

# New Research on High-Frequency Circuits and Electromagnetic Radiation

**[Abstract]** In this paper, capacitance is divided into radiation capacitance and energy storage capacitance, and inductance is divided into radiation inductance and energy storage inductance. The distributed capacitance and distributed inductance existing in high-frequency circuits are mainly distributed radiation capacitance and distributed radiation inductance. Further, this paper reveals that the capacitance and inductance in LC resonance must be energy storage capacitance and energy storage inductance; Kirchhoff's Law is no longer valid in high frequency circuits. Based on these new findings it is pointed out that the transmission line equation is not valid and a new transmission line equation is proposed. This paper provides a new theoretical basis for electromagnetic field radiation, electromagnetic compatibility and antenna design.

**[Keywords]** high-frequency circuits; distribution parameters; radiation capacitance; storage capacitance; radiation inductance; storage inductance; LC resonance; transmission line equation; electromagnetic radiation; Kirchhoff's Law.

## 1. Introduction

Passive components in high-frequency circuits can be broadly categorized into two types: aggregate-parameter components and distributed-parameter components.

Collective parameters passive components are mainly resistors, capacitors and inductors (inductive coils) and connecting wires. An actual resistor, capacitor and inductor coil, at low frequencies, is mainly characterized by resistance, capacitance or inductance characteristics, i.e. nominal characteristics. For example, the electrical characteristic of a resistor is its resistance value  $R$  and is independent of frequency. The electrical characteristics of a capacitor or inductor coil are its capacitance value  $C$  and inductance value  $L$ . Connecting wires have zero resistance, capacitance and inductance at low frequencies. However, at high frequencies, their electrical characteristics change and these changes are mainly due to the distribution parameters of the components. Therefore, not only do the values of the nominal characteristics of these components change when they are used at high frequencies, but they also exhibit impedance characteristics that are not present in the nominal characteristics. These characteristics reflected by the distribution parameters affect the high-frequency characteristics of the components. Distribution parameters that affect the electrical characteristics of passive components are mainly loss resistance, distribution capacitance and distribution inductance.

The theory of the above distribution parameters is often inconsistent with the actual engineering applications, especially when the connecting wires and circuit board dimensions are comparable to the wavelength of the high-frequency signals, the theory and the actual applications are even opposite.

## 2. High frequency circuits

### 2.1 Capacitance

Capacitance is the ability of a conductor to store charge. The capacitance of a conductor is defined as the ratio of the charge to the potential.

$$C = q / u$$

The unit of capacitance is F, which is a very large unit, and the Earth's capacitance is about 710  $\mu$ F.

A conductor with an electric charge that generates an electric field and electric flux around it.

### 2.1.1 Isolated capacitance/radiation capacitance

When a single conductor is surrounded by free space and is far from other objects, such a conductor is an isolated conductor. The capacitance of an isolated conductor is related only to its shape and size, and is an intrinsic property of the isolated conductor, independent of charge and potential.

The capacitance of an isolated conducting sphere of radius R:

$$C = 4 \pi \epsilon_0 R$$

Shown in Fig. 1, is a sphere of insulated conductors, and let its radius  $R = 0.1$  m. The capacitance  $C = 10$  pF of the isolated conductor sphere.

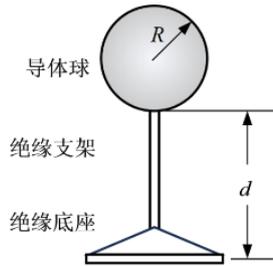


Fig. 1 Insulated conductor sphere

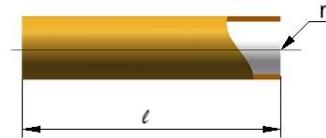


Fig. 2 Isolated conducting cylinder

As shown in Fig. 2, an isolated conducting cylinder of radius  $r$  and length  $l$ . Let the charge be distributed only on the surface of the cylinder, the capacitance of the isolated conducting cylinder:

$$C = 2 \pi \epsilon_0 (2 r l)^{1/2} \quad (2-1)$$

The electric field generated by an isolated conductor is distributed in the open free space around it, and when the charge  $q(t)$  on the isolated conductor is time-varying, the isolated conductor will radiate electric field waves to the free space around it, so the isolated capacitance is also known as open-radiation capacitance, or radiation capacitance.

