The Origin of Inertia and the Equivalence Principle

V. Nardozza*

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Abstract

In this paper we propose a possible reason why, in Newtonian gravity, gravitational and inertial mass have the same numerical value, as stated by the Equivalence principle. Our reasoning is based on two assumptions, the second of which is an ad hoc and non falsifiable assumption. According to Popper, we are therefore doing pseudo-science. However, we believe that the result is interesting and it deserves attention.

Key Words: Newtonian gravity, equivalence principle.

1 Introduction

Two objects that gravitate in the same way, have the same inertia. This is known as the equivalence principle in its weak formulation. The universal low of gravitation, where gravitational mass is defined, and the second low of dynamics, where the inertial mass is defined, are two completely separate theories and there is no obvious reason why the two mass should be equivalent. This paper is an attempt to find this reason.

The result of this paper are applicable to Newtonian gravity or at least to a small region of space-time where space can be considered flat and the local Lorentz invariance of General Relativity applies. In such a region, we may use classical Newtonian gravity, as a good approximation of the real theory, and for the theory to be more effective, we may modify it to remove the fact that Newtonian Gravity is not Lorentz invariant, that perturbations in gravitational filed propagate at infinite speed and that there is no kinetic energy density associated to fields variation despite the fact that we know from Classical Mechanics that this is not the case and fields always propagate at finite speed and have kinetic energy. This is the starting point of our reasoning which leads to the definition of two assumptions (presented in the next section). For a different approach to the same problem see also [1].

2 Assumptions

This paper is based on two assumptions.

- In empty space, perturbations in gravitational field propagate according to the scalar wave equations in the potential ϕ:

$$-\frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} + \nabla\phi = 0 \tag{1}$$

  where c is the speed of light and where the gravitational field is equal to $\mathbf{g} = -\nabla\phi$, as in standard Newtonian Gravity.

- Elementary masses are point masses and, for a point mass m, the gravitational filed is equal to:

$$\mathbf{g}(r) = \begin{cases} -\frac{Gm}{r^2}\hat{i}_r & \text{for } r > R = \Lambda m \\ \\ 0 & \text{for } r < R = \Lambda m \end{cases} \tag{2}$$

  where Λ is a new constant of nature and G is the gravitational constant.

*Electronic Engineer (MSc). Lytham, UK. mailto: vinardo@nardozza.eu

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The first assumption is simply our modification of Newtonian Gravity to take into account that perturbations in the field propagate at the speed of light. This is consistent with Newtonian gravity because, in the static case, the time derivatives in the wave equation vanish and we go back to the Laplace equation which describes Newtonian Gravity in empty space. Although we have not done it, we are convinced that the first assumption can be derived by General Relativity in the weak field approximation. This would be a circular reasoning because we want to use this equation for the Equivalence Principle which in turn is one of the main principles of General Relativity.

A direct consequence of the first assumption is that gravitational fields have both potential and kinetic energy associate with amplitude of gradient and rate of change of the gravitational potential:

$$\mathcal{E}_V = \frac{1}{2} K_V |\nabla\phi|^2 \quad ; \quad \mathcal{E}_T = \frac{1}{2} K_T \left(\frac{\partial\phi}{\partial t}\right)^2 \tag{3}$$

This is true because it is true for any field described by the wave equation. Since we know that the constant associated to the potential energy stored in the gravitational field is (see [2]):

$$K_V = \frac{1}{4\pi G} \tag{4}$$

And we know that for the scalar wave equation $c = \sqrt{K_V/K_T}$, we can easily evaluate K_T. We have:

$$\mathcal{E}_V = \frac{1}{2} \overbrace{\left(\frac{1}{4\pi G}\right)}^{K_V} |\mathbf{g}|^2 \quad ; \quad \mathcal{E}_T = \frac{1}{2} \overbrace{\left(\frac{1}{4\pi G c^2}\right)}^{K_T} (\dot{\phi})^2 \tag{5}$$

It is worth to note that this assumption makes the theory Lorentz invariant.

The second assumption is the statement that, for a point mass, fields inside the ball B_R of radius R and centred on the mass are vanishing and the potential is constant. We will see that, given the strength of the gravitational field, this radius R turns out to be several order of magnitude smaller of the radius of know particle and even of the Planck length. However, this paper is concerned with classical mechanics rather then quantum mechanics and this fact will be simply ignored.

Moreover, the second assumption is ad hoc and not falsifiable. According to Popper, we are doing pseudo-science. However, the fact that the assumption is not falsifiable is a common problem of more important theories such as String Theory or Quantum Loop Gravity. And the reason is that gravity acts at a scale which is beyond our capability of testing. We have somehow accept that or stop searching for theories of gravity. Note that, we are not comparing the content of this paper with String Theory or Quantum Loop Gravity. It would be like to compare the light of a candle with that of the Sun! However, as in our case, these theories are not falsifiable.

