

e/P Mass Ratio (β) and F.S.C. (α) Calculation from the Hydrogen Ground State

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Abstract

The fine structure constant and electron/proton mass ratio can be reasonably derived from the hydrogen ground state wherein mass and inertia test against electric charge in the most fundamental configuration. The contention is that because these numbers are 'pure' numbers, we can dispense with physics altogether and solve the problem of their numeric quantity from a purely logical-probabilistic-mathematical perspective.

Using the equality of centripetal and electrostatic force and the Bohr radius, we can obtain the known force equation expressed in terms of the fine structure constant and e/P mass ratio.

$$F = \alpha^3 \beta^2 = 1.152599 \times 10^{-13}$$

This number is derived as follows ...

$$\frac{m_e v^2}{r} = k_e \frac{q_1 q_2}{r^2} \quad \boxed{\text{Bohr radius} = \frac{\hbar}{m_e c \alpha}}$$

centripetal force electrostatic force

$$\frac{m_e v^2}{r} = \text{Force (ground state)}$$

$$\boxed{\frac{\hbar}{m_e c \alpha} = r \text{ (Bohr radius)}}$$

$$F = \frac{m_e v^2}{\frac{\hbar}{m_e c \alpha}} = \frac{m_e^2 c \alpha v^2}{\hbar}$$

$$\boxed{\begin{array}{l} c = 1 \text{ and } \alpha = v \text{ (in the hydrogen} \\ \text{ground state)} \\ \hbar = 1/2\pi R; \quad 1/\sqrt{2\pi R} = \text{proton} \\ \text{mass} \\ \beta = 1/1836.1527 \quad \alpha = 1/137.035999 \end{array}}$$

$$F = \frac{m_e^2 c \alpha v^2}{\hbar} = \frac{m_e^2 \alpha^3}{\hbar} = \left[\frac{\beta \times 1}{\sqrt{2\pi R}} \right]^2 \alpha^3$$

$$F = \beta^2 \alpha^3 = 1.152599 \times 10^{-13}$$

In the derived equation ... α and β are "pure numbers". We can therefore *ignore all dimensions* and treat this purely as a probabilistic mathematical difficulty. The Bohr radius can be expressed as ... $\hbar / (\beta \alpha \times M_p c)$... so that $\hbar / M_p c$... is 'unit length' in this scale (proton Compton radius). The Bohr radius is then $1/\beta \alpha = 251,619.0162$.

Our question is why 1/137 and 1/1836? ... and not some other numbers that would also generate F? How are parameters and exponents logically adjudicated? We have two parameters, α and β , and two constant exponents (3,2). Then, given F, what are the most probable values of α and β ? How do we solve a 'double variable' uniquely?

If we give each parameter $F^{1/2}$, the computed values of α and β are: 1/143 and 1/1716 ... about a 5% error.

And this should be the case for all α and β when the exponents are equal (by the impartiality of logic). However, the exponents here are unequal ($3 > 2$). Therefore α should take more than $F^{1/2}$ and β less.

We then take the force equation and give each parameter values of $F^{1/2}$ multiplied by a factor Q and 1/Q yielding ...

$$Q F^{1/2} \times F^{1/2} / Q = F$$

So that,

$$[QF^{1/2}]^{1/3} = \alpha \dots \text{and} \dots [F^{1/2}/Q]^{1/2} = \beta$$

We know, by the impartiality of logic, that if both parameters in the equation have equal exponents, Q must have a value of 1. Thus, for all

$$Q\alpha^n \times \beta^n/Q = F \dots Q=1$$

We may then calculate the necessary "Q" values from their ratio.

$$\frac{Q}{\frac{1}{Q}} = Q^2 = \frac{2}{2} = \frac{\alpha\alpha}{\beta\beta} \quad \boxed{\frac{Q \times Q}{\frac{1}{Q}} = Q^3 = \frac{3}{2} = \frac{\alpha \times \alpha \alpha}{\beta \beta}}$$

$$Q = \sqrt[3]{\frac{3}{2}} \quad \text{and} \quad \frac{1}{Q} = \sqrt[3]{\frac{2}{3}}$$

$$\left[\left(\frac{3}{2} \right)^{\frac{1}{3}} F^{\frac{1}{2}} \right]^{\frac{1}{3}} = \frac{1}{137.0317}$$

$$\left[\left(\frac{2}{3} \right)^{\frac{1}{3}} F^{\frac{1}{2}} \right]^{\frac{1}{2}} = \frac{1}{1836.2365}$$

