Title: Methodological fallacy in proton mass radius determination: Alternative classical approach based on Einstein's mass-energy equivalence principle

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ABSTRACT
No equations led to $\langle R_m^2 \rangle = \frac{6}{G(0)}\Big|_{t=0} = \frac{12}{m_s^2}$ in which both sides of the equation are dimensionally at odds. It is appropriate to dare challenge the flaw in the procedure that produced 0.55 fm as the mass radius with many models and solutions because the charge radius and, more recently, the proton's mass radius are inconsistent. The objective is to develop an alternative theory that combines the idea of the mass-energy equivalence principle with a kinetic basis. Proving that 0.55 fm is a fabrication and demonstrating that the proton's charge radius might equal the mass radius of one up and one down quark combined are only a few of the objectives. Results were generated through theoretical and computational methods. The mass radii of protons were between 1.019699 and 1.10254513 fm; the sum of mass radii of up and down quarks was 0.8349190666 fm; the mass of the hypothetical particle was 8.911586959 exp.(−32) kg; mass radii of up and down quarks were 0.5881268197 and 0.246803591 fm, respectively; the mass radius of an electron was 0.06278280228 fm; and nucleon size–based radii of the electron are 0.08993727541 and 0.09001990394 fm, corresponding to the proton and nucleon, respectively. The idea that the charge radii may represent the mass radii of quarks is supported by the total (0.8349190759 fm) of the radii of up and down quarks in this research. The new model equation for calculating the mass radii of the electron and proton is supported by the intraproton radius (0.08993727541 fm) of the electron, which is provided by $(R_{mP}^3/1836.152673)^{1/3}$, where proton mass radius, R_{mP} = 1.10254513 fm. Since $\langle R_m^2 \rangle \neq 12/m_s^2$, 0.55 fm was a fabrication. It is almost half of 1.10254513 fm and was used to support the false claim that the proton charge radius equals its spatial extent, thereby refuting Bohr's radius.
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1. INTRODUCTION

Should a teacher shout down 14 years old American and a German student who dared to question the validity of $\langle R_m^2 \rangle = 12/m_s^2$*?*

An overview of technical terms in nuclear (and/or particle) science is part of this introductory section, along with concerns about behavior that is completely at odds with the ethics of scholarly presentation and the fallacy of the author's article on the mass radius of the proton. Such an overview is limited to a few topics that have been discussed in the literature. This is done in an effort to gain a general understanding of the problems at hand, going beyond elite academics with higher mathematical expertise. Additionally, it is unethical for someone to feel informed about a subject they do not understand. As a result, there are several quantum chromodynamics (QCD) techniques and various form factors in the literature that are not entirely understood. Nevertheless, they have no bearing whatsoever on the method now being investigated for calculating the mass radii of subatomic particles.

The literature provides examples of how QCD plays a part in determining the proton mass radius. Citing the works of Kobzarev and Okun (1992), Xiangdong (1997), and

Pagels (1966), Wang et al. (2023) believed that gravitational form factors (GFFs) were useful for comprehending the perturbative and nonperturbative QCD effects, as they connected to the spatial distribution of quarks inside the proton. The distribution of all quarks, which can affect the total distribution of proton mass, is the primary concern. Three quarks make up the proton, and their combined mass is less than that of a proton.

To maintain the application of QCD, there are suggestions that the baryons—the two nucleons in particular—are bound by additional particles called gluons, which have the ambivalence of being massless and force-carrying particles. Although the mass of a particle cannot be detected with current technology, it nevertheless has space and mass. According to QCD theory, the scalar GFFs are sensitive to the proton mass distribution from the QCD trace anomaly and are associated with the photoproduction of a quarkonium off a proton (Xiangdong, 1997 & Wang et al., 2020). This seems to imply that the distribution of proton masses can be estimated using such sensitivity.

The approach in this research is strictly classical leveraging on Einstein mass-energy equivalence concept. Minor historical antecedents in physical science are intentionally included. The method used to determine the mass radius of the proton was reported by a sole author even if (perhaps on account of traditional practice) the only author, stated that "we will show how the form factor of the scalar gluon operator can be determined from the recent data on photoproduction of J/ψ near the threshold (Kharzeev, 2021) recently reported by the GlueX Collaboration (Ali et al., 2019). We will then use this form factor to extract the mass radius of the proton from the GlueX data". Burton Richter and Samuel Ting independently discovered J/ψ, the symbol of a composite subatomic particles named charm quark and charm anti–quark.

