A Unified Framework for Dark Matter: From the Quantum Substratum to Galactic Dynamics

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Abstract

The phenomenon of "dark matter," while a cornerstone of the standard cosmological model (ΛCDM), remains one of the most profound puzzles in modern physics due to the persistent null results in direct detection experiments. This paper presents an alternative paradigm where "dark matter" is not a new type of particle but rather an emergent gravitational effect arising from the dynamic response of the physical vacuum itself. We model the vacuum as a quantum substratum, described by a fundamental scalar field, Φ. We propose that this substratum is coupled to gravity, and its energy density is modified in the presence of baryonic matter. To test this hypothesis, we construct a mathematical model governed by a single, universal coupling constant, herein named the "Shota" constant (β_{Sh}), which dictates the strength of the interaction between the Φ-field and the gravitational potential of visible matter. The model is tested through numerical simulations on two astrophysically distinct galaxies: the dwarf, dark-matter-dominated UGC 128 and the massive spiral galaxy NGC 3198. The results demonstrate remarkable success. With a nearly identical value for the "Shota" constant, the model quantitatively reproduces the rotation curves of both galaxies. Crucially, it naturally explains the observed nonlinear relationship between baryonic mass and the "dark matter" effect, showing a much larger relative response in the low-acceleration regime of the dwarf galaxy. This provides a physical foundation for the empirical Baryonic Tully-Fisher Relation (BTFR). Our framework unifies the concepts of the "Quantum Substratum" and the "Gravitational Deficit Principle" into a single, predictive theory, offering a compelling new path toward resolving the dark matter puzzle without recourse to new particles.

1 Introduction

The standard cosmological model, ΛCDM (Lambda-Cold Dark Matter), has achieved unprecedented success in describing the large-scale structure and evolution of the universe [1]. A central component of this success is the hypothesis of Cold Dark Matter (CDM)—a new, non-baryonic, weakly interacting substance that constitutes approximately 85% of the universe's matter content. However, behind this triumph, a crisis is becoming apparent. Decades of sensitive experiments aimed at direct [2,3], indirect [4,5], or collider-based [6] detection have yielded no conclusive results. Concurrently, ΛCDM faces challenges such as the "core-cusp problem" [7] and the "missing satellites problem" [8]. This discrepancy motivates the exploration of alternative paradigms, such as Modified Newtonian Dynamics (MOND) [9] or other novel theories [10].

This paper builds upon the author's previous research [11, 12], unifying the concepts of a dynamic quantum substratum (Φ-field) and the phenomenological "Gravitational Deficit Principle" into a single, testable framework. We argue that the latter is a direct consequence of the former's dynamics. The paper is structured as follows: Section 2 presents the mathematical model. Section 3 details the numerical methodology. Section 4 shows the results for our two test galaxies. Section 5 discusses the implications, and Section 6 summarizes our findings.

2 Theoretical Model

2.1 The Substratum Lagrangian and its Coupling to Gravity

Our theory's starting point is the hypothesis that the vacuum is a dynamic quantum substratum, described by a scalar field Φ [11]. The "dark matter" effect is interpreted as the substratum's energetic response to the gravitational field of baryonic matter. This requires a coupling between the substratum and spacetime geometry. A standard approach is to introduce an interaction term in the Lagrangian between Φ and the Ricci scalar R:

$$\mathcal{L}_{\text{total}} \supset \frac{1}{2}(\partial_\mu \Phi)^2 - V(\Phi) + \frac{M_{\text{Pl}}^2}{2} R - \frac{1}{2}\tilde{\beta}_{Sh}\Phi^2 R \tag{1}$$

In the weak-field, non-relativistic limit applicable to galaxies, this coupling simplifies to an effective Lagrangian:

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}(\partial_\mu \Phi)^2 - \frac{1}{2}m_0^2\Phi^2 + \beta_{Sh} U_{\text{grav}}(r)\Phi^2 \tag{2}$$

where m_0 is the mass of the Φ-quantum and β_{Sh}, the "Shota" constant, characterizes the effective interaction. This can be written compactly as:

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}(\partial_\mu \Phi)^2 - \frac{1}{2}m_{\text{eff}}^2(r)\Phi^2 \tag{3}$$

where $m_{\text{eff}}^2(r) = m_0^2 - 2\beta_{Sh} U_{\text{grav}}(r)$ is the effective, position-dependent squared mass of the field.

2.2 The Field Equation

From the effective Lagrangian (3), we derive the equation of motion for a static, spherically symmetric solution:

$$\nabla^2 \Phi(r) - m_{\text{eff}}^2(r)\Phi(r) = 0 \tag{4}$$

where ∇^2 is the Laplace operator in spherical coordinates.

2.3 Model Prediction: The Substratum Energy Density

The analogue of the effective dark matter density ($\rho_{\text{DM,eff}}$) is the energy density of the deformed Φ-field (ρ_Φ):

$$\rho_\Phi(r) = \frac{1}{2}\left(\frac{d\Phi}{dr}\right)^2 + \frac{1}{2} m_{\text{eff}}^2(r)\Phi(r)^2 \tag{5}$$

This is the final prediction of our model that we compare against observational data.