For all equations of this type, the Q value is dependent on the exponential ratio ... proven as mathematically significant by this limit equation:

$$\lim_{\Delta x \rightarrow 0} \left(\frac{\left[\frac{x+\Delta x}{x} \right]^{>\omega} - 1}{\left[\frac{x}{x+\Delta x} \right]^{\omega} - 1} \right) = \frac{>\omega}{\omega}$$

$$\Delta x \rightarrow 0 \left| \left[\frac{x+\Delta x}{x} \right]^{>\omega} - 1 \right| / \left| \left[\frac{x+\Delta x}{x} \right]^{\omega} - 1 \right| = \frac{>\omega}{\omega}$$

in general, for all real ω
greater than or equal to 1
in the form

$$\boxed{\alpha^{>\omega} \beta^{\omega} = F} \quad Q = \left[\frac{>\omega}{\omega} \right] \frac{1}{(>\omega)-\omega+2}$$

Where,

$$Q \text{ maximum is } 1.3210997\dots = 3.591\dots^{1/4.591\dots}$$

and,

$$\frac{3.591\dots}{1.321\dots} = \frac{3.591\dots}{1.321\dots} = 2.71828\dots = e$$

The Adjudication of Dependent Equations

On September 14th, 2023, I was fortunate to discover a reason for the fact that, in the unit scale, the centripetal force in the ground state of hydrogen is equal to the square root of the angular momentum (the square root of \hbar). This problem plagued me since the late 1980s when I derived the fine structure constant and electron-proton mass ratio from that ground state by assuming the above "squared" relationship between angular momentum and centripetal force.

I independently discovered the relationship $\alpha^3 \times \beta^2 = \text{ground state force}$ (already known on the internet). By 'playing around with the numbers', I came upon 1.499 which I observed was the exponential ratio between the parameters α and β . And, by musing upon this, was able to reason out a logical adjudication of the relationship between parameters and exponents ... yielding the FSC and e/P ratio. However, I had no based reason to set the centripetal force as the square root of the angular momentum in the hydrogen ground state. It remained unsubstantiated until decades later ... but clearly the derivation of FSC and e/P is dependent upon this necessary relationship.

Essentially, functionally speaking, why should the centripetal force be the square root of the angular momentum and not any other number? The answer is that the fundamental relationship is not "functional", i.e. it is non-physical. It is a "logical" relationship.

Here is the relationship:

Let $MVR = MVR$... then, quantify all with a universal "1". So that $1x1x1 = 1x1x1$. Clearly, we could use any positive real number and nothing would change. We've made a trivial observation. What do we do with something non-trivial?

Like ... **MVR** is logically and functionally related to **MV^2/R** . How is this to be quantified at the elementary logical level? Angular momentum presumes centripetal force as its necessary logical concomitant. It is impossible to have angular momentum without centripetal force and vice versa.

Let the equations be separated into their "parameters of variance" leaving those of non-variance with a coefficient of '1' (preserving identity).

$$\begin{array}{l} (M=1) VR \text{ <?> } (M=1) V^2/R \\ VR \text{ <?> } V^2/R \end{array}$$

Now, if we quantify all these parameters by assigning the number "1", we get nothing meaningful as expected. However, we must be mindful that changing a parameter to "one" by fiat (for convenience) will hide any variant information (like $mc = mc^2$ if $c=1$). So, we may use any real number greater than zero (just not '1' which only returns identity).

Then the answer is revealed.

$$\begin{array}{l} (M=1) VR \text{ <?> } (M=1) V^2/R \\ (M=1) 5x5 \text{ <?> } (M=1) 5^2/5 \\ (M=1) 25 \text{ <?> } (M=1) 5^2/5 \\ (M=1) 25 \text{ <?> } (M=1) 5 \\ 25 \text{ <?> } 5 \\ N \text{ <?> } N^{1/2} \end{array}$$

Therefore, at the most elementary level, centripetal force must be the square root of the angular momentum. This feature of existence, the relationship between dependent equations MUST BE adjudicated by logic and is not a result of mechanical functions.

Note: "Logical adjudication" is NOT "mathematics" but rather supersedes it. "Something" ... theologians would say "God" ... must adjudicate a general logical problem before mathematics can resolve individual instances ... the programmer/program problem. Or, as a scientist would see it ... abstract logic as an overseer of mechanical functions.

Summation

The original problem gives accuracies of 1 part in ~32,000 and ~21,000. These values generated for $\alpha = (1/137.0317)$ and $\beta = (1/1836.2365)$ appear to require correction. However, as part of an extended theory of existence as ... logic ... counting (1,2,3...n) ... the present values have variations depending on where in space and when in time the measurements are taken, i.e. the FSC will be greater in large void spaces and lesser in high concentrations of matter (galactic clusters). The speed of light will therefore be greater In large voids and less around clumps of matter.

