Restoring Consistency to Quantum Mechanics by Interpreting the Observer

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Abstract

We set forward a set of axioms to interpret the observer in quantum mechanics. Our axioms treat observers as quantum states and provide rules for what they can observe. We demonstrate that our framework, which we call the NEW interpretation of Quantum Mechanics, provides a resolution of the Frauchiger-Renner paradox [1], restoring consistency to Quantum Mechanics. The resolution is brought about by a careful definition of 'Consistency', Assumption (C). With this precise definition, our interpretation satisfies all three Assumptions (Q), (C), and (S), without logical contradictions. The scope of this paper only extends to setting forward our axioms and addressing the Frauchiger-Renner paradox. Additional context for the NEW worldview, along with a matching NEW conjecture, is provided elsewhere [2].

1 Introduction

The Frauchiger-Renner paradox demonstrated the limitations of popular interpretations of quantum mechanics [1]. A no-go theorem they derive asserts that three natural-sounding assumptions, (Q), (C), and (S), cannot all be valid. Assumption (Q) captures the universal validity of quantum theory (or, more specifically, that an agent can be certain that a given proposition holds whenever the quantum-mechanical Born rule assigns probability 1 to it). Assumption (C) demands consistency, in the sense that the different agents' predictions are not contradictory. Finally, (S) is the requirement that, from the viewpoint of an agent who carries out a particular measurement, this measurement has one single outcome.

In this paper, we propose a NEW interpretation of quantum mechanics. Our axiomatic framework, which is better conceptualized as an interpretation of observers in quantum mechanics, is laid out in Section 2. We propose observers as effective quantum states capable of internal and external measurements. We analyze the Gedanken experiment proposed by Frauchiger and Renner in the language of the NEW interpretation in Section 3. We find that as long as we are precise in our definition of 'agents', our framework satisfies all three assumptions, (Q), (C), and (S). While the focus of this paper is the Frauchiger-Renner paradox, our proposal for interpreting observers has profound and vast implications, especially when combined with a NEW conjecture outlined in previous work [2]. We conclude with a brief discussion of this context in Section 4.

2 Axioms

- **Axiom 1:** The universe as a whole has its wavefunction $|U^W\rangle$ that evolves unitarily forever.
- **Axiom 2:** This whole wavefunction can be written as a superposition in some basis, with $|U\rangle$ as one of the terms in the superposition. $|U^W\rangle = |U\rangle \oplus \dots$ For convenience, we will refer to $|U\rangle$ as the universe and specify 'the whole universe' to refer to $|U^W\rangle$.

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- **Axiom 3:** There exist sybsystems of the universe $|U\rangle$, 'qumans' $|H\rangle$ that are capable of measurement, such that $|H\rangle$ is not a subsystem of any other terms in the direct sum of Axiom 2. We write $|U\rangle = c|H\rangle \otimes |U_H\rangle$. The measurements can be directed outwards or inwards, towards subsystems of $|H\rangle$. The overall constant c does not play a role in measurements made by $|H\rangle$, except in the context of Axiom 9 below.

- **Axiom 4:** External measurements performed by $|H\rangle$ will yield results as applied to the subsystem $|U_H\rangle$.

- **Axiom 5:** There is a special form of measurement between two qumans $|H_1\rangle$ and $|H_2\rangle$ called a 'communication' between them.

- **Axiom 6:** Two qumans $|H_1\rangle$ and $|H_2\rangle$ can only communicate if we can write $|U\rangle = c|H_1\rangle \otimes |H_2\rangle \otimes |U_{H_1H_2}\rangle$. Here, $|U_{H_1}\rangle = |H_2\rangle \otimes |U_{H_1H_2}\rangle$ and $|U_{H_2}\rangle = |H_1\rangle \otimes |U_{H_1H_2}\rangle$.

- **Axiom 7:** The result of each individual measurement m on $|U_H\rangle$ is as dictated by quantum mechanics, in the following sense. Suppose S is a subsystem of $|U_H\rangle = |S\rangle \otimes |E_S\rangle$. If $|S\rangle = \sum_i c_i|\alpha_i\rangle$, where $|\alpha_i\rangle$ are eigenstates of measurement m, then after measurement, $|H\rangle \otimes |U_H\rangle$ splits into $\bigoplus_i c_i|H_{\alpha_i}\rangle \otimes |\alpha_i\rangle \otimes |E_S\rangle$.