### 2.1.2 Capacitors/storage capacitors

Two conductors that are close to each other with a layer of non-conductive insulating medium sandwiched between, form a capacitor. Capacitors are components that store electricity and energy, playing an important role in circuits such as tuning, bypassing, coupling, and filtering.

$$C = \frac{Q}{U_A - U_B} = \frac{\epsilon_r S}{4\pi k d}$$

The most common type of capacitor is the parallel plate capacitor, which consists of two metal conductor pole plates parallel to each other, separated by a dielectric material. Formula for calculating the capacitance of a parallel plate capacitor:

Where  $U_A - U_B$  is voltage between the two parallel plates,  $\epsilon_r$  is relative dielectric constant,  $k$  is electrostatic force constant,  $S$  is the area directly opposite the two plates, and  $d$  is the distance between the two plates. The electric field between two parallel plates of a parallel plate capacitor is a uniform electric field with electric field strength  $E = U / d$ .

Parallel plate capacitor is a storage of power and electrical energy components, generally considered to have its electric field energy closed, concentrated between the two metal parallel plates, do not radiate electric field energy to free space. Capacitors are also known as enclosed energy storage capacitors, energy storage capacitors or capacitors.

In summary, capacitors are categorized into radiation capacitors and energy storage capacitors. In LC resonant circuits, capacitor C is also an energy storage capacitor, and the electric field energy stored in capacitor C and the magnetic field energy stored in inductor L are converted to each other. In high-frequency circuits, there is a non-negligible radiation capacitance in the lines of the PCB board, and even for capacitors, there is also a non-negligible radiation capacitance in their pins. In high-frequency circuits, EMC electromagnetic compatibility, transmission line and antenna design and application, lead distribution capacitance, in consideration of the energy storage distribution capacitance at the same time, must consider the impact of radiation distribution capacitance; especially for antennas, distribution capacitance is mainly manifested as radiation distribution capacitance.

Figure 2A shows the identification of the storage capacitor in the circuit and Figure 2B shows the identification of the radiation capacitor.



Fig. 2A Energy storage capacitance

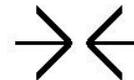


Fig. 2B Radiation capacitance

## 2.2 Inductance

When a metal wire is passed through an electric current, a magnetic field and magnetic flux are created around it. The inductance of a wire is defined as the ratio of magnetic flux to current.

$$L = \Phi / I$$

The unit of inductance is H (Henry).

### 2.2.1 Isolated inductance/radiation inductance

When a long metallic straight wire is surrounded by free space and is located away from other objects, such a wire is an isolated wire. This is shown in Figure 3.

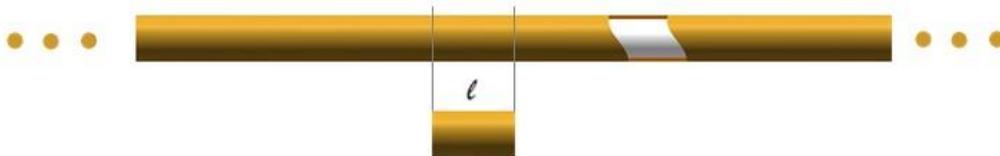


Fig. 3 Inductance of an isolated wire

In the metal long straight wire to take the length of l a section of the wire, then the section of the isolated wire, the isolated inductance of the approximate formula:

$$L = \mu_0 l / 2 \tag{2-2}$$

An isolated inductor produces a magnetic field that is distributed in the open free space around it. When the current in the isolated inductor is time-varying, the isolated inductor will generate an alternating magnetic field and radiate magnetic field waves into free space, so the isolated inductor is also called an open radiation inductor, or radiation inductor.

### 2.2.2 Inductors/Storage Inductors

An inductor is a component that converts electrical energy into magnetic energy and stores it. The most common type of inductor is a metal wire coil, which is generally made of enameled wire, and the inductance of the coil is proportional to its number of turns. The inductance of a coil is simply a parameter related to the number of turns, size and shape of the coil and the dielectric; it is an intrinsic characteristic of the coil independent of the applied current.

Inductors are capable of converting electrical energy into magnetic energy and storing it as a component, and are generally considered to have their magnetic energy enclosed and concentrated inside the center of the coil, without radiating magnetic energy to the outside free space. Inductors are also known as closed energy storage inductors, energy storage inductors or inductors.

