One single metric model's unifying perspective on four issues with galactic dynamics: B-TF relation, rotation curves, spirals and SMBH-growth.

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In this paper, we intend to replace four mysteries of galactic dynamics with one. This one mystery is introduced as a postulate: the constant Lagrangian metric as a space-time background for galactic dynamics. The four mysteries of galactic dynamics are the B-TF relation, the rotation curves, the spiral structure and SMBH-growth. Why the galactic space-time background behaves like a giant disk with directionally synchronized metronomes on it (the constant Lagrangian metric) remains an open question.

Keywords: alternative gravity theory, galaxies, rotation curves, spirals, SMBH

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I. A SYNTONIZED METRIC BACKGROUND FOR GALAXIC DYNAMICS

A. Lagrangians and syntonisation: from Gravity Probe B and GNSS to galaxies.

In GNSS, synchronization (= run at equal time) of clocks is an important issue, for which Einstein's theories of relativity, both the special and the general, play a key role. The frequency shift of two clocks p and q with mass m and rest-energy U_0 in some position and motion around a central mass can be related to the Lagrangian L of the system with those two clocks. In first order in Φ/c^2 accuracy, the results for all sets of two clocks p and q will be [2] :

$$\boxed{\frac{\Delta\nu_{pq}}{\nu_p} = \frac{\Delta L_{pq}}{U_0}}. \tag{1}$$

For each clock separately, we have the relation between reference-time t and individual clock-time τ:

$$\left(\frac{d\tau}{dt}\right)^2 = 1 + \frac{2\Phi}{c^2} - \frac{v_{orbit}^2}{c^2} = 1 - \frac{2L}{U_0}. \tag{2}$$

In this formula, we add a scalar potential Φ to a vector-based-velocity-turned-scalar-kinetic-energy. This Newtonial scalar potential is then the low intensity field limit of Einstein's space-curvature. So the formula adds curvature to speed in a superposition way. To avoid that, in the velocity-of-space theory of gravity, the potential energy is replaced by intrinsic kinetic energy, with a velocity-of-space equal to the escape velocity at position r but with the opposite direction: $-V = K_{esc} = K_{intrinsic}$.

From an Einsteinian GR perspective, in the velocity-of-space theory of gravity, curvature is replaced by intrinsic velocity. The advantage is that we add speed and speed or velocity and velocity , instead of speed and potential or speed and curvature, in order to get the total relativistic clock frequency shift. The one velocity is intrinsic, velocity-of-space, and the other velocity can be extrinsic, as in orbital velocity of a clock in a satellite. We then get

$$-\left(\frac{d\tau}{dt}\right)^2 = 1 - \frac{v_{esc}^2}{c^2} - \frac{v_{orbit}^2}{c^2} = 1 - \frac{2L}{U_0}. \tag{3}$$

with a Lagrangian as in $L = K_{orbit} - V = K_{orbit} + K_{esc} = K_{orbit} + K_{intr}$. This was used in [2] to derive the relativistic GNSS clock-rate corrections and in [1] to derive the geodetic precesion.

This last application, using the velocity-of-space theory of gravity to derive the geodetic precession, showed that the approach of adding intrinsic velocity to extrinsic velocity also worked with the vector aspects of those velocities, so with spin precession effects in three dimensions. In the literature there is still a debate as how to explain the geodetic precession, either as a special relativity Thomas precession effect, or as a curvature only effect, or as the summation of a time like Thomas precession and a space like Schouten precession due to curvature, resulting in the correct precession rate [1]. The ongoing discussion shows that there is no clarity as how to add a special relativity effect to a curvature effect, how to add a Thomas precession to a Schouten precession so to say. But in the velocity-of-space theory of gravity, both effects were reduced to velocity of the spinning ball relative to local space and then those velocities, the one intrinsic and the other extrinsic, were added as vectorial rapidities [1]. The same approach was used in [2] to derive formula (1). In this paper we will apply this approach to galactic dynamics, using both the scalar addition method as the vector addition method.