- **Axiom 8:** $|H_{\alpha_i}\rangle$ above are new qumans, who can make an internal measurement m_I to know that S is in state $|\alpha_i\rangle$ in $|U_{H_{\alpha_i}}\rangle = |\alpha_i\rangle \otimes |E_S\rangle$.

- **Axiom 9:** If the measurement m in Axiom 7 is repeated n times by $|H\rangle$ on identical subsystems S^i of $|U_H\rangle = |S_1\rangle \otimes |S_2\rangle \cdots \otimes |S_n\rangle \otimes |E_S\rangle$, $|H\rangle$ will find themselves in state $|H_{\beta_1,\beta_2,\cdots,\beta_n}\rangle$, such that the value α_i appears in the set $(\beta_1, \beta_2, \cdots, \beta_n)$ with probability $|c_i|^2$. This is equivalent to a confirmation of the Born rule by versions of $|H\rangle$.

We refer to this axiomatic framework where observers are themselves quantum states allowed to make measurements in their corresponding 'universes' as the NEW interpretation of quantum mechanics.

3 Frauchiger-Renner Paradox in the NEW worldview

Let us consider the experimental setup in the Frauchiger-Renner paper in the language of the NEW framework.

The universe contains systems R and S, qumans $\bar{F}$, F, $\bar{W}$, W, and environment E.

$$|U\rangle = |R\rangle \otimes |S\rangle \otimes |\bar{F}\rangle \otimes |F\rangle \otimes |\bar{W}\rangle \otimes |W\rangle \otimes |E\rangle \tag{1}$$

We start with system R in the state

$$|\mathsf{init}\rangle_R = \sqrt{\frac{1}{3}}|\mathsf{heads}\rangle_R \oplus \sqrt{\frac{2}{3}}|\mathsf{tails}\rangle_R \tag{2}$$

At time n:00, $\bar{F}$ makes a measurement in the $(|\mathsf{heads}\rangle_R, |\mathsf{tails}\rangle_R)$ basis and sets S in the chosen state based on their observed state of R. In the notation from the original paper, we have

$$U^{00\to 10}_{\mathrm{R}\to\bar{\mathrm{L}}\mathrm{S}}|\mathsf{init}\rangle_{\mathrm{R}} = \sqrt{1/3}|\bar{h}\rangle_{\bar{L}} \otimes |{\downarrow}\rangle_{\mathrm{S}} + \sqrt{2/3}|\bar{t}\rangle_{\bar{L}} \otimes |{\rightarrow}\rangle_{\mathrm{S}} \tag{3}$$

We can also write this state as

$$U^{00\to 10}_{\mathrm{R}\to\bar{\mathrm{L}}\mathrm{S}}|\mathsf{init}\rangle_{\mathrm{R}} = \sqrt{\frac{1}{3}}|\bar{h}\rangle_{\bar{L}}|\downarrow\rangle_S \oplus \sqrt{\frac{1}{3}}|\bar{t}\rangle_{\bar{L}}|\downarrow\rangle_S \oplus \sqrt{\frac{1}{3}}|\bar{t}\rangle_{\bar{L}}|\uparrow\rangle_S \tag{4}$$

Before measurement, the universe where quman $\bar{F}$ could perform measurements was $|U_{\bar{F}}\rangle = |R\rangle \otimes |S\rangle \otimes |F\rangle \otimes |\bar{W}\rangle \otimes |W\rangle$. After measurement, per Axiom 7, we get two 'universes' with a version of the original quman $\bar{F}$ in each. Let $|\bar{F}_{\mathsf{heads}_R}\rangle$ to denote the quman $\bar{F}$ who has made a measurement and observed R in state heads, and $|\bar{F}_{\mathsf{tails}_R}\rangle$ to denote the quman $\bar{F}$ who has made a measurement and observed R in state tails. These two qumans can only make measurements in their corresponding universes, with

$$|U_{\bar{F}_{\mathsf{heads}_R}}\rangle = |\mathsf{heads}\rangle_R|\downarrow\rangle_S \otimes |F\rangle \otimes |\bar{W}\rangle \otimes |W\rangle \otimes |E\rangle \tag{5}$$

and

$$|U_{\bar{F}_{\mathsf{tails}_R}}\rangle = |\mathsf{tails}\rangle_R(|\uparrow\rangle + |\downarrow\rangle)_S \otimes |F\rangle \otimes |\bar{W}\rangle \otimes |W\rangle \otimes |E\rangle \tag{6}$$

.