In summary, inductors are categorized into radiation inductors and energy storage inductors. In LC resonant circuits, inductor L is an energy storage inductor, and the magnetic field energy stored in inductor L and the electric field energy stored in capacitor C are converted to each other. In high-frequency circuits, there is a non-negligible radiation inductance in the lines of the PCB board. In high-frequency circuits, EMC electromagnetic compatibility, transmission line and antenna design and application, the introduction of the aggregate parameter and the distribution parameter inductance, in the consideration of the energy storage inductance at the same time, it is necessary to consider the impact of radiation inductance; especially for antennas, the radiation inductance is the main conversion of energy of its magnetic field energy.

FIG. 4A shows the identification of the storage inductor in the circuit and FIG. 4B shows the identification of the radiation inductor in the circuit.



Fig. 4A Energy storage inductor



Fig. 4B Radiation inductor

### 2.3 Connecting Cables

Connection lines are often referred to as transmission lines in RF circuits and are used to transmit electrical energy and electrical signals. At low frequencies, the resistance, capacitance and inductance of a connecting line are zero. However, at high frequencies, the connection line must be considered for its distributed resistance, distributed capacitance and distributed inductance. Distributed resistance of the connecting line produces heat loss, distributed capacitance and distributed inductance of the connecting line is mainly manifested in the distribution of radiation capacitance and distribution of radiation inductance, which produces radiated electric field and radiated magnetic field, i.e., radiation loss. For a single conductor connection line, its distributed radiation capacitance is approximated by the formula (2-1), and the distributed radiation inductance is approximated by the formula (2-2).

In order to reduce the loss of the connecting line radiated electric field and radiated magnetic field, RF circuits are often used in parallel dual conductor, parallel multi-conductor, coaxial, strip line, etc., so that the distribution capacitance of the transmission line and the distribution of inductance is more often manifested as a distribution of energy storage capacitance and distribution of energy storage inductance.

## 2.4 Resonant circuits

A passive one-port network containing capacitive, inductive, and resistive elements, the circuit is in resonance when the port equivalent impedance is resistive. Common resonant circuits are RLC series resonant circuits and parallel resonant circuits. Figure 5 shows an RLC series resonant circuit.

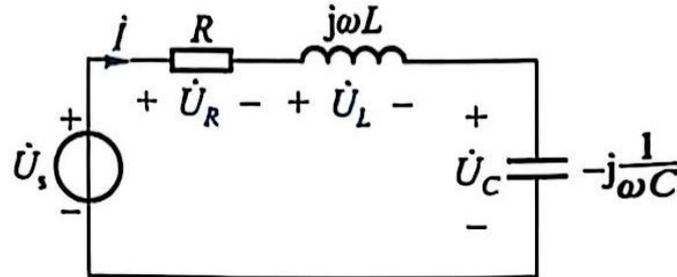


Fig. 5 RLC series resonant circuit

Its equivalent impedance at the power supply side:

$$Z(\omega) = R + j(\omega L - 1/\omega C)$$

When  $(\omega L - 1/\omega C) = 0$ , the circuit is in resonance. The angular frequency at this point is called the resonant angular frequency:

$$\omega_0 = (LC)^{1/2}$$

When the circuit is in resonance, it has the following characteristics:

(1) At resonance the port impedance is resistive and the amplitude of the port current is maximized. The electric field energy of the capacitor forms a periodic oscillation with the magnetic field energy of the inductor.

$$\dot{U}_L(\omega_0) + \dot{U}_C(\omega_0) = 0$$

(2) The total series voltage of the inductor and capacitor at resonance is equal to zero, which is equivalent to a short circuit.

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R} = \frac{1}{R} \sqrt{\frac{L}{C}}$$

(3) Overvoltage may occur on the inductor and capacitor at resonance, such that

$Q$  is the quality factor of the circuit. When  $Q \gg 1$ , then  $U_L(\omega_0) = U_C(\omega_0) \gg U_s$ , the voltage of the inductor and capacitor is much higher than the supply voltage, which is known as an overvoltage phenomenon in power systems, and poses an overvoltage hazard to power system equipment. However, in communication systems, series resonance amplifies weak signals.