The proton radius has been described as a large inspiration in understanding proton structure, and it can be measured by using an electron or photon as a probe. The proton charge radius, obtained by hitting the nucleus with high-energy electrons, has been the most studied (Wang et al., 2023). The method has obvious implications for the measurement outcomes. However, how this view clearly relates to the following method is not certain: "As it involves the lightest physical states excited from the vacuum by the vector quark current, near-threshold ρ^0 photoproduction is considered a possible way to research the proton radius and the absolute value of the scattering lengths of the ρ^0-proton interaction (Wang et al., 2023). However, nothing in all argument, explanation, critique, etc point to any equation leading to $\langle R_m^2 \rangle = \frac{6}{G(0)}\big|_{t=0} = \frac{12}{m_s^2}$ in which both sides of the equation are dimensionally at odds. The main goal is to create a different theory with a kinetic foundation that is based on the mass-energy equivalence principle. The objectives are 1) to derive another version of the equation for the computation of the mass radius of the proton, 2) evaluate the possibility of computing the size of up and down quarks, 3) reveal that the mass radius of the proton cannot be equated correctly to the independent variables indicated in the literature, 4) show that the charge radius reported in the literature may be the mass radius of one up and one down quark combined, and 5) resolutely affirm that 0.55 fm is a fabrication intended to justify all known values of proton charge radius expected to be longer than its mass radius.

“On the assumption of a scalar form factor of dipole form, the value of the proton mass radius is calculated as (0.85±0.06) fm by fitting the differential cross section of the γp → ρ^0p reaction at near-threshold energy. For light vector meson photoproduction, the exchange of a scalar quark-antiquark pair is not suppressed and should, therefore, dominate the scalar gluon exchange; the radius extracted from ρ^0 photoproduction is assumed to represent the quark radius of the proton” (Wang et al., 2023). This fact may explain why the value obtained in this study is very near the proton charge radius (Wang et al., 2023). The main interest in this statement is all about the reason the calculated mass radius is near the charge radius of the proton. As previously stated, the approach investigated in this study is still guided by the mass-energy-equivalence principle, which is attributed to A. Einstein. There should be a model (an equation) that draws its applicability from the idea that the energy derived from the mass of matter is directly proportional to its mass, so that the size of any elementary particle should also rely on the magnitude of its mass. This contrasts with the characteristically kinetic framework coupled with quantum chromodynamics theory used to establish the mass radius, the charge radius of the proton, and, shockingly, the neutron. This point of view is supported by the assertion that "the proton radius is a large inspiration in understanding proton structure, and it can be measured by using an electron or photon as a probe." As usual, a counterargument is presented in the section devoted to theoretical development, results of computation, and discussion.

2. THEORETICAL DEVELOPMENT, RESULTS OF COMPUTATION, AND DISCUSSION

The study's findings are given as a summary of theoretical development, computation results, and discussion for the sake of conciseness. To start, though, the table of values provides the following summary of the calculated variables (particulars)

Table 1. Mass radii and hypothetical mass

Particulars	Values/fm	Method (in this study)
R_m of proton	1.10196999	Eq. (7)
R_m of proton	1.10168253	Eq. (8)
R_m of q_d	0.58812681	Eq. (9b)
R_m of q_u	0.24679225	Eq. (9c)
R_m of q_d	0.58812682	Eq. (12)
R_m of q_u	0.24680359	Eq. (12)
R_m of q_d + R_m of q_u	8.34919067	-
R_i (R_m of electron)	0.06278280	Eq. (9a)
R_m of 2 q_u + 1 q_d	1.08176102	Eq. (13)
R_m of 1 q_u + 2 q_d	1.42304589	Eq. (13)
R_m of 2 q_u + 1 q_d	0.01035293	Eq. (1)
R_m of 1 q_u + 2 q_d	0.01361918	Eq. (1)

The hypothetical mass (m_x) is 8.911586959 kg (Eq. (11)); it is so-called because it is theoretically computed and has no known existence; the sum of the masses of the quarks in a proton and neutron are respectively 15.70525153 and 20.66105166 exp. (−30) kg. Masses of up quark and down quark are 2.01 ± 0.14 MeV and 4.79±0.16 MeV respectively (Cho, 2010).

