Time and Possibilities in Quantum Mechanics

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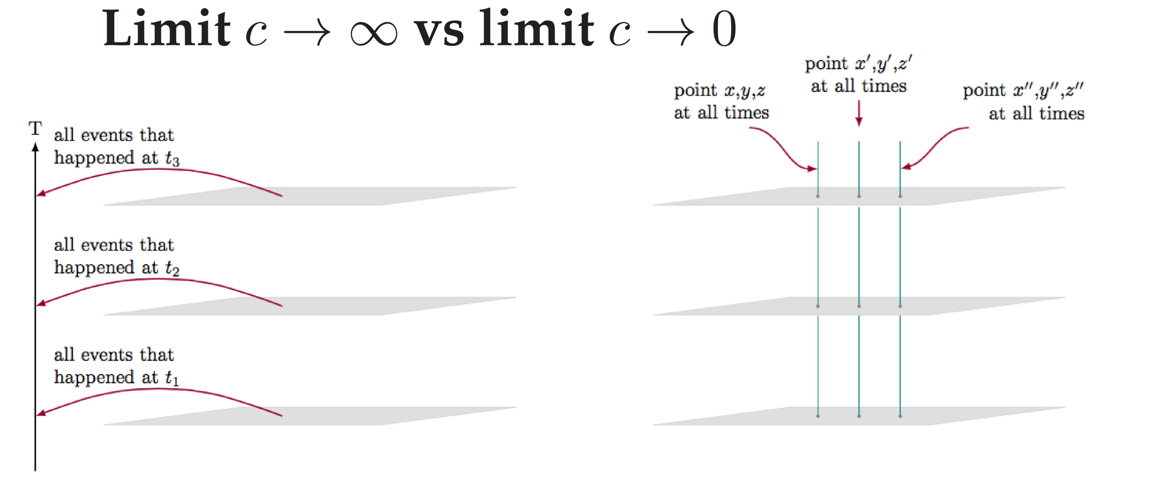
Abstract

If we start with Minkowski Spacetime and take the global limit $c \to 0$ seriously, it yields a spacetime which is utterly unfamiliar to our intuitions, yet consistent and intelligible. In this spacetime, motion in space is impossible and a novel coordinate transformation I call the time-Galilean transformation describes aging without motion. I argue that this transformation fits the role of unitary time evolution in quantum mechanics because a) quantum states can always be decomposed in terms of a superposition of stationary states (energy eigenstates), and b) quantum motion is fundamentally different from classical motion. I take this as evidence for a modal distinction between the classical and quantumworlds. I concretize this idea with the introduction of what I call the Heisenberg Interpretation, which interprets quantum states as a certain kind of pure physical possibility. The interpretation makes novel predictions with respect to the interface between quantum theory and Einstein's general relativity arising from the distinction between possibilities and actualities in spacetime.

A Limit Distinction

Limit $c \to \infty$ vs limit $c \to 0$



The relation of simultaneity "decomposes" spacetime into 3-dimensional pieces, each representing "all of space at a given time," by projecting spacetime onto time, i.e., by identifying spacetime points that have the same time coordinates.

Similarly, one can think of the notion of "same place" as projecting spacetime onto space, i.e., by identifying spacetime points that have the same spatial coordinates; each of the trajectories thus singled out represents "a given place at all times.

From: DiSalle, Robert, "Space and Time: Inertial Frames", The Stanford Encyclopedia of Philosophy (Winter 2020 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/win2020/entries/spacetime-iframes/>

Currently we suppose that the two spacetimes resulting from these two limits are equivalent, in the sense that the Galilean transformations are taken to apply to both. But conceptually, this does not make sense: the limit $c \to \infty$ implies an upper speed limit of infinity, whereas the limit $c \to 0$ implies an upper speed limit of zero. In such spacetime, motion in space is *impossible!*

I believe that because of the seeming absurdity of this implication, the limit $c \to 0$ has not been taken seriously and as a result, the spacetimes resulting from taking these two limits have been incorrectly lumped together. If we can define the relevant coordinate transformation, then it puts the intuitive picture on a firm mathematical grounding. The transformation that correspondes to the picture above on the right is what I will call the time-Galilean transformation.