In summary, the essence of LCR circuit resonance is that the electric field energy in the capacitor and the magnetic field energy in the inductor is converted to each other, increasing and decreasing, and fully compensated. The sum of the electric field energy and magnetic field energy remains constant at any moment. Therefore, the capacitance and inductance in the above resonant circuit must be energy storage capacitance and energy storage inductance, and the radiation capacitance and radiation inductance are not applicable to the above resonant circuit analysis.

### 3. transmission line theory and challenges to transmission line theory

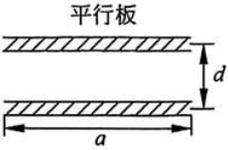
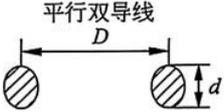
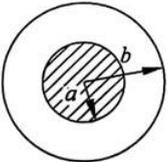
#### 3.1 Transmission Line Theory

Transmission line means a connector capable of transmitting electrical energy and electrical signals. In low-frequency circuits, the transmission line serves only to connect the electrical components. The distribution parameters of the transmission line itself are negligible. For example, a table lamp power line is 2 meters long, the operating frequency of its power supply is 50Hz, and the wavelength is 6000 kilometers. This power line is very short in relation to the wavelength and the effect of the wavelength does not need to be considered.

In the case of high-frequency circuits, where the signal wavelength and the dimensions of the connecting line can be compared with each other, there is already a significant fluctuation effect on the connecting line. The current and voltage on the transmission line are not only a function of time, but also of spatial position. At this point the distributed parameter effects of the parameter lines themselves must be considered.

According to the existing transmission line theory: the wire flows through the current, around the high frequency magnetic field will be generated, and thus the points along the wire will exist in series distribution inductance  $L$ ; between the two wires plus the voltage, there is a high-frequency electric field between the wires, so the line between the distribution of capacitance will be generated in parallel  $C$ ; conductor through the current will be heated, and the high frequency due to the skin effect of the resistance will be increased, this is the distribution of resistance  $R$ ; conductor current leakage, this These distribution parameters can be ignored at low frequencies, but the high frequency caused by the voltage along the line, the current amplitude changes, as well as phase lag, which is known as the distribution parameter effect. Table 3-1 shows the distribution parameters of parallel plates, parallel twin conductors and coaxial lines.

Table 3-1 Distribution parameters of common transmission lines

传输线的横截面 单位长度的分布参数	平行板 	平行双导线 	同轴线 
$R/(\Omega \cdot m^{-1})$	$\frac{2}{a} \sqrt{\frac{\pi f \mu_0}{\sigma_0}}$	$\frac{2}{\pi d} \sqrt{\frac{\omega \mu_0}{\sigma_0}}$	$\sqrt{\frac{f \mu_0}{4 \pi \sigma_1}} \left( \frac{1}{a} + \frac{1}{b} \right)$
$G/(S \cdot m^{-1})$	$\frac{\sigma a}{d}$	$\frac{\pi \sigma}{\ln \frac{D + \sqrt{D^2 - d^2}}{d}}$	$\frac{2 \pi \sigma}{\ln \frac{b}{a}}$
$L/(H \cdot m^{-1})$	$\frac{\mu_0 d}{a}$	$\frac{\mu_0}{\pi} \ln \frac{D + \sqrt{D^2 - d^2}}{d}$	$\frac{\mu_0}{2 \pi} \ln \frac{b}{a}$
$C/(F \cdot m^{-1})$	$\frac{\epsilon d}{a}$	$\frac{\pi \epsilon}{\ln \frac{D + \sqrt{D^2 - d^2}}{d}}$	$\frac{2 \pi \epsilon}{\ln \frac{b}{a}}$

In the table,  $\epsilon$  and  $\sigma$  are the dielectric constant and conductivity of the medium between the conductors, respectively;  $\sigma_1$  is the conductivity of the conductor, and  $\mu_0$  is the magnetic permeability of the conductor and the medium.

The following transmission line equations are derived from the parallel twin conductor. A microelement of length  $\Delta z$  is intercepted on the parallel twin conductor, and the equivalent circuit of the distributed parameters of this microelement segment is shown in Fig. 6. Where the voltages and currents at the two ends of the equivalent circuit are  $u(z, t)$ ,  $i(z, t)$ ,  $u(z+\Delta z, t)$ ,  $i(z+\Delta z, t)$ , respectively. It has a resistance  $R\Delta z$  (in  $\Omega/m$ ), a distributed inductance  $L\Delta z$  (in H/m), a distributed capacitance  $C\Delta z$  (in F/m) and a leakage conductance  $G\Delta z$  (in S/m).

