On the Gravitational Analog of Continuous Spacetime Dimensions

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Abstract

In this brief note we point out that a spacetime endowed with continuous dimensions can be regarded as dual to classical gravitation in four dimensions. This observation provides a foundation for the gravitational behavior of Cantor Dust (CD), under the assumption that CD represents a cluster of Dark Matter ultralight objects such as self-interacting bosons, axions, fuzzy condensates or 3D anyons.

Key words: effective gravitational coupling, Dark Matter, Cantor Dust, continuous spacetime dimensions.

A straightforward comparison of the Newton potential

$$V_N = -\frac{G_N m^2}{r} \tag{1}$$

with the Coulomb potential expressed in natural units ($\hbar = c = 1$)

$$V_C = -\frac{e^2}{4\pi r} = -\frac{\alpha}{r} \tag{2}$$

falls in line with the concept of Planck mass defined as

$$M_{Pl} = \sqrt{\frac{\hbar c}{G_N}} = \sqrt{\frac{1}{G_N}} \tag{3}$$

By analogy with the fine structure constant α, (1) – (3) suggest the introduction of the *effective gravitational coupling* [1]

$$\alpha_G = \frac{m^2}{M_{Pl}^2} \tag{4}$$

The usual interpretation of (4) is that gravitational effects are practically irrelevant to the low-energy sector of Quantum Field Theory (QFT) and the Standard Model of Particle Physics. Since, in QFT mass runs with the observation scale (μ), a natural extrapolation of (4) is given by

$$\alpha_G(\mu) = \frac{m^2(\mu)}{M_{Pl}^2} \tag{5}$$

Following [2 - 3], dimensional regularization of QFT indicates that the continuous deviation from $D=4$ spacetime dimensions takes the form

$$\varepsilon(\mu) = 4 - D(\mu) = O(m^2(\mu)/\Lambda_{UV}^2) << 1 \tag{6}$$

in which Λ_{UV} is the high energy cutoff of the theory. By (5), under the identification $\Lambda_{UV} = O(M_{Pl})$ and taking the far asymptotic limit $m << M_{Pl}$ yields

$$\boxed{\alpha_G(\mu) = O(\varepsilon(\mu))} \tag{7}$$

Consider now the metric term describing the Newtonian potential created by a point-source of mass m at a distance R

$$g_{00} = 1 + 2V_N = 1 - 2G_N \frac{m}{R} \tag{8}$$

or, on account of (3),

$$g_{00} = 1 - 2\frac{m}{M_{Pl}^2 R} \tag{9}$$

For a particle-antiparticle pair produced by vacuum fluctuations and separated by distance having the same order of magnitude as their Compton wavelength λ_C, that is, $R = O(\lambda_C) = O(1/m)$, (9) turns into

$$g_{00}(\mu) = 1 - 2\frac{m^2(\mu)}{M_{Pl}^2} \approx 1 - 2\varepsilon(\mu) \tag{10}$$

or

$$\boxed{\varepsilon(\mu) \approx \frac{1}{2}(1 - g_{00}(\mu))} \tag{11}$$

Loosely speaking, (11) may be associated with the *gravitational coupling of a particle-antiparticle pair separated by a distance scale comparable with their Compton wavelength.* This observation hints to a foundation for the gravitational content of Cantor Dust (CD), under the assumption that CD is a large-scale cluster of ultralight Dark Matter particles such as self-interacting bosons, axions, fuzzy condensates or 3D anyons [4].

References

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