A Novel Transformation

The **Galilean Transformations** are, in full generality

$$\begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ v_1 & R_{11} & R_{12} & R_{13} \\ v_2 & R_{21} & R_{22} & R_{23} \\ v_3 & R_{31} & R_{32} & R_{33} \end{pmatrix} \begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} + \begin{pmatrix} T_0 \\ T_1 \\ T_2 \\ T_3 \end{pmatrix}$$

Where R_{ij} are orthogonal matrix elements representing rotation and the T_i represent translation.

The **time-Galilean transformations**

$$\begin{pmatrix} \tau \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} v & 0 & 0 & 0 \\ 0 & R_{11} & R_{12} & R_{13} \\ 0 & R_{21} & R_{22} & R_{23} \\ 0 & R_{31} & R_{32} & R_{33} \end{pmatrix} \begin{pmatrix} \tau' \\ x' \\ y' \\ z' \end{pmatrix} + \begin{pmatrix} T_0 \\ T_1 \\ T_2 \\ T_3 \end{pmatrix}$$

where v is a time boost. As can be verified, this transformation permits rotation in space and translation in space and time, but no motion in space. The symbol τ is used because it is very different from our usual concept of time. Allowing differential time boosts across space implies that in some frames, a system can be observed to "age" backward. For instance, suppose we have two systems with distinct time boosts v_1 and v_2, such that $v_1 > v_2$ in some reference frame, then relative to system 1, system 2 goes further back into the past. Is there any model of reality which can accommodate such a seemingly strange implication?

The Distinction in QM

The relation of a general time boost v to QM is still under investigation, but the special case $v = 1$ can be straightforwardly related: any quantum state can be decomposed into basis states, and if we regard these as prior to the superposition state, it may lead to a concept of time without motion. Thus, consider the decomposition

$$\Psi(\mathbf{r}, t) = \sum_{1}^{N} c_n \psi_n(\mathbf{r}, t) = \sum_{1}^{N} c_n \psi_n(\mathbf{r}) e^{-i\omega t}$$

which can be interpreted as the passage of time at distinct points, one for each stationary state $e^{-i\omega t}$ associated with a $\psi_n(\mathbf{r})$. That quantum "motion" (which arises out of phase interference) has to be thought of in terms of a probability current suggests to me that quantum states should be fundamentally thought of as **possibilities**, whereas classical states are to be thought of fundamentally as states of systems that exist as **actualities**. The **Heisenberg Interpretation of Quantum Mechanics** attempts to formally implement the distinction. It formally separates classical and quantum domains in terms of a distinction between possibilities and actualities, and the argument just given suggests that this can also be expressed in terms of the distinction between t and τ.

The Heisenberg Interpretation of Quantum Mechanics

Assume that a possibility is conceptually a labeled unit I will call an *actualizability*. Then,

$$\text{Quantum State} = |\psi\rangle = |\psi\rangle\,\langle\psi|\psi\rangle = \text{label} \circ \text{Actualizability} = \text{Quantum Possibility}$$

Now, we need to define extra mathematical structure to introduce distinction into the mathematics:

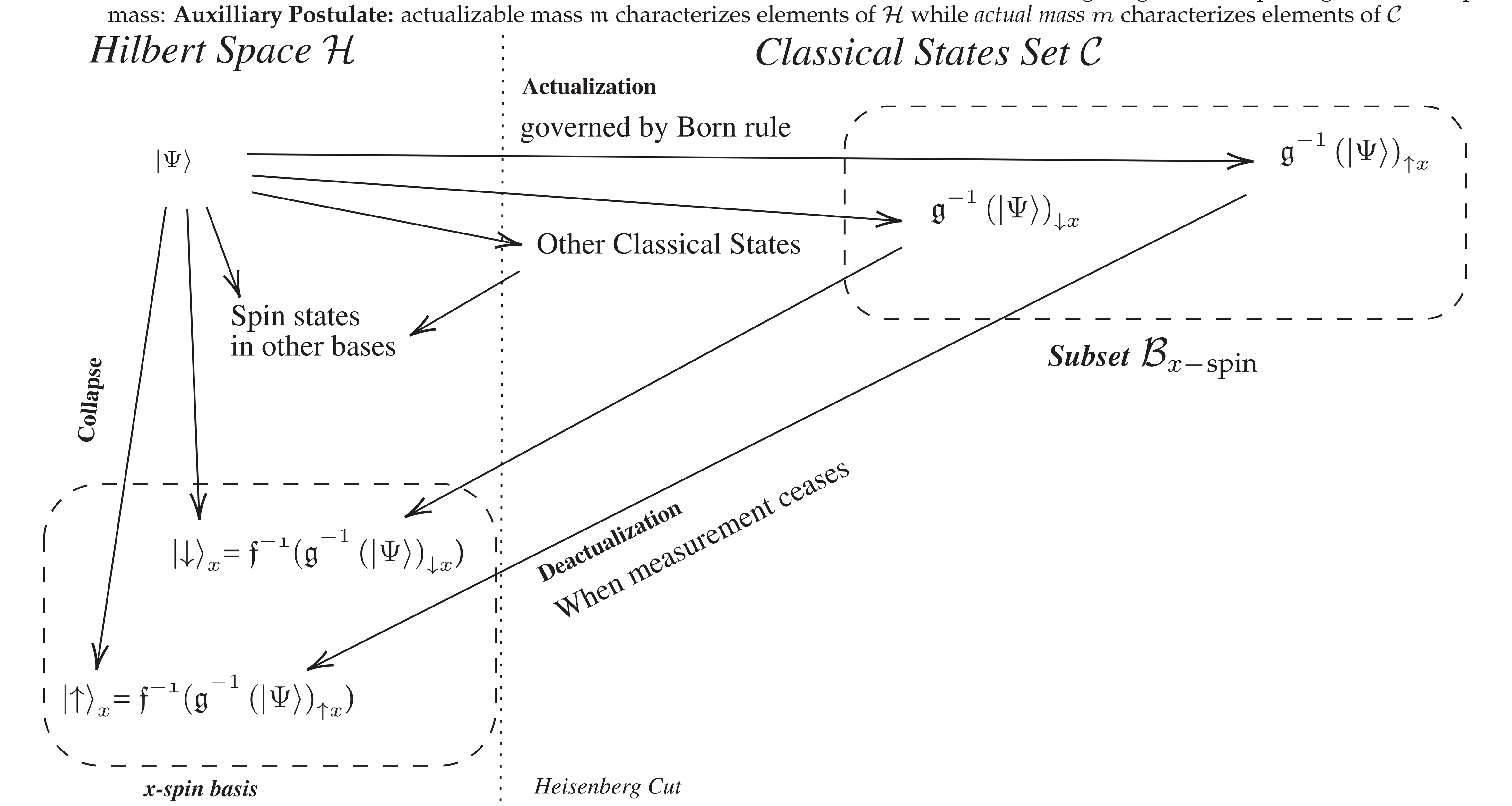
- a family $\mathfrak{E}$ of partial maps I call the *eigenvector maps* $\mathfrak{e} : \mathcal{H} \longrightarrow \mathcal{H}$, each with a domain of definition that includes a subset of a complete set of basis states in a given measurement basis, and its range containing the single element which is the superposition state.
- An unstructured set disjoint from Hilbert Space which collects *states under measurement*, which are now interpreted as states of *actual physical systems*. It will be called the *classical States Set* $\mathcal{C}$. The product set $H \times \mathcal{C}$ relates elements via two sets of partial maps, symbolized by $\mathfrak{f}$ and $\mathfrak{g}$, respectively.
- A family $\mathfrak{F}$ of partial maps $\mathfrak{f} : \mathcal{H} \longrightarrow \mathcal{C}$, called the *possibility-actuality correspondence*. The domain of definition of each f is a subset of a complete set of basis states in a given measurement basis α which is identical to the domain of definition of at least one eigenvector map $\mathfrak{e}$. Its range $\mathcal{B}_\alpha \subset \mathcal{C}$ includes the corresponding states under measurement, which it relates via a one-to-one correspondence. There are as many partial maps $\mathfrak{f}$ as eigenvector maps $\mathfrak{e}$, and conceptually, $\mathfrak{f}$ ensures that each possibility *in a measurement basis* of $\mathcal{H}$ matches exactly one actual system in $\mathcal{C}$.
- A family $\mathfrak{G}$ of partial maps $\mathfrak{g}$ called the *unactualized certainty-actuality correspondence*. The domain of definition of each g is a subset $\mathcal{B}_\alpha \subset \mathcal{C}$ which is identical to the range of at least one partial map f, and includes in its range a single element of $\mathcal{H}$, namely that which is related to a set of eigenvectors in basis α which stand in a one-to-one correspondence to the elements of $\mathcal{B}_\alpha$.