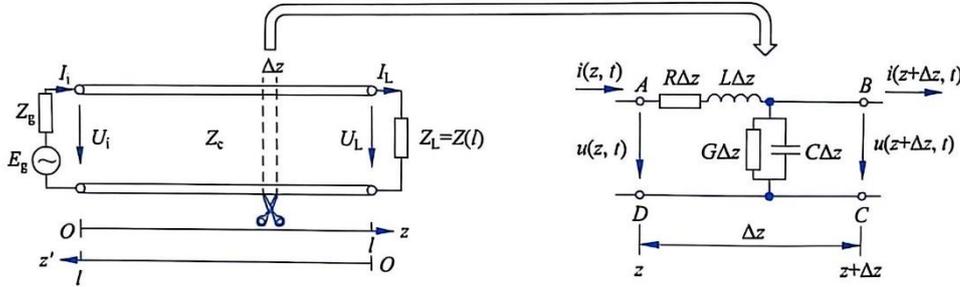


Fig. 6 Distributed parameter equivalent circuit for length  $\Delta z$  parallel twin conductors

From Fig. 6, this is obtained from Kirchhoff's voltage and current laws:

$$\begin{aligned} u(z, t) - R\Delta z i(z, t) - L\Delta z \frac{\partial i(z, t)}{\partial t} - u(z + \Delta z, t) &= 0 \\ i(z, t) - G\Delta z u(z + \Delta z, t) - C\Delta z \frac{\partial u(z + \Delta z, t)}{\partial t} - i(z + \Delta z, t) &= 0 \end{aligned} \quad (3-1)$$

makes  $\Delta z$  converge to 0 and further partial derivation of  $z$  is obtained:

$$\begin{aligned} \frac{\partial u(z, t)}{\partial z} &= -Ri(z, t) - L \frac{\partial i(z, t)}{\partial t} \quad (a) \\ \frac{\partial i(z, t)}{\partial z} &= -Gu(z, t) - C \frac{\partial u(z, t)}{\partial t} \quad (b) \end{aligned} \quad (3-2)$$

so that the voltage and current at point  $z$  are  $U(z)$ ,  $I(z)$ ,  $Z = R + j\omega L$ ,  $Y = G + j\omega C$ , which can be obtained from equation (3-2):

$$\begin{aligned} \frac{d^2 U(z)}{dz^2} - ZYU(z) &= 0 \quad (a) \\ \frac{d^2 I(z)}{dz^2} - ZYI(z) &= 0 \quad (b) \end{aligned} \quad (3-3)$$

Eq. (3-3) is the well-known transmission line equation, which is a "second-order chi-squared ordinary differential equation".

### 3.2 Questioning the transmission line theory

In the derivation of equation (3-3) transmission line equation, the distributed capacitance and distributed inductance are energy storage capacitor and energy storage inductor. In fact, parallel twin conductors flow through the current, the resulting high-frequency magnetic field, almost all radiated to the space around it, stored in the parallel twin conductors between the magnetic energy is approximated to be 0, that is, the parallel twin conductors of the distribution of inductance for the distribution of radiation inductance. Similarly, the distributed capacitance of the parallel twin conductor is mainly manifested as the distributed radiation capacitance.

In addition, Kirchhoff's laws apply to linear DC circuits. In low and medium frequency AC circuits, Kirchhoff's law can be used as an engineering approximation; in high frequency circuits, especially when the transmission line distribution parameters cannot be neglected, Kirchhoff's law no longer holds (see Appendix A). Therefore, the transmission line equation of Eq. (3-3) is not valid.

In parallel twin conductors, the transmission line distribution parameters are almost entirely distributed radiation capacitance and distributed radiation inductance, and the distributed energy storage capacitance and distributed energy storage inductance can be ignored. High-frequency signals in the parallel twin conductor transmission process, radiation loss is larger. Coaxial line there is a large distributed energy storage capacitance, high-frequency signal in the coaxial line transmission process, the radiation loss is relatively small. The transmission line equation of coaxial line must consider the distributed radiation capacitance, distributed radiation inductance and distributed energy storage capacitance.