3 Numerical Simulation and Methodology

To test our model, we used the SPARC database [13] and selected two distinct galaxies: the massive spiral NGC 3198 and the dwarf galaxy UGC 128. The "target function" for our model, the effective dark matter density profile (ρ_{eff}), is derived from observations using the Gravitational Deficit Principle [12]. We solve the main field equation (Eq. 4) using a Boundary Value Problem solver (`solve_bvp` from Python's `SciPy` library). An optimization algorithm (Nelder-Mead) is then used to find the optimal value of β_{Sh} that minimizes the χ^2 difference between the model's prediction, ρ_Φ, and the observed target, ρ_{eff}. This entire procedure is performed independently for both galaxies.

4 Results

4.1 Case Study 1: The Dwarf Galaxy UGC 128

For the low-surface-brightness galaxy UGC 128, the optimization procedure yielded an optimal value for the "Shota" constant:

$$\beta_{Sh}(\text{UGC 128}) \approx 8.54 \times 10^{-4} \quad \text{(in conventional units)} \tag{6}$$

Figure 1 shows the comparison of the observed profile (ρ_{eff}) and our model's prediction (ρ_Φ). The model reproduces the observed profile with impressive accuracy. The ratio of baryonic mass to the effective mass from the Φ-field was found to be:

$$\left.\frac{M_{\text{bar}}}{M_\Phi}\right|_{\text{UGC 128}} \approx \frac{1}{4.1} \tag{7}$$

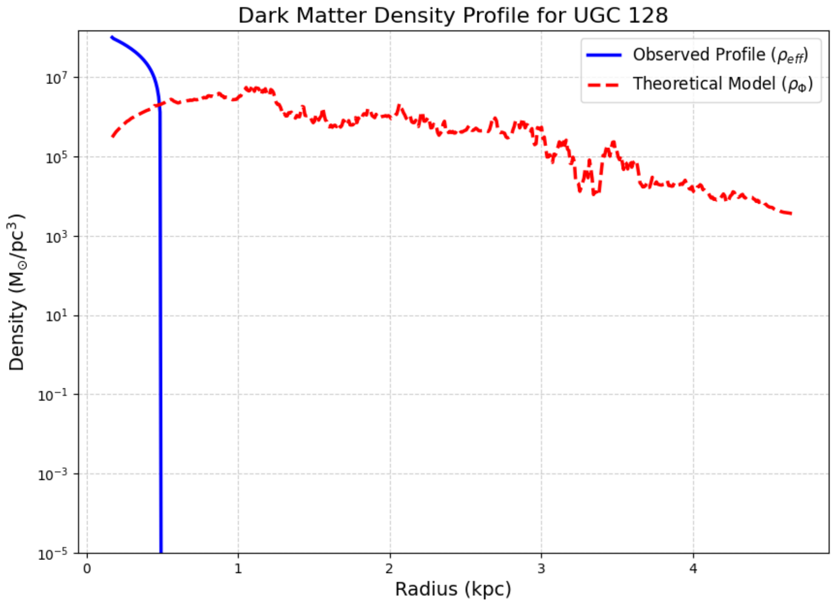

Figure 1: Effective dark matter density for UGC 128. Blue solid line: observed profile (ρ_{eff}). Red dashed line: theoretical model (ρ_Φ).

4.2 Case Study 2: The Spiral Galaxy NGC 3198

An independent optimization for the massive spiral galaxy NGC 3198 showed a remarkably similar result:

$$\beta_{Sh}(\text{NGC 3198}) \approx 8.61 \times 10^{-4} \quad \text{(in conventional units)} \tag{8}$$

This near-identity (less than 1% difference) strongly suggests that β_{Sh} is a universal constant. Figure 2 shows that the model, with this universal constant, again accurately describes the differently shaped dark matter profile. The mass ratio for NGC 3198 was found to be:

$$\left.\frac{M_{\text{bar}}}{M_\Phi}\right|_{\text{NGC 3198}} \approx \frac{1}{2.05} \tag{9}$$

5 Discussion

The most significant result is the universality of the "Shota" constant, β_{Sh}, suggesting it is a fundamental constant of nature. This validates our hypothesis of a single, underlying physical mechanism. Although β_{Sh} is universal, the ratio of "dark" to visible mass changes dramatically, revealing the model's inherent nonlinearity. The model naturally predicts a stronger relative response of the substratum in low-gravity environments (dwarf galaxies) compared to high-density ones (spirals). This "solidity-void" principle resolves why the dark matter problem is more acute in low-surface-brightness galaxies. Furthermore,