The key *eureka*-moment for this paper's approach was the idea to extend the clock syntonization (= equal clock frequencies) perspective from GNSS to galaxies. When I connected this clock-rate formula to the problem of the galactic rotation curve, I realized that the flat rotation curve implies atomic clock syntonization in those areas where $v_{orbit} \approx v_{final}$ and $v_{esc} \approx 0$. In those outer regions, the gravitational potential can be assumed to be approximately zero and the orbital velocity constant. It is intriguing to realize that you can jump from orbit to orbit and still encounter a constant clock-rate on all the orbiting satellites you encounter on an imaginary voyage through the outer regions of galaxies. Those flat rotation rate zones are the GNSS engineer's dream come true. This implies that precisely in those regions where the classical virial theorem seems in trouble, $L \simeq constant$, not just in one single orbit *but also between different orbits*. This made me curious as to the clock-rate status in the inner regions.

B. The Lagrangian inside and at the boundary of the galactic bulge

In order to study the relativistic clock-rate behaviour in the inner regions of galaxies, I had to construct a model galaxy. My model galaxy is build of a model bulge with mass M and radius R and, analogue to Schwarzschild metric, an empty space around it. In such a mode galaxy, the Newtonian gravitational potential is fully determined by the bulge. The model bulge has constant density $\rho_0 = \frac{M}{V} = \frac{3M}{4\pi R^3}$ and its composing stars rotate on geodetics in a quasi-solid way. So all those stars in the bulge have equal angular velocity on their geodetic orbits, with $v_{orbit} = \omega r$. So we also have $v_{orbit}^2 = \omega^2 r^2$ and

$$\frac{K_{orbit}}{m} = \frac{\omega^2 r^2}{2} \tag{4}$$

On the boundary between the quasi solid spherical bulge and the emptiness outside of it, the orbital velocities are behaving smoothly. So the last star in the bulge and the first star in the region outside of the bulge have equal velocities and potentials. I also assume that the Newtonian potential itself is unchanged and unchallenged, remaining classical in the whole galaxy and its surroundings. In the velocity-of-space theory of gravity, the Newtonian potential is needed to calculate the escape velocity and thus the intrinsic inbound velocity of space at position r around M. Such a model galaxy doesn't, for the moment, have a SMBH in the centre of its bulge and it only has some very lonely stars with test mass m in the space outside the bulge.

The gravitational potential energy in such a case is well known. Outside the sphere the potential energy equals the Newtonian potential energy V_N:

$$\frac{V_N}{m} = -\frac{GM}{r}. \tag{5}$$

Inside the sphere the potential energy is

$$\frac{V}{m} = -\frac{GM}{2R}\left(3 - \frac{r^2}{R^2}\right) = -\frac{1}{2}\frac{GM}{r}\left(\frac{3}{R} - \frac{r^3}{R^3}\right) = -\frac{1}{2}\frac{V_N}{m}\left(\frac{3}{R} - \frac{r^3}{R^3}\right). \tag{6}$$

At the center of the rotating sphere, $K = 0$ and we have $\frac{L}{m} = -\frac{V}{m} = \frac{3GM}{2R}$. At the boundary between the bulge and empty space we have $r = R$ so

$$\frac{V}{m} = -\frac{GM}{R} = \frac{V_N(R)}{m}. \tag{7}$$

Also at the boundary, the virial theorem should hold, because we have build an ordiinary, Newtonian quasi solid bulge in empty space. Deviations should appear at $r < R$, not at $r = R$. So at the boundary we have $\frac{K}{m} = -\frac{1}{2}\frac{V_N}{m}$, so

$$\frac{K_{orbit}}{m} = \frac{\omega^2 R^2}{2} = \frac{GM}{2R} \tag{8}$$

and thus

$$\omega^2 = \frac{GM}{R^3} \tag{9}$$

at the boundary. This results in a Lagrangian at the boundary of

$$\frac{L}{m} = \frac{K_{orbit}}{m} - \frac{V}{m} = \frac{GM}{2R} + \frac{GM}{R} = \frac{3GM}{2R} = constant. \tag{10}$$

Because ω is constant in the quasi solid bulge, we get for $0 < r < R$:

$$\frac{K_{orbit}}{m} = \frac{\omega^2 r^2}{2} = \frac{GMr^2}{2R^3} \tag{11}$$

and thus for $0 < r < R$ we get a Lagrangian:

$$\frac{L}{m} = \frac{K_{orbit}}{m} - \frac{V}{m} = \frac{GMr^2}{2R^3} + \frac{GM}{2R}\left(3 - \frac{r^2}{R^2}\right) = \frac{3GM}{2R} = constant. \tag{12}$$

As a result, inside such a model bulge, L is a constant of the motion of a mass m, not only in one orbit but also between orbits. All the clocks inside such a model bulge would be syntonized. So at the outer edges of galaxies, the Lagrangian is a constant and inside the bulge the Lagrangian is also constant. This made me wonder what kind of model Galaxy I would get if I declared the Lagrangian to be a constant at all r, so from $r = 0$ to $r = \infty$.