In the NEW worldview, by axiom 7, we write the system in Eq.4 as

$$|R, S, \bar{F}\rangle = \sqrt{\frac{1}{3}}|\mathsf{heads}\rangle_R|\bar{F}_{\mathsf{heads}_R}\rangle|\downarrow\rangle_S \oplus \sqrt{\frac{1}{3}}|\mathsf{tails}\rangle_R|\bar{F}_{\mathsf{tails}_R}\rangle|\downarrow\rangle_S \oplus \sqrt{\frac{1}{3}}|\mathsf{tails}\rangle_R|\bar{F}_{\mathsf{tails}_R}\rangle|\uparrow\rangle_S \tag{7}$$

We provide a quick reference for the translation between the original and NEW notation in Table 1.

Original notation	NEW notation
$\lvert\bar{h}\rangle_{\bar{L}}$	$\lvert\mathsf{heads}\rangle_R\lvert\bar{F}_{\mathsf{heads}_R}\rangle$
$\lvert\bar{t}\rangle_{\bar{L}}$	$\lvert\mathsf{tails}\rangle_R\lvert\bar{F}_{\mathsf{tails}_R}\rangle$
$\lvert\frac{1}{2}\rangle_L$	$\lvert\uparrow\rangle_S\lvert F_{\uparrow_S}\rangle$
$\lvert\frac{-1}{2}\rangle_L$	$\lvert\downarrow\rangle_S\lvert F_{\downarrow_S}\rangle$

Table 1: Translation between Frauchiger-Renner and NEW notation. The original paper used a notation for 'labs', which captured the inanimate systems R and S together with observers making measurement. We want to clarify this in our notation by explicitly writing out terms involving systems R and S. We can then use $\bar{F}$ and F to exclusively denote the 'qumans' making measurements, instead of whole labs.

The *Statement* $\bar{\mathrm{F}}^{n:02}$*:* "I am certain that W will observe $w = \mathsf{fail}$ at time n:31" holds in $|U_{\bar{F}_{\mathsf{tails}_R}}\rangle$. The state of system S in $|U_{\bar{F}_{\mathsf{tails}_R}}\rangle$ is a symmetric combination of $|\uparrow\rangle_S$ and $|\downarrow\rangle_S$. After L measures S, in the notation of the orginal paper, the state in $|U_{\bar{F}_{\mathsf{tails}_R}}\rangle$ will be a symmetric combination of $|\frac{1}{2}\rangle_L$ and $|\frac{-1}{2}\rangle_L$, which is orthogonal to the state $w = \mathsf{ok}$.

Note, the original universe U from Eq.1 can also be written as $|W\rangle \otimes |U_W\rangle$, with

$$|U_W\rangle = |R\rangle \otimes |S\rangle \otimes |\bar{F}\rangle \otimes |F\rangle \otimes |\bar{W}\rangle \otimes |E\rangle. \tag{8}$$

Quman W makes measurements in U_W. The statement $\bar{\mathrm{F}}^{n:02}$ does not hold in $|U_W\rangle$, as we will soon demonstrate.

At time n:01, F measures system S.

$$U^{10\to 20}U^{00\to 10}|\mathsf{init}\rangle_{\mathrm{R}} = \sqrt{\frac{1}{3}}|\bar{h}\rangle_{\bar{L}}|\frac{-1}{2}\rangle_L \oplus \sqrt{\frac{1}{3}}|\bar{t}\rangle_{\bar{L}}|\frac{-1}{2}\rangle_L \oplus \sqrt{\frac{1}{3}}|\bar{t}\rangle_{\bar{L}}|\frac{1}{2}\rangle_L \tag{9}$$

Invoking axioms 7 and 8 in the NEW worldview, in place of original quman $|F\rangle$ before measurement, we have two qumans $|F_{\uparrow_S}\rangle$ and $|F_{\downarrow_S}\rangle$ who can only make measurements in their respective universes, with

$$|U_{F_{\downarrow_S}}\rangle = (|\mathsf{heads}\rangle_R|\bar{F}_{\mathsf{heads}_R}\rangle|\downarrow\rangle_S \oplus |\mathsf{tails}\rangle_R|\bar{F}_{\mathsf{tails}_R}\rangle|\downarrow\rangle_S) \otimes |\bar{W}\rangle \otimes |W\rangle \otimes |E\rangle \tag{10}$$

and

$$|U_{F_{\uparrow_S}}\rangle = |\mathsf{tails}\rangle_R|\bar{F}_{\mathsf{tails}_R}\rangle|\uparrow\rangle_S \otimes |\bar{W}\rangle \otimes |W\rangle \otimes |E\rangle \tag{11}$$

.