The point of this extra mathematical structure is to be able to express a given eigenvector map inside Hilbert space in terms of a composition of two partial maps which relate elements of Hilbert Space to those of the Classical states set. Thus, for any given vector in a particular measurement basis, we can find partial maps $\mathfrak{e} \in \mathfrak{E}$, $\mathfrak{f} \in \mathfrak{F}$ and $\mathfrak{g} \in \mathfrak{G}$ such that $\mathfrak{e} = \mathfrak{g} \circ \mathfrak{f}$.

The Heisenberg Interpretation postulates:

1. The elements of quantum mechanical Hilbert space $\mathcal{H}$ represent quantum possibilities
2. The time evolution of the elements of $\mathcal{H}$ obeys the Schrödinger equation.
3. Measurements are represented by linear Hermitian operators that are functions of the position and/or momentum operator acting on elements of $\mathcal{H}$
4. Quantum collapse is expressed in terms of the *collapse relation* $\mathfrak{e}^{-1} : \mathcal{H} \longrightarrow \mathcal{H}$, which can be expressed as a composition of two relations
   (a) The *actualization relation* $\mathfrak{g}^{-1} : \mathcal{H} \longrightarrow \mathcal{C}$
   (b) The *deactualization map* $\mathfrak{f}^{-1} : \mathcal{C} \longrightarrow \mathcal{H}$
   where the co-domain $\mathcal{C}$ is the classical states set. The range for each relation is the complete set $\mathcal{B}_\alpha \subset \mathcal{C}$ of those elements which are classical counterparts to the elements of $\mathcal{H}$ in a given measurement basis α.
5. The actualization relation $\mathfrak{g}^{-1}$ is subject to the Born Rule.

The formal distinction between unmeasured states and states under measurement also necessitates on logical grounds a splitting in the concept of mass: **Auxilliary Postulate:** actualizable mass $\mathfrak{m}$ characterizes elements of $\mathcal{H}$ while *actual mass* m characterizes elements of $\mathcal{C}$



The composition $\mathfrak{e}^{-1} = \mathfrak{f}^{-1} \circ \mathfrak{g}^{-1}$ illustrated by a spin $1/2$ state for the x-spin basis.

Testable Implications

The interpretation leaves predictions within the domain of QM unchanged, but changes the relationship to gravity. One prediction which may be possible to test in the near future is a null result for the BMV effect, see C. Marletto, V. Vedral,"Answers to a few questions regarding the BMV experiment" *quantum-ph:1907.08994*