C. A Constant Lagrangian Model Galaxy and the resulting orbital velocity

I postulated $L = K_{orbit} - V_N = constant$ in the entire galaxy, without changing the Newtonian potential. What would the implications be for the orbiting velocity of a test mass m placed in the space outside the galactic bulge? And how do we interpret the resulting situation in the context of the velocity-of-space theory of gravity?

Outside the model bulge, where $R \leq r \leq \infty$, we have

$$\frac{L}{m} = \frac{K_{orbit}}{m} - \frac{V}{m} = \frac{v_{orbit}}{2} + \frac{GM}{r} = \frac{3GM}{2R} \tag{13}$$

so, we have for the square of the orbital velocity:

$$v_{orbit}^2 = \frac{3GM}{R} - \frac{2GM}{r} = v_{final}^2 - v_{esc}^2 = v_{final}^2 - v_{intr}^2. \tag{14}$$

In the last part of this equation, we used the intrinsic velocity-of-space $\vec{v}_{intr}$ as the negative of the escape velocity. We then get the vector summation:

$$\vec{v}_{final} = \vec{v}_{orb} + \vec{v}_{intr}. \tag{15}$$

How do we interpret this result? And does it work?

D. A constant Lagrangian metric, the velocity-of-space and spiral galaxies

In the last section we ended with a Constant-Lagrangian based expression for the orbital velocity of a test mass in our model galaxy as:

$$v_{orbit}^2 = \frac{3GM}{R} - \frac{2GM}{r}. \tag{16}$$

In Fig.(1) I sketched the result, with vertically the orbital kinetic energy K_{orbit} in units of $-V/2$ and horizontally the distance from the border of the bulge in units of the bulge radius R. The bold line gives the orbital kinetic energy curve and the dotted bold line gives the $-V = +K_{escape} = K_{intrinsic}$ curve.