The transmission characteristics of a high-frequency signal in an independent single conductor are analyzed below. A microelement of length  $\Delta z$  is intercepted in the independent conductor, and the equivalent circuit of the distribution parameters of this microelement segment is shown in Fig. 7. Where the instantaneous voltages and currents at both ends of the equivalent circuit are  $u(z, t)$ ,  $i(z, t)$ ,  $u(z+\Delta z, t)$ ,  $i(z+\Delta z, t)$ , respectively. It has a resistance  $R\Delta z$  (in  $\Omega/m$ ), a distributed radiation inductance  $L\Delta z$  (in H/m), and a distributed radiation capacitance  $C\Delta z$  (in F/m).

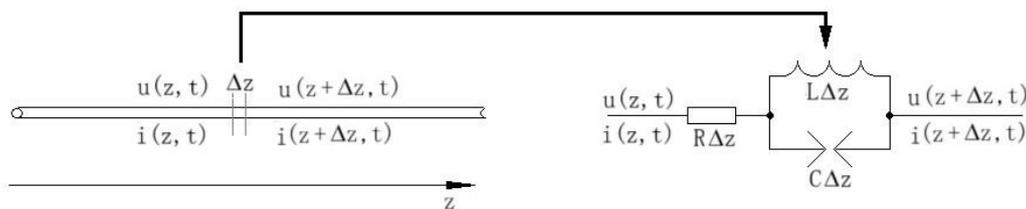


Fig. 7 Distribution parameter equivalent circuit for length  $\Delta z$  independent conductor

Let the effective values of  $u(z, t)$ ,  $i(z, t)$ ,  $u(z+\Delta z, t)$ ,  $i(z+\Delta z, t)$  be  $U(z)$ ,  $I(z)$ ,  $U(z+\Delta z)$ ,  $I(z+\Delta z)$ , respectively; the power of thermal loss on the distributed resistance  $R\Delta z$  on the heat loss power is  $W_R$ , the radiation loss power on the distribution radiation inductance  $L\Delta z$  is  $W_L$ , and the radiation loss power on the distribution radiation capacitance  $C\Delta z$  is  $W_C$ , and based on the principle of conservation of energy, then the following equations are available:

$$U(z) I(z) = W_R + W_L + W_C + U(z+\Delta z) I(z+\Delta z) \quad (3-4)$$

Formula (3-4) explains: high-frequency current in the transmission line flow project, due to the distribution of resistance of the thermal loss, distribution of radiation inductance of the magnetic field radiation loss as well as distribution of radiation capacitance of the electric field radiation loss, along with the lengthening of the transmission line, the effective value of its current and potential (or the peak value) due to the loss becomes smaller. In addition, different locations of the transmission line, a moment of instantaneous current and potential there is a phase difference, that is, different locations of the transmission line, a moment of instantaneous current and potential are not equal. Figure 8 shows the loss attenuation line of a 100 MHz high-frequency signal on a transmission line, which is generally an exponential curve.

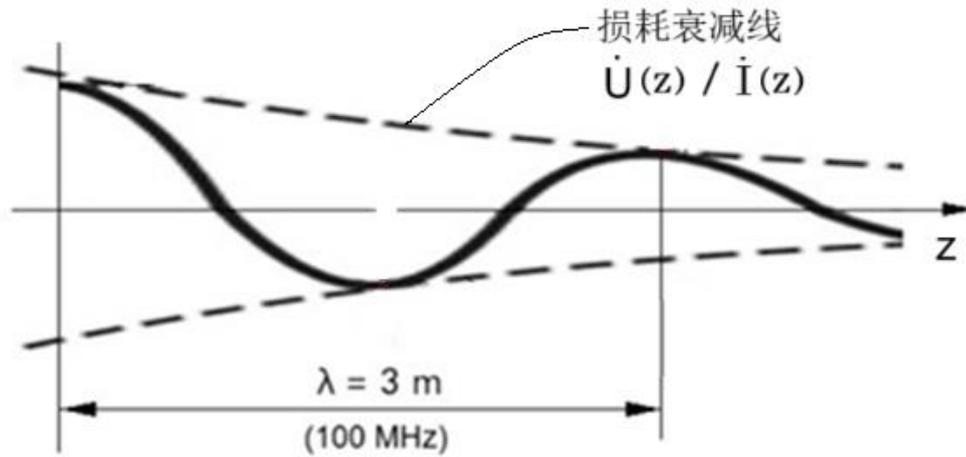


Fig. 8 Attenuation of 100 MHz RF signal on transmission line

The purpose of the antenna is to radiate electric and magnetic field waves into free space, and the distribution parameters on the antenna are mainly the distributed radiation inductance and distributed radiation capacitance, which can be completely ignored for the distributed energy storage capacitance and distributed energy storage inductance.