This is conjectured to be the case because the FSC is the measure of the “springiness of space” (Dirac). And, that springiness has then a gradient that pushes matter together in the same direction as gravity (an extra force). This ‘force’ is directly analogous to Cheerios clumping up in a bowl of milk when there aren’t enough left to cover the milk. The FSC field gradient is then a 3-dimensional analog of surface tension. This conjectured tension gradient is what the FSC measures. Thus, the speed of light may slow down or speed up, depending on where and when it is measured.

These values vary over time ... the FSC varying inversely as the 1/12 root of the age or radius of the universe and the e/P mass ratio varying as the 1/8 root. [Note: Using the ‘unit scale’]

The foregoing conjectures stem from a simplistic model of the universe developed over the course of the last six decades where existence is initiated in the manner of the set of integers in set theory. In general, if everything is removed from existence there yet remains two concepts ... the quality of the state (0) and its quantity (1). This makes three concepts in ‘nothing’ (0,1),2 and this makes 4 ... (0,1),2,3 ... which then makes 5 and so on to infinity. Existence is then the instantiation of the ‘count’ employing the set of ‘quals’ we call “geometry”.

In this model ... everything **MUST** change over time ... gradually. If the universe is as I suspect fundamentally, just ... Logic, counting ... then this must be so. Here (left table below) are the approximate rates of change of parameters on a “unit scale” where unit length is the distance between two baryons if all were spread out uniformly (~22 inches). Unit time is the time taken for light to cross the unit length (~10⁻⁸ seconds). Unit mass (1) is the initial mass now diminished to 10⁻¹³ unit masses. And instead of space expanding ... we take the opposite perspective that matter is shrinking.

Note that most parameters are exponential multiples of ‘13’. I encountered an old table of values last year by Dicke with other universal parameters having exponential multiples of ‘13’. So, this is not unprecedented. That it is possible to adopt such a set of changing values with orderly exponents, indicates that such models are worthy of examination.

baryon number	10 ⁷⁸ (pure number)	cc R³
particle production rate	10 ⁵² n/unit time	cc R²
reaction velocity	10 ³⁹ ul/ut	cc R^{3/2}
radius of universe	10 ²⁶ unit length	cc R¹
Compton wavelength	10 ⁻¹³ ul	cc R^{-1/2}
particle mass	10 ⁻¹³ unit mass	cc R^{-1/2}
elementary charge ‘e’	10 ⁻¹³ unit charge	cc R^{-1/2}
Planck constant	10 ⁻³⁹ um·ul ² /ut	cc R⁻¹
gravitational constant	10 ⁻³⁹ ul ³ /um·ut ²	cc R^{-3/2}
hydrogen ground state force	10 ⁻¹³ um·ul/ut ²	cc R^{-1/2}
e/P mass ratio	1/1836 (pure number) ..	cc R^{-1/8}
fine structure constant	1/137 (pure number) ..	cc R^{-1/12}
speed of light	1/137 ul/ut	cc R^{-1/12}

Quantity		unit		epoch
Absolute time	<i>t</i>	s	<i>t</i>	10 ⁵²
Speed of light	<i>c</i>	m/s	<i>t</i> ^{-1/2}	10 ⁻²⁶
Wavelength	λ	m	<i>t</i> ^{-1/4}	10 ⁻¹³
Frequency	<i>f</i>	1/s	<i>t</i> ^{-1/4}	10 ⁻¹³
Current time step	τ	s	<i>t</i> ^{1/4}	10 ¹³
Speed	<i>v</i>	m/s	<i>t</i> ^{-1/2}	10 ⁻²⁶
Acceleration	<i>a</i>	m/s ²	<i>t</i> ^{-3/4}	10 ⁻³⁹
Inertial mass	<i>m</i>	kg	<i>t</i> ^{-3/4}	10 ³⁹
Cosmic horizon	<i>R</i>	s	<i>t</i> ^{1/2}	10 ²⁶
Observed horizon	<i>R'</i>	s	<i>t</i> ^{3/4}	10 ³⁹
Perceived time	<i>t'</i>	s	<i>t</i> ^{3/4}	10 ³⁹
Volume universe	<i>V</i>	m ³	<i>t</i> ^{3/2}	10 ⁷⁸
Number of particles	<i>N</i>	–	<i>t</i> ^{3/2}	10 ⁷⁸

from Robert Dicke

*“For the generation of all things ...
one principle is sufficient” - Leibniz*