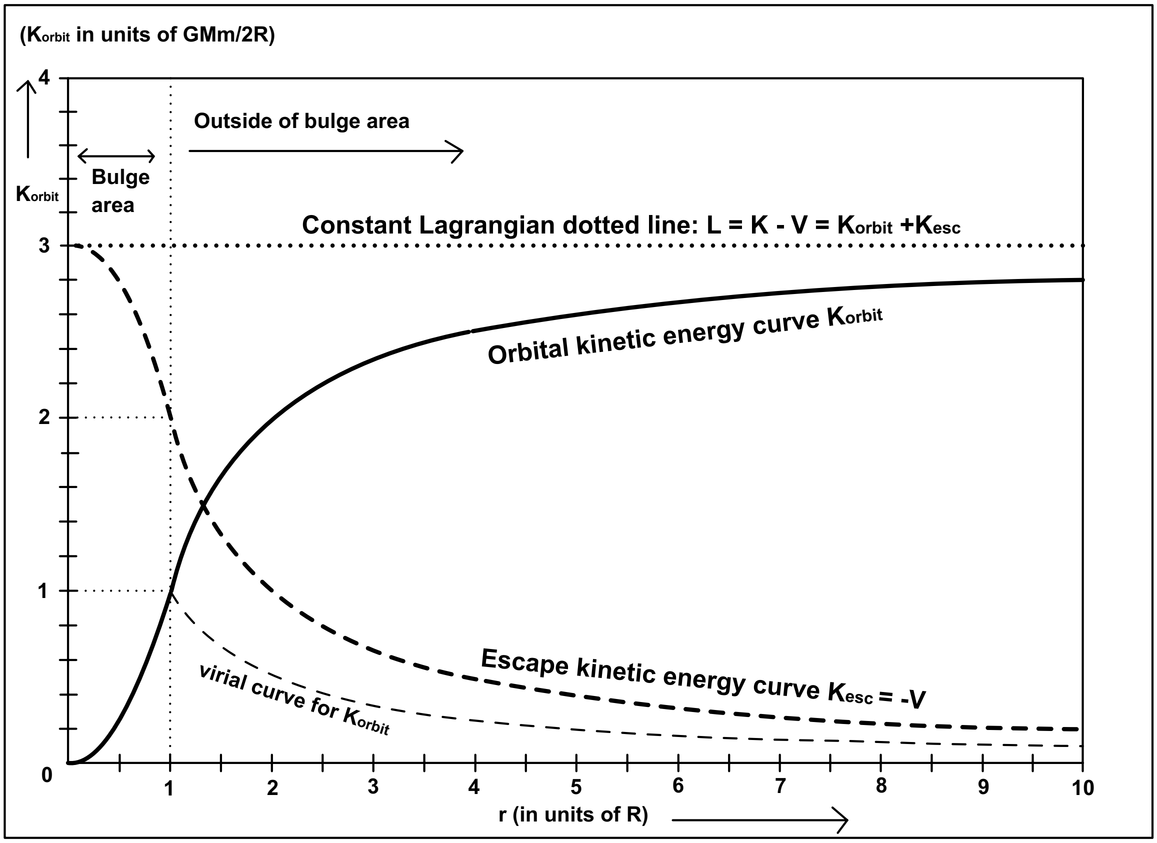

FIG. 1. The orbital kinetic energy in the model galaxy with $L = constant$ set against the radius.

We tested this formula for the orbital velocity using the SPARC database comprising the experimental velocity rotation data of 175 LTG galaxies [4]. Of those 175 galaxies, 77 allowed a fit of the rotation curve within the measurement uncertainties, so about 44 percent, see Fig. (2) and Fig(3) and ref. [3]. Another 18 galaxies could almost be plotted within the measurement uncertainties But for the other galaxies in the database, the data didn't fit the model without additional ad-hoc assumptions. For 13 galaxies of the database, we got "two fits" which gave the impression that those galaxies underwent a 'bulge reset' due to too much mass being added in the disk. To be short, our model was too smooth and didn't allow for real galactic disks including somewhat chaotic and large mass accumulation.

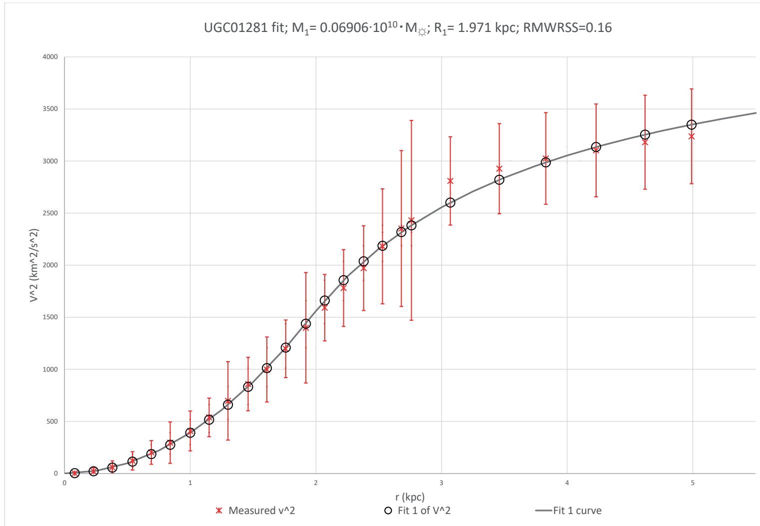

FIG. 2. The weighted residual graph.

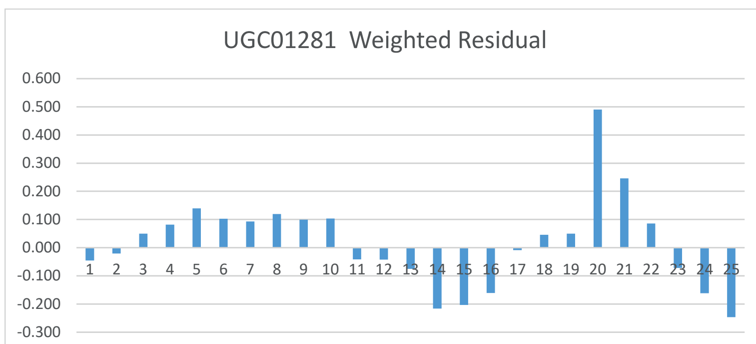

FIG. 3. The RMWRSS value quantify the quality of the fitting curve.