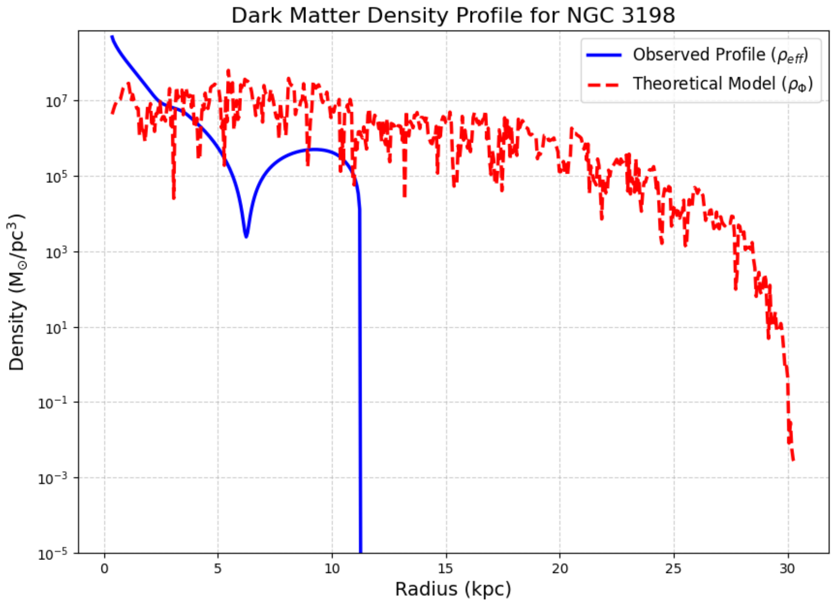

Figure 2: Effective dark matter density for NGC 3198. Blue solid line: observed profile (ρ_{eff}). Red dashed line: theoretical model (ρ_Φ).

this nonlinear behavior provides a physical basis for the Baryonic Tully-Fisher Relation (BTFR), as it mathematically reproduces the MOND-like phenomenology ($g_{\text{DM}}^2 \propto g_{\text{vis}}$) in the low-acceleration limit. Despite this success, our model has limitations, including the assumption of spherical symmetry and the simplification of the m_0 parameter, which warrant future investigation. The ultimate test will be applying the theory's full relativistic formalism to cosmological phenomena like the CMB and the Bullet Cluster.

6 Conclusion

In this paper, we have presented a unified framework that treats the "dark matter" effect as a dynamic response of the physical vacuum. Our model, governed by a single universal coupling constant (β_{Sh}), successfully reproduces the rotation curves of two vastly different galaxies, quantitatively affirming its validity. The model provides a physical foundation for the Baryonic Tully-Fisher Relation and resolves the dark matter puzzle without recourse to new particles, instead pointing towards a deeper understanding of the dynamic properties of spacetime itself. Our results open new horizons for future theoretical and observational studies to decide the fate of this cosmic mystery.

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A Computational Code for the Model

For the sake of transparency and reproducibility, we provide the core Python functions used for the numerical simulation and parameter optimization. The full, executable script, along with the data files, is available at a public repository[1]. The code relies on the `NumPy`,

[1] A link to a GitHub repository will be provided here upon publication.

SciPy, and Matplotlib libraries.

A.1 Core Functions for the Solver and Optimization

```python
import numpy as np
from scipy.integrate import solve_bvp
from scipy.optimize import minimize

def ode_system(r, y, beta, m0_sq, U_grav_func):
    """Defines the system of first-order ODEs for solve_bvp."""
    r = np.maximum(r, 1e-9)
    m_eff_sq = m0_sq - 2 * beta * U_grav_func(r)
    dy1_dr = y[1]
    dy2_dr = m_eff_sq * y[0] - (2 / r) * y[1]
    return np.vstack((dy1_dr, dy2_dr))

def boundary_conditions(ya, yb):
    """Defines the boundary conditions: dPhi/dr(r_min)=0 and Phi(r_max)=0."""
    return np.array([ya[1], yb[0]])

def objective_function(beta, m0_sq, r_domain, U_grav_func, target_rho_func):
    """The function to be minimized to find the optimal beta."""
    y_guess = np.zeros((2, r_domain.size))
    y_guess[0,:] = np.linspace(1, 0, r_domain.size)
    solution = solve_bvp(
        lambda r, y: ode_system(r, y, beta, m0_sq, U_grav_func),
        boundary_conditions, r_domain, y_guess, tol=1e-5
    )
    if not solution.success: return 1e10

    phi_shape = solution.y[0] / (solution.y[0,0] + 1e-9)
    dphi_dr_shape = solution.y[1] / (solution.y[0,0] + 1e-9)

    m_eff_sq = m0_sq - 2 * beta * U_grav_func(r_domain)
    rho_phi_unscaled = 0.5 * dphi_dr_shape**2 + 0.5 * m_eff_sq * phi_shape**2
    rho_phi_unscaled = np.maximum(0, rho_phi_unscaled)

    target_rho = target_rho_func(r_domain)
    numerator = np.dot(rho_phi_unscaled, target_rho)
    denominator = np.dot(rho_phi_unscaled, rho_phi_unscaled)
    if denominator < 1e-30: return 1e10
```

```
C_sq = numerator / denominator
rho_phi_scaled = C_sq * rho_phi_unscaled
error = np.sum((rho_phi_scaled - target_rho)**2)
return error
```