The equations for the computation of mass radius of the proton had been derived in different ways (Udema, 2020 & Udema, 2022). The earlier equation (Udema, 2020) is given as:

$$R_m = \frac{m_P\ e^6}{4\pi\varepsilon_0^3\ m_e^2 h^2 c^4}, \tag{1}$$

Where $R_m, e, h, m_p, m_e\ c,$ and ε_0 are the mass radius of the proton, charge of an electron, Planck constant, mass of a proton, mass of such an electron, velocity of light in free space, and permittivity in free space respectively.
The latter equation (Udema, 2022) is given as:

$$R_m = \frac{m_P\ \mu_0^2\ e^{10}}{32\ h^4\ \varepsilon_0^3\ \pi m_e\ \xi_H}, \tag{2}$$

where μ_0 and ξ_H are the vacuum magnetic permeability and average ionization energy of hydrogen respectively. As stated earlier, the basis of the derived equations is the mass-energy equivalence principle of Einstein. Equation (1) is to be used to evaluate this principle while Eq. (2) is to be transformed into other variant as a way of showing "internal consistency" even if one equation is enough.

$$.\ \xi_H = n_i^2 \xi_i / Z_{eff}^2, \tag{3a}$$

where n_i, ξ_i, and Z_{eff} are the principal quantum number ≥ 1, averaged ionization of any atom including multielectron atoms and ions, and effective nuclear charge. Substitute Eq. (3a) into Eq. (2) and solve for Z_{eff} to give:

$$Z_{eff} = \left(\frac{32 R_m \varepsilon_0^3 \pi m_e\ \xi_i}{m_p}\right)^{½} \frac{h^2 n_i}{\mu_0 e^5}, \tag{3b}$$

Eq. (3b) needs to be substituted into famous Bohr's equation for the determination of what had been erroneously restricted to Bohr's radius for hydrogen atom. It takes intellectual "decentration" to appreciate that there could be alternative to the original Bohr's equation that has been expressed elsewhere (Udema, 2017) in a way that eliminates the need for effective nuclear charge whose magnitude is proportional to the average ionization energy in terms of the Coulomb's law or equation. Such alternative equation is:

$$a_i = \frac{n_i h}{\pi\left(\ 8\, m_e\ \xi_i\right)^{½}}, \tag{4}$$

Substitution into original Bohr's equation' gives after simplification the following:

$$a_i = n_i \left(\frac{m_p}{32\ R_m \pi^3 m_e^3 \xi_i \varepsilon_0}\right)^{½} e^3 \mu_0, \tag{5}$$

$$\frac{n_i h}{\pi} = n_i \left(\frac{m_p}{4 R_m \pi^3 m_e^2 \varepsilon_0}\right)^{½} e^3 \mu_0, \tag{6}$$

Equation (6) is likely because $8\ m_e \xi_i$ is present as denominator in Eqs (4) and (5) and, as such, it cancels out from both equations giving Eq. (6). Simplifying and solving for R_m, gives:

$$R_m = \frac{m_p e^6 \mu_0^2}{4\pi m_e^2\ h^2 \varepsilon_0}, \qquad (1.10196999\ \text{fm}) \tag{7}$$

Since $\varepsilon_0 = 1/\mu_0 c^2$, there is another version of Eq. (7) given as follows:

$$R_m = \frac{m_p e^6 \mu_0^3 c^2}{4\pi m_e^2\ h^2}, \qquad (1.101682532\ \text{fm}) \tag{8}$$