Eq. (3-4) considers only the incident wave on the transmission line, not the reflected wave.

## Appendix A

### Kirchhoff's law does not apply to AC circuits

Kirchhoff laws (Kirchhoff laws) is the voltage and current in the circuit follow the basic laws, is the analysis and calculation of complex circuits, in 1847, Kirchhoff) in its epoch-making circuit theory text "on the study of the linear distribution of the current obtained by the solution of the equation ". Kirchhoff's laws include Kirchhoff's current law (KCL) and Kirchhoff's voltage law

(KVL). Kirchhoff's laws were initially used for the analysis of DC linear circuit, then developed for the analysis of AC circuit, and extended to the analysis of steady-state and transient circuits of LCR nonlinear circuit.

Does Kirchhoff's law apply to alternating circuits? The answer is no. In low and medium frequency AC circuits, Kirchhoff's law can be used as an approximate formula; in high frequency circuits, Kirchhoff's law no longer holds.

Kirchhoff's first law, also known as Kirchhoff's current law and abbreviated as KCL, is a manifestation of the continuity of the current in lumped parameter circuit, and its physical background is the axiom of conservation of charge. Kirchhoff's current law is a law that determines the relationship between the currents in each branch of a circuit at any node. Assuming that the current into a node is positive and the current out of this node is negative, the algebraic sum of all currents involved in this node equals zero. Expressed as an equation,

$$\sum_{k=1}^n i_k = 0$$

Kirchhoff's current law states that the sum of all currents entering a node is equal to the sum of all currents leaving this node.

The following proves that Kirchhoff's current law does not hold in an AC circuit. As shown in Fig. A1, a sinusoidal AC circuit is connected in series with a resistor R. The sinusoidal signal has an angular frequency  $\omega$ , a wavelength  $\lambda$ , and a sinusoidal current with a peak value of  $I_p$ . A microelement with a length of  $\Delta z$  is intercepted in the AC circuit wire, and the microelement  $\Delta z$  has only one current input,  $i_{in}(t)$ , and one current output,  $i_{out}(t)$ , which is the simplest node.

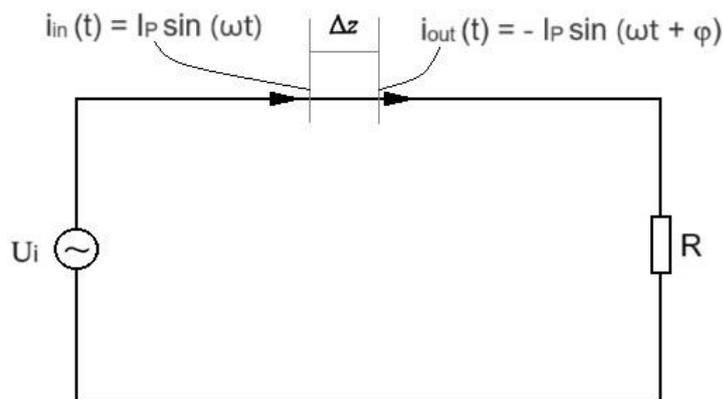


Fig. A1  $\Delta z$  microelement simple node

Let the input current at moment  $t$  of the node of the micro element  $\Delta z$ :

$$i_{in}(t) = I_P \sin(\omega t) \quad (A-1)$$

Then the output current of the node of the micro element  $\Delta z$  at the moment  $t$ :

$$i_{out}(t) = - I_P \sin(\omega t + \varphi) \quad (A-2)$$

where  $\varphi = 2\pi(\Delta z/\lambda)$  is the phase difference.