We begin to see how the logical chain behind inferred statements unravels.

Consider the *Statement* $\mathrm{F}^{n:12}$*:* "I am certain that $\bar{\mathrm{F}}$ knows that $r = \mathsf{tails}$ at time n:01.". The quman $|F_{\uparrow S}\rangle$ can only make statements in $|U_{F_{\uparrow_S}}\rangle$. Indeed, this statement is valid in $|U_{F_{\uparrow_S}}\rangle$, since there is no term involving $|\bar{F}_{\mathsf{heads}_R}\rangle$ in $|U_{F_{\uparrow_S}}\rangle$.

However, quman $|\bar{F}_{\mathsf{tails}_R}\rangle$ cannot communicate with quman $|F_{\uparrow S}\rangle$. Note that the universe $|U_{\bar{F}_{\mathsf{tails}_R}}\rangle$ in Eq.12 contains $(|\uparrow\rangle + |\downarrow\rangle)_S$, hence a superposition of $|F_{\uparrow S}\rangle$ and $|F_{\downarrow S}\rangle$ after time n:01. By Axiom 6, while quman $|\bar{F}_{\mathsf{tails}_R}\rangle$ could communicate with quman $|F\rangle$ before measurement at time n:01, after measurement, this quman ceases to exist. Instead we have two new qumans, and the universe $|U_{\bar{F}_{\mathsf{tails}_R}}\rangle$ from Eq.12 is direct sum of the universes where these qumans and system S are entangled:

$$|U_{\bar{F}_{\mathsf{tails}_R}}\rangle = |\mathsf{tails}\rangle_R|\uparrow\rangle_S \otimes |F_{\uparrow S}\rangle \otimes |\bar{W}\rangle \otimes |W\rangle \otimes |E\rangle \oplus |\mathsf{tails}\rangle_R|\downarrow\rangle_S \otimes |F_{\downarrow S}\rangle \otimes |\bar{W}\rangle \otimes |W\rangle \otimes |E\rangle \tag{12}$$

.

This is no longer of the form $|U\rangle = c|H_1\rangle \otimes |H_2\rangle \otimes |U_{H_1H_2}\rangle$, so quman $|\bar{F}_{\mathsf{tails}_R}\rangle$ cannot communicate either with quman $|F_{\uparrow S}\rangle$ or quman $|F_{\downarrow S}\rangle$.

The first step for quman $|\bar{F}_{\mathsf{tails}_R}\rangle$ to try to communicate with quman $|F_{\uparrow S}\rangle$ is to make a measurement on F in basis $(|F_{\uparrow S}\rangle, |F_{\downarrow S}\rangle)$. Axiom 7 applies again, and we get two qumans $|\bar{F}_{\mathsf{tails}_R, F_{\uparrow_S}}\rangle$ and $|\bar{F}_{\mathsf{tails}_R, F_{\downarrow_S}}\rangle$. Only quman $|\bar{F}_{\mathsf{tails}_R, F_{\uparrow_S}}\rangle$ can communicate with quman $|F_{\uparrow S}\rangle$.

This quman can only make statements in

$$|U_{\bar{F}_{\mathsf{tails}_R, F_{\uparrow_S}}}\rangle = |\mathsf{tails}\rangle_R|\uparrow\rangle_S|F_{\uparrow S}\rangle \otimes |\bar{W}\rangle \otimes |W\rangle \otimes |E\rangle \tag{13}$$

Consider *Statement* $\mathrm{F}^{n:13}$*:* "I am certain that $\bar{\mathrm{F}}$ is certain that W will observe $w = \mathsf{fail}$ at time n:31.".

The reasoning breaks down once we clarify which version of $\bar{\mathrm{F}}$ we are talking about. The quman that inhabits $|U_{F_{\uparrow_S}}\rangle$ is $|\bar{F}_{\mathsf{tails}_R, F_{\uparrow_S}}\rangle$, while the quman who can infer $\bar{\mathrm{F}}^{n:02}$ is a different quman $|\bar{F}_{\mathsf{tails}_R}\rangle$ who cannot communicate with quman $|F_{\uparrow S}\rangle$ according to Axiom 6.

We can similarly see that $\bar{\mathrm{F}}^{n:02}$ is not valid in $|U_W\rangle$.