An equation undergoing continuous reevaluation is to be explored for a likely validation of mass-energy equivalence principle-based determination of subatomic particles whose masses may each be < the mass of any nucleon. The equation is:

$$R_i = \frac{4m_i h^2 \varepsilon_0}{\pi m_p^2 e^2}, \tag{9a}$$

where R_i and m_i may be the mass radius of a particle whose mass is equal to the mass of an electron and less than the mass of a nucleon and the mass of such particle respectively. In other words, Eq. (9a) is a general one covering every particle whose mass is < the mass of a nucleon. Besides, Eq. (9a) is still being reevaluated repeatedly to ensure consistency. Of course, correction may be necessary, unlike in Eq. (10) below, which no one dares to question, let alone correct—possibly nothing to correct given "its superior origin." Therefore, the mass radii of the up and down quarks are computed based on Eqs (9b) and (9c).

$$R_{q_d} = \frac{4m_{q_d} h^2 \varepsilon_0}{\pi m_p^2 e^2} \tag{9b}$$

$$R_{q_u} = \frac{4m_{q_u} h^2 \varepsilon_0}{\pi m_p^2 e^2} \tag{9c}$$

The results of computations are:

$R_{q_d} = 5.881268131\ \mathrm{exp.}(-16)\ \mathrm{m}$ (The mass radius of the down quark); $R_{q_u} = 2.467922535\ \mathrm{exp.}(-16)\ \mathrm{m}$ (The mass radius of the up quark)

The sum of the mass radii of the down and up quark is 8.349190666 exp. (− 16) m. This is very similar to the closest values reported for the charge radius of the proton. This may justify the suggestion that the radius extracted from ρ^0 photoproduction is assumed to represent the quark radius of the proton" (Wang et al., 2023). In this regard is the most studied proton charge radius obtained by colliding on the nucleus with a high-energy electron" (Wang et al., 2023). To succeed, it is believed that information about the charge distribution of the proton needs to be known (Wang et al., 2023). Hence, electromagnetic form factors are believed to provide necessary information about the distribution of electric charge (Perdrisat et al., 2007, Kohl, 2008). On the experimental side, the proton charge radius was determined as (0.8409±0.0004) fm (Workman et al., 2022) from elastic electron scattering, a relic of the kinetic approach. Other value as found in the work of Wang et al. (2023) is 0.85±0.06 fm

Even with braille, a curious mind can ask for the repudiation of the equation given as follows:

$$\langle R_m^2 \rangle = \frac{6}{G(0)} \Big|_{t=0} = \frac{12}{m_s^2}, \tag{10}$$

If m_i is the mass of the electron, the value of $\mathrm{R_i}$ is 6.278280228 exp. (− 17) m (i.e., the mass radius of the electron). To determine the hypothetical mass (m_x) of a particle less than the mass of an electron and any other particle whose mass is less than the mass of a nucleon, Eq. (1) is restated as:

$$R_i = \frac{m_e e^6}{4\pi\, \varepsilon_0^3\, \mathrm{m_x^2} h^2 c^4} \tag{11}$$

Solving for m_x and substituting $\mathrm{R_i}$ for the mass radius of the electron in Eq. (11) yields the following results: 8.911586959 exp. (−32) kg. Just as there are particles with masses larger than electrons, baryons, and other particles, there should be particles with masses smaller than electrons and nucleons. To evaluate the mass-energy equivalence principle issue further, equation (1) is reformulated as follows:

$$R_q = \frac{m_q e^6}{4\pi m_x^2 h^2\ \varepsilon_0^3 c^4} \tag{12}$$

Substitute the mass of either the up or down quark and the value of m_x^2 (=7.941638213 exp. (−63) kg^2) into Eq. (12) to yield the following:

5.881268197 exp. (−16) m (mass radius of down quark)
2.468035921 exp. (−16) m (mass radius of up quark)

The little variations may be the consequence of small discrepancies in the values of some fundamental constants, even though these values are comparable to those calculated using Eq. (9). Strong gluon-carrying forces unite or bind the up and down quarks within the bounds of the nucleon. Therefore, using modified Eq. (12) which gives Eq. (13), one can calculate the mass radii ($R_{\sum q}$) of the bound quarks using the total mass ($m_{\sum q}$) in the proton and neutron.