II. NEW INTERPRETATION OF THE CONSTANT LAGRANGIAN MODEL GALAXY

I realized that the problem with my model galaxy might be in the interpretation of orbital kinetic energy and the related orbital velocity as calculated with the constant Lagrangian postulate. Following the GNSS and geodetic models, I interpreted the orbital velocity as extrinsic, so the velocity of the test mass m in the empty space around the bulge. And I interpreted the negative of the escape velocity as the intrinsic velocity of space. This last velocity defines, in the velocity-of-space theory of gravity, the radial free fall around standard central massess, see Fig(4).

It occurred to me that the syntonization implied in the constant Lagrangian model could only refer to the metric and not to test-masses located in that metric. The metric in a constant Lagrangian model galaxy behaves like in the phenomenon of metronome synchronization, where all

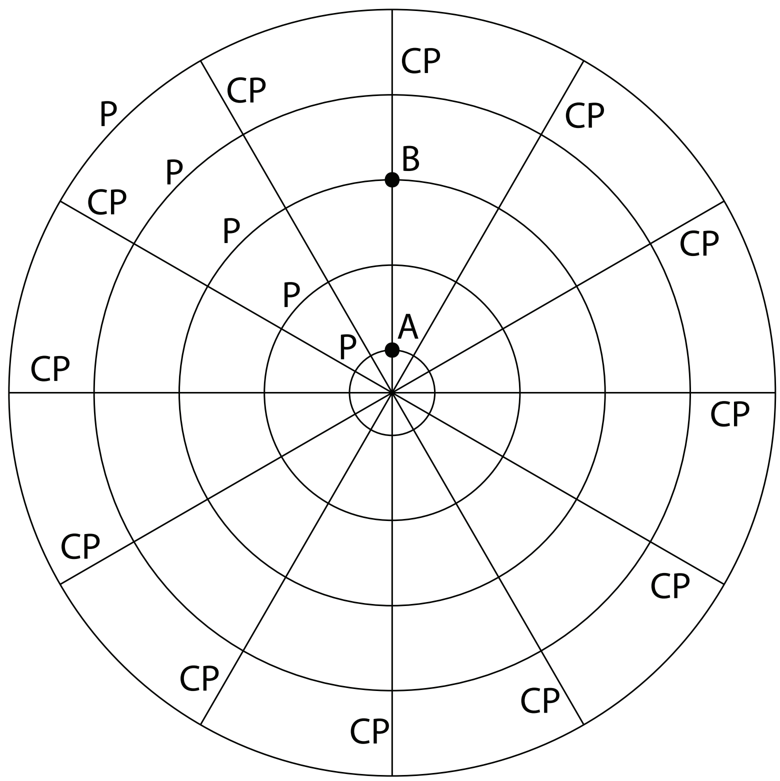

FIG. 4. The GNSS radial CP and orbital P geodesics.

the loosely connected metronomes eventually align their rhythms and beat in unison. In this situation, according to my postulate, the whole galactic metric behaves like an immense collection of space metronomes. But in that case, the orbital velocity as calculated with the constant Lagrangian model should also be interpreted as an intrinsic velocity of space, just as the radial intrinsic velocity. And the sum of those two intrinsic velocities should then also be an intrinsic velocity, a velocity-of-space, which we dub the Lagrangian intrinsic velocity. In the velocity-of-space theory of gravity, this Lagrangian velocity $\vec{v}_L$ equals the sum of the sum of the orbital velocity $\vec{v}_{orbit}$ and the negative of the escape velocity $\vec{v}_{radial}$:

$$\vec{v}_L = \vec{v}_{orb} + \vec{v}_{rad}. \tag{17}$$

But due to the intrinsic kinetic energies we also have

$$v_L^2 = v_{orb}^2 + v_{rad}^2. \tag{18}$$

And because in our constant Lagrangian model galaxy we know the value and directions of $\vec{v}_{orbit}$

and $\vec{v}_{radial}$, we can calculate the angle α between these intrinsic velocities. We get:

$$v_L = \sqrt{\frac{3GM}{R}} \tag{19}$$

and

$$tan(\alpha) = \frac{v_{rad}}{v_{orb}} = \frac{\sqrt{\frac{2GM}{r}}}{\sqrt{\frac{3GM}{R} - \frac{2GM}{r}}} = \sqrt{\frac{2}{\frac{3r}{R} - 2}}, \tag{20}$$

which, in units of R, gives

$$\alpha = arctan\left(\sqrt{\frac{2}{3r-2}}\right). \tag{21}$$

This is valid in our model for $R < r < \infty$. Inside the bulge, for $0 < r < R$, we get

