Monty Hall game simplified

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abstract
The game show problem aka Monty Hall problem [1], originated when
Craig Whitaker posed a question of a winning strategy for a 3 door game show
to Marilyn Savant who wrote articles for Parade magazine.
Her 1990 response was to switch doors when given the option. [2] [3]
The debate of probability of success as 2/3 vs 1/2 has continued until today.
This paper reveals errors in her response.

Whitaker's question
Suppose you're on a game show, and you're given the choice of three doors. Behind one door is a car, behind the others, goats. You pick a door, say #1, and the host, who knows what's behind the doors, opens another door, say #3, which has a goat. He says to you, "Do you want to pick door #2?" Is it to your advantage to switch your choice of doors?

Marilyn Savant's 1990 response [3]
Yes; you should switch. The first door has a 1/3 chance of winning, but the second door has a 2/3 chance.
The benefits of switching are readily proven by playing through the six games that exhaust all the possibilities. For the first three games, you choose #1 and "switch" each time, for the second three games, you choose #1 and "stay" each time, and the host always opens a loser...
For the first three games, you choose #1 and "switch" each
time, for the second three games, you choose #1 and "stay" each time, and the host always opens a loser. Here are the results.

	DOOR 1	DOOR 2	DOOR 3	RESULT
GAME 1	AUTO	GOAT	GOAT	Switch and you lose.
GAME 2	GOAT	AUTO	GOAT	Switch and you win.
GAME 3	GOAT	GOAT	AUTO	Switch and you win.
GAME 4	AUTO	GOAT	GOAT	Stay and you win.
GAME 5	GOAT	AUTO	GOAT	Stay and you lose.
GAME 6	GOAT	GOAT	AUTO	Stay and you lose.

fig.1

the simple game

The player wins 1 of 3 prizes, car, goat1, or goat2.
The player chooses door x. The host opens door x to verify the prize won.
The statistic used to measure success is w/n, with w=choose a door containing a car, and n=all possible choices. For this game, w/n=1/3. There is no 2nd choice.

the modified game

game rules
The game rules as proposed by Whitaker.
rule 1. the host cannot open the door from the players 1st guess.
rule 2. the host cannot open a door containing a car.
rule 3. the host must offer the player a 2nd guess.
Initial conditions: the player does not know the location of the car, thus they can only make a random guess.
The host knows the location of the car, thus their choice is not random, acknowledged by Savant:

"So let's look at it again, remembering that the original answer defines certain conditions, the most significant of which is that the host always opens a losing door on purpose. (There's no way he can always open a losing door by chance!)"

e	p	h	r
1	g1	g2	c
2	g2	g1	c
3	c	g1	g2
4	c	g2	g1

fig.2

In fig.2, e is event/game number, p is player 1st choice (always door 1), h is door opened by the host per the game rules, and r is the remaining closed door.

There are 3 distinct prizes, car c, goats g1 and g2. Fig.2 is essentially a table of host choices relative to each prize.

Rule 1 restricts the host from opening door 1.

If a goat is behind door 1, the host can open the door containing the other goat.

If c is behind door 1, the host can open both doors containing goats, but in separate games. Opening both in 1 game would reveal the location of the car, defeating the purpose of the game. The game rules require a 4th game. With Whitaker's modified game, per the rules, the host does not open the player's 1st chosen door, but opens a goat door removing it from play. The 3-door game is not completed, and the host has reduced the 3-door game to a 2-door game.

The player's 2nd choice is now between column p and column r.

Comparing p to r, there is no advantage.

Savant's game

N	e	p	h	r
n	1	g1	g2	c
n	2	g2	g1	c
n/2	3	c	g1	g2
n/2	3	c	g2	g1

fig.3

A column N has been added to fig.3 for number of games played.

If c is behind door 1, Savant only considers the host opening a g1 door half of games 3, and opening a g2 door for the other half of games 3.

I.e. (g1 or g2) instead of (g1 and g2). This produces a bias with c/n ratio of 1/3 for column p (stay) and c/n ratio of 2/3 for column r (switch).

All games should have the same format, and there is no reason to play one game differently from the others. Since the statistics for a 2-door game are the same as for a coin toss (1 of 2 outcomes), there should be no bias.

choice by prize

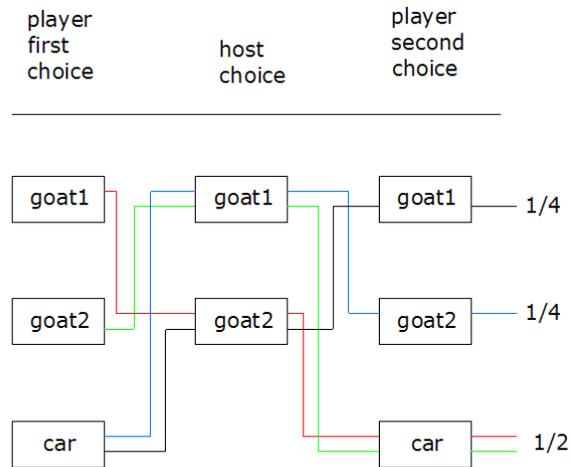| player first choice	host choice	player second choice	
goat1	goat1	goat1	1/4
goat2	goat2	goat2	1/4
car		car	1/2

fig.4

The success ratio (guess a car door)/(all possible guesses) relates to guesses, NOT locations, as in fig.4. Each colored path is 1 of 4 possible games, with the player switching. The 1st column is the stay prize. A comparison shows no advantage.

conclusion

The answer to Whitaker's question is no.

reference

[1} The American Statistician, August 1975, Vol. 29, No. 3
[2] game show problem, Wikipedia Sep 2024
[3] Marilyn vos Savant, https://web.archive.org/web/20130121183432/http://marilynvossavant.com
[4] Wikipedia, Gambler's fallacy, Apr 2025