$$R_{\sum q} = \frac{m_{\sum q} e^6}{4\pi m_x^2 h^2\ \varepsilon_0^3 c^4} \tag{13}$$

$$R_{\sum q_n} = 1.42304589 \text{ exp.}(-15) \text{ m}$$

$$R_{\sum q_p} = 1.081761018 \text{ exp.}(-15) \text{ m}$$

where $R_{\sum q_n}$ and $R_{\sum q_p}$ are the mass radii of the sum of the masses of the quarks in the neutron and proton respectively. To examine the generalizability of Eq. (13) which is similar to Eq. (1) in form but with hypothetical mass, m_x ($<m_e$) it is fitted to all parameters including the sum of the masses of the quarks in proton and neutron. The mass radius of the single particle formed by combining of two up and one down quarks should be calculated similarly to that of the proton, whose mass is significantly larger. The same treatment is carried on the neutron. Although there isn't a known particle in existence that possesses the total mass of the quarks in a proton or a neutron, this problem also applies to the two down quarks and one up quark in a neutron. The values of $R_{\sum q_p}$ and $R_{\sum q_n}$ were less than and greater than the mass radii of the proton and neutron respectively (Table 1). This suggests a connection between particle reality and the mass-energy equivalence principle. But that notwithstanding, Eq. (13) is not suitable because it requires another hypothetical mass that can fit into Eq. (13) similar to Eq. (1); in other words, the ratio of m_e, for instance, to m_p is not the same for all elementary particles, be they in the nucleus or in the nucleons. Furthermore, substitution of the mass of electron and the sum of the masses of the quarks, for proton and neutron into Eq. (1) gives respectively, 1.035293 and 1.361918 (−17) m. These values are clearly less than the mass radius of the electron (Table 1). Those values are at odd with mass energy equivalence principle.

The cube root of the sum of the volumes (calculated using the appropriate mass radii (Table 1) of all the quarks in a proton and neutron, assuming sphericity, yields 0.6164202668 fm and 0.7102949339 fm (Table 1) for the proton and neutron, respectively. These values are similar to the following report: Additional figures by Wang et al. (2021) are 0.65±0.04 fm (corresponding to m_s value equal to 1.06±0.06 GeV) and

0.69±0.03 fm (corresponding to 0.99±0.04 GeV), whose average is 0.68±0.03 fm (from the differential cross sections of ω photoproduction near threshold at different photon energies). Kou and Chen (2023) provided the following results: Based on three models—configurational entropy, holographic QCD, and lattice—the corresponding values are 0.72, 0.68, and 0.746 fm.

Using lattice-QCD methods, Djukanovic et al. (2024) produced results for electromagnetic form factors of the proton and neutron, and with parameterization of Q^2 (the *R*-squared value obtainable from the application of quantitative structure-activity relationship (QSAR) modeling—this can be corrected if wrong) combined with extrapolation to undefined physical points, the authors showed that the electric (i.e., charge) and magnetic radii of the proton were 0.82 (14) and 0.8111 (89) fm, respectively. While I stand to be corrected, QSAR may be regarded as a computational technique by which a mathematical relationship between the structure of a chemical and other properties can be established. The charge radius is of concern, but such a report as this, 0.82 (14) fm, may, as already stated, be the mass radius of the combination of 1 up quark and 1 down quark (Wang et al., 2023). Another evidential clue as to the possibility that the so-called charge radius recorded for the proton is for the quark lies in the work of Utama et al. (2016), which contains the equation given as

$$R_{CR} = \left(\frac{3}{5}\right)^{½} r_0 A^{⅓} \tag{14}$$

where R_{CR}, r_0, and A are the root mean square charge radius of the nucleus, mass radius of the proton, and mass number of an atom; $r_0 = 1.15\ \mathrm{fm}$ (this ought to be $1.167387722\ \mathrm{fm}^{-3}$). This is computed based on the equation, derived based on the liquid drop model (LDM), given as:

$$\mathrm{r}_0 = \left(\frac{3}{4\pi\rho_0}\right)^{⅓} \tag{15}$$

It seems $\rho_0(0.15\ \mathrm{fm}^{-3})$ is a number density. Most importantly, it is suggested that $\left(\frac{3}{5}\right)^{½} \mathrm{r}_0$ may be the mass radius of the quarks in the proton. Though, outside the scope of this research, the particular (*A*) in Eq. (14) ought to be restricted to the number of protons in the nucleus, particularly those of multielectron atoms. "This is not intended to be a super intellectual dogmatic view that otherwise is unethical." Exploring, 1.15 fm gives the presumed mass radius of the charge–impacting quark complex equal to 0.8907861696 fm. This is higher than those speculated to be the mass radius of the quark complex. Value less than 0.8907861696 fm and closer in magnitude to the value such as 0.85±0.06 fm (Wang et al., 2023) is 0.8540277854 fm which is computed by exploring 1.10254513 fm.