$$v_L = \sqrt{\frac{3GM}{R}} \tag{22}$$

and

$$\alpha = arctan\left(\sqrt{\frac{3R^2}{r^2} - 1}\right), \tag{23}$$

and in units of R

$$\alpha = arctan\left(\sqrt{\frac{3}{r^2} - 1}\right). \tag{24}$$

The constant value of the total intrinsic velocity of space in our constant Lagrangian model galaxy is, in units of GM/R, given by the simple $v_L = \sqrt{3}$. The inward angle α can be calculated for every value of r, see Fig.(5). If one calculates a range of directed values for $\vec{v}_L$ and arranges them head-tail, a spiral results, four of which we depicted in Fig. (6) for the region of $0 < r < 18R$. These spiral curves are the free fall geodesics of out constant Lagrangian model galaxy. In Fig.(7) we added the orbits and radials around the bulge with $R = 1$ as the first circle in the middle of the figure. So the free fall velocity-of-space spirals inwards, with $\alpha \approx 90^o$ for $r \approx 0$. In Fig.(8), we fitted two of the constant Lagrangian spirals onto the spiral model of [6], their fig. 2.

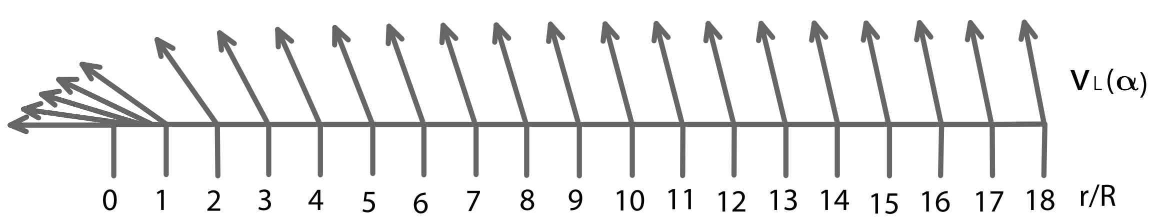

FIG. 5. $\vec{v}_L$ for several values of r/R, with α calculated with Eq.(24) and Eq.(21).

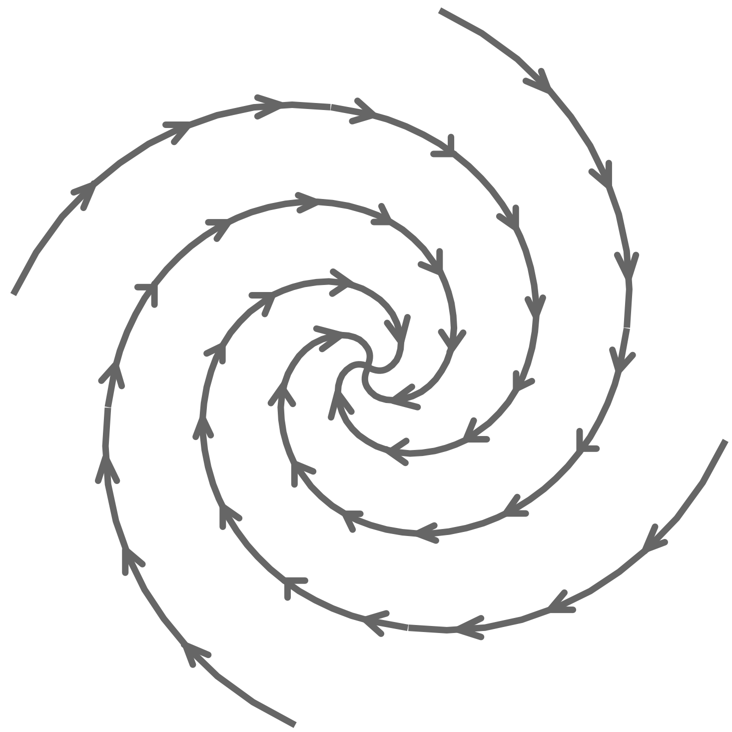

FIG. 6. Free fall spirals in the constant Lagrangian model galaxy from $r/R = 0$ to $r/R = 18$.