From the perspective of agent W who has been isolated from all systems so far, we have

$$|U\rangle = |R, \bar{F}, S, F\rangle \otimes |\bar{W}\rangle \otimes |W\rangle \otimes |E\rangle, \tag{14}$$

so that

$$|U_W\rangle = |R, \bar{F}, S, F\rangle \otimes |\bar{W}\rangle \otimes |E\rangle. \tag{15}$$

Once quman L makes their measurement, by axiom 7, we have

$$\begin{aligned} |R, S, \bar{F}, F\rangle \quad = \quad & \sqrt{\frac{1}{3}}|\mathsf{heads}\rangle_R|\bar{F}_{\mathsf{heads}_R}\rangle|\downarrow\rangle_S|F_{\downarrow S}\rangle \oplus \sqrt{\frac{1}{3}}|\mathsf{tails}\rangle_R|\bar{F}_{\mathsf{tails}_R}\rangle|\downarrow\rangle_S|F_{\downarrow S}\rangle \\ & \oplus \sqrt{\frac{1}{3}}|\mathsf{tails}\rangle_R|\bar{F}_{\mathsf{tails}_R}\rangle|\uparrow\rangle_S|F_{\uparrow S}\rangle \end{aligned} \tag{16}$$

Writing this in the basis $|\mathsf{ok}\rangle_L$ and $|\mathsf{fail}\rangle_L$:

$$\begin{aligned}|R,\bar{F},S,F\rangle &= \sqrt{\frac{1}{6}}|\bar{h}_{\bar{F}}\rangle(|\mathsf{ok}\rangle_L+|\mathsf{fail}\rangle_L)\oplus\sqrt{\frac{1}{6}}|\bar{t}_{\bar{F}}\rangle(|\mathsf{ok}\rangle_L+|\mathsf{fail}\rangle_L)\oplus\sqrt{\frac{1}{6}}|\bar{t}_{\bar{F}}\rangle(|\mathsf{fail}\rangle_L-|\mathsf{ok}\rangle_L)\\ &= \sqrt{\frac{1}{6}}[|\bar{h}_{\bar{F}}\rangle|\mathsf{ok}\rangle_L+(|\bar{h}_{\bar{F}}\rangle+2|\bar{t}_{\bar{F}}\rangle)|\mathsf{fail}\rangle_L]\\ &= \sqrt{\frac{1}{6}}[|\mathsf{heads}\rangle_R|\bar{F}_{\mathsf{heads}_R}\rangle(|\downarrow\rangle_S|F_{\downarrow S}\rangle-|\uparrow\rangle_S|F_{\uparrow S}\rangle)\\ &\quad\oplus(|\mathsf{heads}\rangle_R|\bar{F}_{\mathsf{heads}_R}\rangle+2|\mathsf{tails}\rangle_R|\bar{F}_{\mathsf{tails}_R}\rangle)(|\downarrow\rangle_S|F_{\downarrow S}\rangle+|\uparrow\rangle_S|F_{\uparrow S}\rangle)]\end{aligned}\tag{17}$$

Once quman W makes a measurement in their chosen basis as specified in Table 2 of the Frauchiger-Renner paper, we get two qumans $|W_{\mathsf{ok}_L}\rangle$ and $|W_{\mathsf{fail}_L}\rangle$, with

$$|U_{W_{\mathsf{ok}_L}}\rangle=|\mathsf{heads}\rangle_R|\bar{F}_{\mathsf{heads}_R}\rangle(|\downarrow\rangle_S|F_{\downarrow S}\rangle-|\uparrow\rangle_S|F_{\uparrow S}\rangle)\otimes|\bar{W}\rangle\otimes|E\rangle,\tag{18}$$

and

$$|U_{W_{\mathsf{fail}_L}}\rangle=(|\mathsf{heads}\rangle_R|\bar{F}_{\mathsf{heads}_R}\rangle+2|\mathsf{tails}\rangle_R|\bar{F}_{\mathsf{tails}_R}\rangle)(|\downarrow\rangle_S|F_{\downarrow S}\rangle+|\uparrow\rangle_S|F_{\uparrow S}\rangle)\otimes|\bar{W}\rangle\otimes|E\rangle.\tag{19}$$