In practical engineering applications, taking the nodal microelement 1.0 mm, the input current  $i_{in}(0) = 0$  from equation (A-1) when the circuit operates at a frequency of 50 Hz and at  $t = 0$ .

At this point, the wavelength of the sinusoidal signal  $\lambda = 6 \times 10^6$  m, then the phase angle  $\varphi = 6 \times 10^{-8}^\circ$ . The output current is obtained from equation (A-2):

$$\begin{aligned} i_{out}(t) &= - I_P \sin(\varphi) \\ &= - 1.05 \times 10^{-9} I_P \end{aligned}$$

The current flowing out of the node is not equal to the current flowing into the node, but they are approximately equal. Therefore, Kirchhoff's current law can be used as an approximate formula under low-frequency AC signals.

When the operating frequency of the circuit is 10 GHz, the input current  $i_{in}(0) = 0$  from equation (A-1) at  $t = 0$ . At this time, the wavelength of the sinusoidal signal  $\lambda = 0.03$  m, then the phase angle  $\varphi = 12^\circ$ . The output current is obtained from equation (A-2):

$$\begin{aligned} i_{out}(t) &= - I_P \sin(\varphi) \\ &= - 0.21 I_P \end{aligned}$$

The current flowing out of the node is clearly not equal to the current flowing into the node. With high frequency AC signals, Kirchhoff's current law no longer holds.

When the node  $\Delta z$  is taken to be 0, the face of current inflow is the same face of current outflow, so  $\Delta z$  cannot be taken to be 0. However, it is possible to calculate the relative rate of change of the algebraic sum of currents at the node as the length of the microelement tends to zero. The algebraic sum of the currents flowing into and out of the node of the microelement  $\Delta z$  at the moment  $t$ :

$$\begin{aligned} i_{in}(t) + i_{out}(t) &= I_P \sin(\omega t) - I_P \sin(\omega t + \varphi) \\ &= I_P (\sin(\omega t) - \sin(\omega t) \cos(2\pi(\Delta z/\lambda)) - \cos(\omega t) \sin(2\pi(\Delta z/\lambda))) \end{aligned}$$

Makes  $\Delta Z$  converge to 0, the relative rate of change of the algebraic sum of the nodal currents. Calculate the following limit:

$$\begin{aligned}
 \lim_{\Delta Z \rightarrow 0} \frac{I_P (\sin(\omega t) - \sin(\omega t) \cos(2\pi(\Delta Z/\lambda)) - \cos(\omega t) \sin(2\pi(\Delta Z/\lambda)))}{\Delta Z} \\
 &= \frac{I_P (\sin(\omega t) - \sin(\omega t) - \cos(\omega t) (2\pi(\Delta Z/\lambda)))}{\Delta Z} \\
 &= \frac{-I_P (2\pi\Delta Z/\lambda) \cos(\omega t)}{\Delta Z} \\
 &= -I_P (2\pi/\lambda) \cos(\omega t) \tag{A-3}
 \end{aligned}$$

In the above derivation, when  $\Delta Z$  tends to 0, one can take  $\cos(2\pi(\Delta Z/\lambda)) = 1$ ,  $\sin(2\pi(\Delta Z/\lambda)) = 2\pi(\Delta Z/\lambda)$ . When  $\omega t = 0$ , or  $\omega t = \pi$ , equation (A-3) takes the maximum absolute value:  $I_P (2\pi/\lambda)$ .

From Eq. (A-3), it also follows that under low-frequency AC signals,  $2\pi/\lambda$  is approximated to be 0, and Kirchhoff's current law can be used as an approximate formula. Under high-frequency AC signals,  $2\pi/\lambda$  can be much larger than 1, and Kirchhoff's current law is no longer valid.

The microelement nodes of the above  $\Delta Z$  have distributed radiation capacitance and distributed radiation inductance under high-frequency operating conditions, and also need to take into account the electric-field radiation loss and magnetic-field radiation loss.

In summary, Kirchhoff's current law applies to linear DC circuits. In low and medium frequency AC circuits, Kirchhoff's current law can be used as an approximate formula; in high frequency circuits, Kirchhoff's current law no longer holds. Again, Kirchhoff's voltage law applies to linear DC circuits. In low and medium frequency AC circuits, Kirchhoff's voltage law can be used as an approximate formula; in high frequency circuits, Kirchhoff's voltage law no longer holds.