III. SOME ISSUES OF GALACTIC DYMAMICS FROM THE PERSPECTIVE OF OUR CONSTANT LAGRANGIAN MODEL GALAXY

In our constant Lagrangian model galaxy, we have a free fall metric following the spiral curves defined by the formulas for the angle α, Eq.(24) and Eq.(21). In our model galaxy, these metric free fall spirals act as galactic conveyor belts of space right into the centre of the galaxy, where it would be nice to have a SMBH to absorb all that space. But at the same time, mass that is caught inside such a spiral,will, due to its own inertia, move along with space, right into the SMBH in the centre, in a dynamic free fall. The free fall metric of our constant Lagrangian model galaxy will be a natural growth engine for central galactic black holes, which are needed to stabilize, by absorption, the dynamics of the continuing inflow of space. What is new in our spiral model compared to other models of galactic spirals is that our free fall spiral continues inside the bulge right towards the centre of the galaxy, where the SMBH is located. So our constant Lagrangian model galaxy will have trapped mass that spirals inwards on the disk and in the bulge, right into a, thus forever growing, SMBH at the centre. The growth of SMBHs at the centre of galaxies has, in

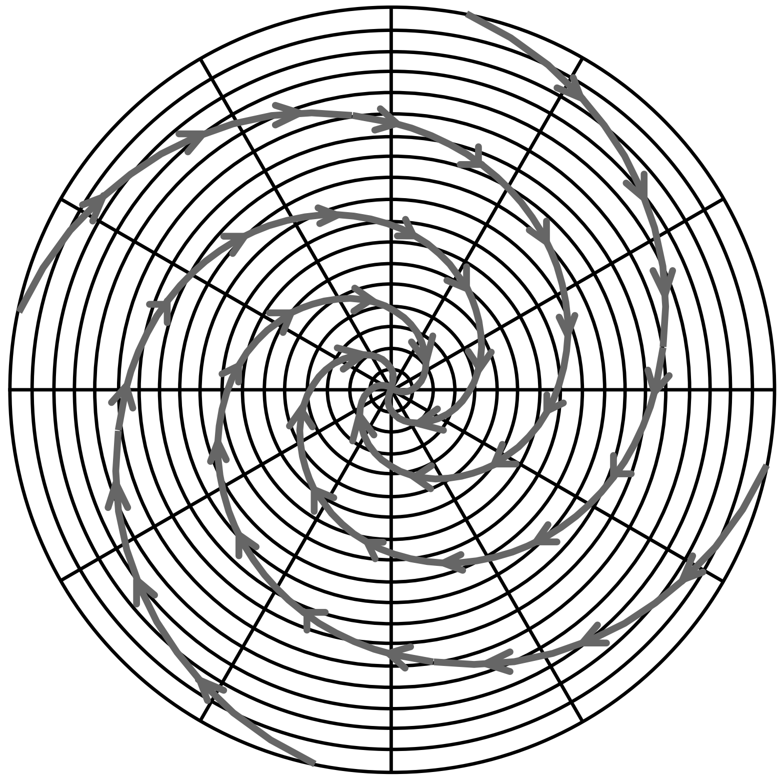

FIG. 7. Free fall spirals with depicted circular orbits and radial lines from $r/R = 0$ to $r/R = 18$.

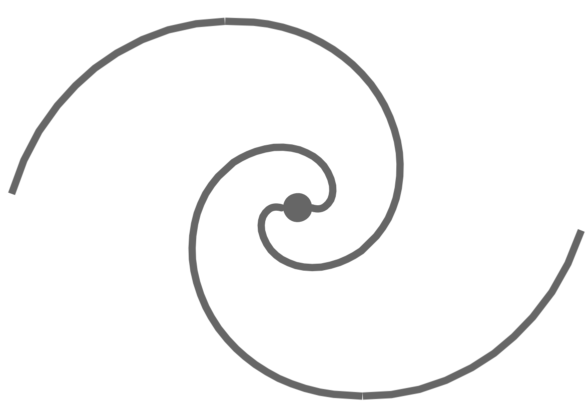

FIG. 8. Two constant-Lagrangian spirals fitted onto the model of [6], fig. 2.

our model, a galactic internal conveyor belt system as its explanation, besides the usual black hole merger growth theory [7].

Our constant Lagrangian model galaxy also provides a natural explanation for the spiral form of many disk galaxies: the combination of the *metric free fall spiral velocity-of-space field* combined with gravitating massess non-homogeneously trapped in such a velocity field can cause the trapped masses to congregate in distinct spiral conveyor belts towards the SMBH at the centre of the galaxy.