Quman $|\bar{F}_{\mathsf{tails}_R}\rangle$ can never communicate with quman $|W_{\mathsf{ok}_L}\rangle$, as we would expect from $\mathrm{F}^{n:02}$. The universe U can be written as

$$|U\rangle=|W_{\mathsf{ok}_L}\rangle|U_{W_{\mathsf{ok}_L}}\rangle\oplus|W_{\mathsf{fail}_L}\rangle|U_{W_{\mathsf{fail}_L}}\rangle.\tag{20}$$

Further, $|U_{W_{\mathsf{ok}_L}}\rangle$ can be written as using original notation for basis $\bar{\mathsf{ok}}$ and $\bar{\mathsf{fail}}$

$$|U_{W_{\mathsf{ok}_L}}\rangle=\frac{1}{\sqrt{2}}(|\bar{\mathsf{ok}}\rangle_{\bar{L}}+|\bar{\mathsf{fail}}\rangle_{\bar{L}})(|\downarrow\rangle_S|F_{\downarrow S}\rangle-|\uparrow\rangle_S|F_{\uparrow S}\rangle)\otimes|\bar{W}\rangle\otimes|E\rangle,\tag{21}$$

After $\bar{W}$ makes their measurement, in the NEW notation, this gives

$$\begin{aligned}|U_{W_{\mathsf{ok}_L}}\rangle &= \frac{1}{\sqrt{2}}[(|\mathsf{heads}\rangle_R-|\mathsf{tails}\rangle_R)(|\downarrow\rangle_S|F_{\downarrow S}\rangle-|\uparrow\rangle_S|F_{\uparrow S}\rangle)\otimes|\bar{W}_{\bar{\mathsf{ok}}_{\bar{L}}}\rangle\\ &\quad+(|\mathsf{heads}\rangle_R+|\mathsf{tails}\rangle_R)(|\downarrow\rangle_S|F_{\downarrow S}\rangle-|\uparrow\rangle_S|F_{\uparrow S}\rangle)\otimes|\bar{W}\rangle_{\bar{\mathsf{fail}}_{\bar{L}}}]\otimes|E\rangle\end{aligned}\tag{22}$$

After making their measurement, without communicating with F or $\bar{F}$, if $|\bar{W}_{\bar{\mathsf{ok}}}\rangle$ asks $|W\rangle$ whether they observed ok or fail, this communication constitutes as a measurement on W. We get two qumans $|\bar{W}_{\bar{\mathsf{ok}}_{\bar{L}},W_{\mathsf{ok}}}\rangle$ and $|\bar{W}_{\bar{\mathsf{ok}},\mathsf{fail}_W}\rangle$. In other words, we can write the original universe U as

$$\begin{aligned}|U\rangle &= \frac{1}{\sqrt{12}}|\bar{W}_{\bar{\mathsf{ok}}_{\bar{L}},W_{\mathsf{ok}}}\rangle\otimes|W_{\mathsf{ok}_L,\bar{W}_{\bar{\mathsf{ok}}}}\rangle\otimes(|\mathsf{heads}\rangle_R|\bar{F}_{\mathsf{heads}_R}\rangle-|\mathsf{tails}\rangle_R|\bar{F}_{\mathsf{tails}_R}\rangle)\\ &\quad\otimes(|\downarrow\rangle_S|F_{\downarrow S}\rangle-|\uparrow\rangle_S|F_{\uparrow S}\rangle)\otimes|E\rangle\oplus\cdots\end{aligned}\tag{23}$$

The factor of $\frac{1}{\sqrt{12}}$ is obtained by combining the factor of $\frac{1}{\sqrt{6}}$ in Eq.17 with the factor of $\frac{1}{\sqrt{2}}$ in Eq.22. By Axiom 9, this result is consistent with the result in the original paper that the combination $\mathsf{ok},\bar{\mathsf{ok}}$ will occur with probability $\frac{1}{12}$. Qumans W and $\bar{W}$ will eventually find themselves in the state $|\bar{W}_{\bar{\mathsf{ok}}_{\bar{L}},W_{\mathsf{ok}}}\rangle\otimes|W_{\mathsf{ok}_L,\bar{W}_{\bar{\mathsf{ok}}}}\rangle$ when the experiment is repeated, as proved in the original paper. There is no contradiction here.

