^2 = \ell^2$$

$$(\Delta\ell)^2 + \left(\ell \cdot \frac{v}{c}\right)^2 = \ell^2$$

$$\Delta\ell = \ell\sqrt{1 - \frac{v^2}{c^2}}$$

This result corresponds to the well-known formula for the Lorentz length contraction [9], which is consistent with the classical Lorentz contraction formula obtained here through a straightforward geometric construction.

5 Conclusion

In this interpretation, the angle φ characterizes the degree of relativistic contraction of the rod’s proper length ℓ with respect to a stationary reference frame. As φ increased from 0° to 90°, the corresponding observed length $\Delta\ell$ decreased.

In summary, this geometric construction reproduces the standard Lorentz contraction [10] formula in a visually intuitive manner.

This visualization helps convey the concept intuitively, without relying on complex coordinate transformations. [11].

While this visualization is designed to support understanding at the educational level, it may also be of conceptual interest to researchers working in the field of spacetime geometry and special relativity.

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