Such an galactic internal dynamics towards spiral congregation of mass has also been eluding the theorists, as illustrates the quote: *The old puzzle of the spiral arms of galaxies continues to taunt theorists* [8].

IV. THE BARYONIC TULLY-FISHER RELATION

Our constant Lagrangian model galaxy can provide an explanation of the B-TF relation in it's MOND version $v_{final}^4 \propto M_{baryonic}$. We have, for v_{final} and with $M = M_{bulge}$:

$$v_{final}^2 = \frac{3GM}{R}, \tag{25}$$

so we also have

$$v_{final}^4 = \frac{9GM}{R^2} GM = 9GMg_{bulge} \tag{26}$$

with

$$g_{bulge} = \frac{GM}{R^2}. \tag{27}$$

So, in short, we have

$$v_{final}^4 = 9g_{bulge}GM. \tag{28}$$

Now, if the gravitational acceleration at the border of the bulge of a large number of bulges has some mean value, and one replaces g_{bulge} by this mean value $\bar{g}_{bulge}$, then we get

$$v_{final}^4 = 9G\bar{g}_{bulge}M. \tag{29}$$

in which $\bar{g}_{bulge}$ has become independent of the mass of an individual bulge M.

MOND explains the B-TF relation using

$$v_{final}^4 = Ga_0M_{bar}. \tag{30}$$

with M_{bar} as the baryonic mass of the entire galaxy [5].

In our constant Lagrangian model galaxy, the B-TF relation is not a fundamental relation. It occurrence has its grounds in eqn.(25) and in the necessity for g_{bulge} to have a mean value with little room for smaller values, because then the bulge would dissipate, and only some room for larger values because then it would collapse on itself.

V. NEWTONIAL DYNAMICS, GENERAL RELATIVITY AND THE STANDARD MODEL IN RELATION TO THE CONSTANT LAGRANGIAN METRIC IN OUR MODEL GALAXY

Our model also solves the problem we had with the earlier version of the constant Lagrangian model galaxy. We can now add ordinary mass to the metric, such as stars and gas-clouds, without restricting them to the constant Lagrangian intrinsic velocity. Stars can still orbit the bulge of the galaxy on circular paths, see Fig.(7). Such an orbit will have the stars move in and out of spirals. Newtonian dynamics, including the virial theorem, can still be used to calculate those orbits, because all that is needed for a star to orbit the bulge is an extra, extrinsic orbital velocity compensating the radial intrinsic free fall velocity. This can be calculated using Newton's laws. The intrinsic orbital velocity doesn't influence this because the star will just by its inertia move along with the orbiting part of the metric. This orbit of the star can be clockwise or counter-clockwise, depending on the history of the star itself. From the perspective of a rotating star, the virial theorem is still valid, relative to local space, in his circular orbit. Because we modified the metric background, we don't need to additionally modify Newtonian dynamics on the foreground.

For larger r, the radial intrinsic velocity will be small and the orbital intrinsic velocity large, so the net orbital rotation velocity of stars will more and more equal to the intrinsic orbital velocity according to the constant Lagrangian model galactic velocity curve of Fig.(1). And one can expect that especially large gas-clouds in the outer ranges of galaxies will be caught in the metric velocity-of-space spiral field. So the more the Newtonian potential becomes insignificant, the more the modified metric will dominate the scene. This rules out the possibility to find a General Relativity cause for our modified metric, because the Einstein Equations are designed to reduce to Newton's theory of gravity in the case of ever weaker fields. But in the constant Lagrangian model galaxy, the intrinsic velocity-of-space is constant from the outer range of the galactic disk to close to the SMBH in the centre of the bulge, only its directional angle changes.

The inapplicability of GR to explain galactic rotation curves motivates the attempts to apply the Standard Model to the mysteries of galactic dynamics, by looking for new particles such as WIMP's (weakly interacting massive particles). But because we replaced several existing mysteries of galactic dynamics by a single new mystery (why should there be a constant Lagrangian metric in our model galaxy), we just unified smaller mysteries into an overarching mystery. Did we solve anything? Can we, on such a basis, rule out the Dark Matter hypothesis? The Dark Matter hypothesis has its reasons in several independent observations, of which the galactic rotation curve mystery is just one. I think that the constant Lagrangian metric is connected to the Hubble expansion of space and therefore might be related to phenomena for which the Dark Energy hypothesis has been formulated.

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