agent	assumed observation	measures in	statement inferred via (Q)	further implied statement	statement inferred via (C)
$\bar{\text{F}}$	$r=\textsf{tails}$ at time n:01	$\lvert U_{\bar{F}_{\textsf{tails}_R}}\rangle$	*Statement* $\bar{\text{F}}^{n:02}$*:* "I am certain that W will observe $w=\textsf{fail}$ at time n:31.". **True** in $\lvert U_{\bar{F}_{\textsf{tails}_R}}\rangle$		
F	$z=+\frac{1}{2}$ at time n:11	$\lvert U_{F_{\uparrow S}}\rangle$	*Statement* $\text{F}^{n:12}$*:* "I am certain that $\bar{\text{F}}$ knows that $r=\textsf{tails}$ at time n:01." **True** in $\lvert U_{F_{\uparrow S}}\rangle$	*Statement* $\text{F}^{n:13}$*:* "I am certain that $\bar{\text{F}}$ is certain that W will observe $w=\textsf{fail}$ at time n:31." **False** in $\lvert U_{F_{\uparrow S},\bar{F}_{\textsf{tails}_R}}\rangle$	*Statement* $\text{F}^{n:14}$*:* "I am certain that W will observe $w=\textsf{fail}$ at time n:31." **False**
$\bar{\text{W}}$	$\bar{w}=\bar{\textsf{ok}}$ at time n:21	$\lvert U_{\bar{W}_{\bar{\textsf{ok}}_{\bar{L}}}}\rangle$	*Statement* $\bar{\text{W}}^{n:22}$*:* "I am certain that F knows that $z=+\frac{1}{2}$ at time n:11." **True** in $\lvert U_{\bar{W}_{\bar{\textsf{ok}}_{\bar{L}}}}\rangle$	*Statement* $\bar{\text{W}}^{n:23}$*:* "I am certain that F is certain that W will observe $w=\textsf{fail}$ at time n:31." **False**	*Statement* $\bar{\text{W}}^{n:24}$*:* "I am certain that W will observe $w=\textsf{fail}$ at time n:31." **False**
W	announcement by agent $\bar{\text{W}}$ that $\bar{w}=\bar{\textsf{ok}}$ at time n:21	$\lvert U_{W_{\bar{W}_{\bar{\textsf{ok}}}}}\rangle$	*Statement* $\text{W}^{n:26}$*:* "I am certain that $\bar{\text{W}}$ knows that $\bar{w}=\bar{\textsf{ok}}$ at time n:21." **True** in $\lvert U_{W_{\bar{W}_{\bar{\textsf{ok}}}}}\rangle$	*Statement* $\text{W}^{n:27}$*:* "I am certain that $\bar{\text{W}}$ is certain that I will observe $w=\textsf{fail}$ at time n:31." **False**	*Statement* $\text{W}^{n:28}$*:* "I am certain that I will observe $w=\textsf{fail}$ at time n:31." **False**

Table 2: The agents' observations and conclusions. The statements that the individual agents can derive from quantum theory depend on the information accessible to them, as described in the original paper [1]. We add a column to specify which universe the agents/qumans measure in, and which statements are true and which are false.

Consistency is restored because qumans $|\bar{W}_{\bar{\textsf{ok}}_{\bar{L}},W_{\textsf{ok}}}\rangle$ and $|W_{\textsf{ok}_L,\bar{W}_{\bar{\textsf{ok}}}}\rangle$ realize that since their measurement basis is rotated with respect to the basis used by F and $\bar{F}$, their universe contains a superposition of qumans $|\bar{F}_{\textsf{heads}_R}\rangle$ and $|\bar{F}_{\textsf{tails}_R}\rangle$, and qumans $|F_{\uparrow S}\rangle$ and $|F_{\downarrow S}\rangle$. A measurement in $|U_{W_{\textsf{ok}},\bar{W}_{\bar{\textsf{ok}}}}\rangle$ is not expected to agree with a measurement in $|U_{\bar{F}_{\textsf{tails}_R}}\rangle$.

What happens if having found themselves to be in state $|\bar{W}_{\bar{\textsf{ok}}_{\bar{L}},W_{\textsf{ok}}}\rangle$ and $|W_{\textsf{ok}_L,\bar{W}_{\bar{\textsf{ok}}}}\rangle$, at time n:41, they ask $\bar{F}$ what they observed? Based on Eq.23, they will sometimes observe $\bar{F}$ to be in state $|\bar{F}_{\textsf{tails}_R,W_{\textsf{ok}},\bar{W}_{\bar{\textsf{ok}}}}\rangle$. Have we reached a contradiction then?