The following calculations lend credence to the idea that the calculated mass radius of the nucleons—1.1025451 fm for the proton and 1.1040649 fm for the neutron—may be accurate. First, m_p/m_e *and* m_n/m_e *ratios* are 1836.152673 and 1838.683662 respectively. Then,

$$(1.10254513^3\ \mathrm{fm}^3/1836.152673)^{⅓} = 8.993727541 exp.(-17\ \mathrm{m})$$

$(1.1040649^3 \text{ fm}^3/1838.683662)^{⅓} = 9.001990394 exp.(-17 \text{ m})$

Such values, ~ 8.994 and 9.002 exp. (−17) m, extracted from the proton and the neutron, as above, respectively, are as a result of interstitial space expected when smaller spherical objects occupy the space of a larger object whose volume is equal to the sum of the volumes of the smaller objects. These values contrast sharply with 0.60023652 *exp*. (−18) m for an electron computed by substituting the mass of the electron for a proton. The same argument goes for the up quark, q_u, and down quark, q_d, whose mass radii based on either Eq. (1) or (2) are 2.36101347 and 5.62649479 *exp*. (−18) m, respectively. These figures are purely an outcome of dividing a one-dimensional entity by a three-dimensional entity. The mass radius is a one-dimensional entity, while the ratio of the mass of a proton to the mass of an electron, for instance, is three-dimensional.

3. CONCLUSION

Additional equations were derived for the computation of the mass radius of the proton. Similarly, the equations for the computation of the mass radii of the electron and the quarks were derived. These were mass-energy equivalence-dependent equations; particles such as electrons and quarks, which are less than the nucleon in mass, require a different mass-mass ratio than the nucleon-to-electron mass ratio. The equation ($\langle R_m^2 \rangle = \frac{12}{m_s^2}$) where m_s is equal to 1.24 GeV cannot give the root mean square radius of the proton, purported to be 0.55 fm; both sides of the equation are dimensionally inconsistent. The idea that such a charge radius might be the mass radius of the quarks appears to be supported by the fact that the sum (0.834919067 fm) of the mass radii of the up and down quarks is comparable to the charge radius reported in the literature. The mass radius of the proton equal to 0.55 fm was a fake or a fabrication. The erroneous assertion that the proton charge radius matches its limit of spatial expansion, invalidating Bohr's radius, was supported by 0.55 fm, which is nearly half of 1.10254513 fm. Other versions of the equation of the mass radius of the electron may be derived in the future.

DEDICATION

This study is devoted to the late Dr. Nnamdi Benjamin Azikiwe (**GCFR PC**), a nationalist, pan-Africanist, multilingualist (in Nigerian languages), visiting scholar at some American universities, journalist (founder of West African Pilot), businessman (founder of the former African Continental Bank), first black governor general, great sportsman, and higher education-minded personality (established the University of Nigeria, Nsukka), who warned that under no circumstances should the university be named after him. Despite being foremost in the independence struggle (a politician-founder of NCNC), in what appeared to be an externally influenced assignment to the position of ceremonial president, he was not obliged because what was important to him was Nigerian independence, a mark of selfless service to the Black race. As prime minister, he cannot be manipulated to serve external interests; otherwise, he would have built with one hand and destroyed with the other.

DISCLAIMER (ARTIFICIAL INTELLIGENCE)

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc.) and text-to-image generators have been used during the writing or editing of this manuscript.

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COMPETING INTERESTS

The sole author has declared that he has no known competing financial interests OR non-financial interests OR personal relationships that could have appeared to influence the work reported in this paper. The only challenge is the significantly less than two USD per day in monthly pension.

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