We have not, any more than we would consider it a contradiction to start with an S_z down electron, measure S_x on it, and find it to be S_z up in a subsequent measurement. There is a term involving quman $|\bar{F}_{\textsf{heads}_R}\rangle$ at time n:01, who puts system S in state $|\downarrow\rangle_S$ measured by quman $|F_{\downarrow S}\rangle$ at time n:11, then $\bar{W}$ and W make their measurements in rotated basis at times n:21 and n:31. After these measurements, there is no contradiction to then find $\bar{F}$ to be in state $|\bar{F}_{\textsf{tails}_R}\rangle$.

Table 2 summarizes which statements hold and which do not. The NEW formalism consistently accounts for the observations by various agents by carefully defining the corresponding qumans and the universe accessible to them by measurement, with consistency defined in the NEW framework as:

Assumption (C)
Suppose that quman $|\text{A}\rangle$ has established that

Statement $\mathrm{A}^{(\mathrm{i})}$*:* "I am certain that quman $|\mathrm{A}'\rangle$ **in my universe** (i.e. $|U\rangle = |A\rangle \otimes |\mathrm{A}'\rangle \otimes |U_{A,\mathrm{A}'}\rangle$, so that qumans $|A\rangle$ and $|\mathrm{A}'\rangle$ can communicate) , upon reasoning within the same theory as the one I am using, is certain that $x = \xi$ at time t."

Then quman $|\mathrm{A}\rangle$ can conclude that

Statement $\mathrm{A}^{(\mathrm{ii})}$*:* "I am certain that $x = \xi$ at time t."

4 Discussion

We have demonstrated how our axiomatic framework, NEW interpretation of Quantum Mechanics, provides a way to consistently interpret observers and their measurements without sacrificing assumptions (Q) or (S). Per axioms 7 and 8, a measurement results in qumans $|H_{\alpha_i}\rangle$, each of whom believe that they have observed a single result $|\alpha_i\rangle$, satisying (S). These axioms along with Axiom 9 also ensure that the Born rule is satisfied. (We explicitly state Axiom 9 for simplicity, however, this should follow from the previous axioms considering the distribution of all possible outcomes.)

So far, we have dispassionately stated our axioms and applied them to the Gendanken experiment without any comment on what the NEW framework means in the big picture. We now briefly highlight how profoundly this framework shifts our worldview - for example, while your friend may see you sitting in a chair and reading this article right now, another quman in another universe might see a peacock perched on a branch at exactly the same spacetime region (x, t) you occupy; your friend just can't see the peacock through quantum measurement. Not in some bubble universe or extra dimensions - this whole drama unfolds on the same spacetime fabric - all the terms in the direct sum are part of the same U_W. The entropy of this whole universe is constant. We believe entropy always increases and experience an arrow of time because we are ourselves constantly changing: $|H\rangle \to |H_{\beta_1}\rangle \to |H_{\beta_1,\beta_2}\rangle \to |H_{\beta_1,\beta_2,\beta_3}\rangle \to \cdots$, so our observable universe is changing $|U_H\rangle \to |U_{H_{\beta_1}}\rangle \to |U_{H_{\beta_1,\beta_2}}\rangle \to |U_{H_{\beta_1,\beta_2,\beta_3}}\rangle \to \cdots$. We feel like we are the same quman because $|H_{\beta_1,\beta_2,\beta_3}\rangle$ can perform an internal measurement and 'remember' past experiences $\beta_1, \beta_2, \beta_3$.

With the NEW hypothesis [2] that gravity 'feels' the whole U_W at once, even more things fall in place. Of course we have to make up concepts like dark matter and dark energy, because our (electromagnetic) observations only measure our universe U, while gravity responds to the whole universe U_W.

The NEW worldview has been explored in broad qualitative strokes in our previous work [2]. We conclude our discussion with just this small taste here, leaving further explorations for future works. NEW framework treats 'qumans' as physical states evolving according to quantum mechanics - leaving little room for concepts like free will. Regardless of whether the reader is prepared to think about dark matter or free will in the NEW framework, the axiomatic framework provides an elegant way to understand the root of the apparent paradox identified by Frauchiger and Renner, and finds a way out by clarifying the consistency assumption.

5 REFERENCES

[1] Frauchiger, D., Renner, R. Quantum theory cannot consistently describe the use of itself. Nat Commun 9, 3711 (2018). https://doi.org/10.1038/s41467-018-05739-8
[2] Tamhankar, S. A NEW Interpretation of Quantum Mechanics with a Conceptual Conjecture for 'Quantum Gravity', https://vixra.org/pdf/2409.0039v1.pdf