Said that, we are actually more concerned with the other aspect of the second assumption. It is clearly an assumption ad hoc and difficult to be accepted as true. This does not prove it automatically to be false though.

3 Inertial Mass

Let m be a mass moving in an inertial reference frame and along the x axis with law of motion given by $y = 0$, $z = 0$ and:

$$x = \sigma(t) \tag{6}$$

If the velocity of the mass $v = \dot{\sigma} \ << c$, the gravitational potential generated by the mass m can be approximated by the potential $\tilde{\phi}(\mathbf{r})$ of a mass at rest and moving along with the mass:

$$\phi(\mathbf{r}, t) = \tilde{\phi}(x - \sigma(t), y, z) \tag{7}$$

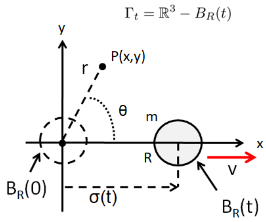

Figure 1: Moving Point Mass

When the mass is passing by, an observer at rest with respect of the reference frame will see the gravitational field at a given point in space to change with time and he will associate a kinetic density of energy to it. If Γ_t is the set $\mathbb{R}^3 - B_R(t)$, where $B_R(t)$ is the ball or radius R and centred on the mass, given Eq. (5), we can evaluate the kinetic energy stored in the whale space as:

$$\begin{aligned} E_T(t) &= \frac{1}{2}\left(\frac{1}{4\pi Gc^2}\right)\int_{\Gamma_t}[\dot{\phi}(x,y,z,t)]^2 dV \\ &= \frac{1}{2}\left(\frac{1}{4\pi Gc^2}\right)\int_{\Gamma_0}\left[\frac{\partial}{\partial t}\tilde{\phi}(x-\sigma,y,z)\right]^2 dV \\ &= \frac{1}{2}\left(\frac{1}{4\pi Gc^2}\right)\int_{\Gamma_0}\left[\frac{\partial\tilde{\phi}}{\partial x}(-\dot{\sigma})\right]^2 dV \end{aligned} \tag{8}$$

Since $\tilde{\phi} = -Gm/\sqrt{x^2+y^2+z^2}$ and $\dot{\sigma} = v$, we have:

$$\begin{aligned} E_T &= \frac{1}{2}\left(\frac{1}{4\pi Gc^2}\right)v^2\int_{\Gamma_0}\left(\frac{\partial}{\partial x}\frac{Gm}{\sqrt{x^2+y^2+z^2}}\right)^2 dV \\ &= \frac{1}{2}\left(\frac{Gm^2}{4\pi c^2}\right)v^2\int_{\Gamma_0}\left(\frac{-x}{(x^2+y^2+z^2)^{\frac{3}{2}}}\right)^2 dv \end{aligned} \tag{9}$$

Converting to Spherical Coordinates, which are more convenient for the calculations, we have:

$$\begin{aligned} E_T &= \frac{1}{2}\left(\frac{Gm^2}{4\pi c^2}\right)v^2\int_0^{2\pi}d\phi\int_0^{\pi}d\theta\int_R^{\infty}\frac{r^2\sin^2\theta}{r^6}r^2\sin\theta dr \\ &= \frac{1}{2}\left(\frac{G}{4\pi c^2}\right)\left(\frac{16\pi}{3R}\right)m^2v^2 \\ &= \frac{1}{2}\left(\frac{4G}{3c^2R}\right)m^2v^2 \end{aligned} \tag{10}$$

From the second assumption we know that $m = \Lambda R$. Substituting this expression in Eq. (10), we have:

$$E_T = \frac{1}{2}\left(\frac{4G}{3c^2R}\right)(\Lambda R)mv^2 = \frac{1}{2}\left(\frac{4G\Lambda}{3c^2}\right)mv^2 \tag{11}$$

We want the quantity in parenthesis to be equal to one. This can be done setting:

$$\Lambda = \frac{3}{4}\frac{c^2}{G} \tag{12}$$

And we finally have:

$$E_T = \frac{1}{2}mv^2 \tag{13}$$

We have shown that the kinetic energy of a point mass, given from classical mechanics, is actually the kinetic energy stored in the gravitational fields varying with time.

This eventually leads to the Equivalence Principle because the very same fields, responsible for masses to attract each other, are also responsible for the kinetic energy of a moving mass.

4 Self Energy of a Mass

Now we want to evaluate the Self Energy of a point mass that is the potential energy stored in the gravitational field and that is required to create the particle. In this case the calculation is slightly simpler. Since for a mass at rest in the origin we have that $\mathbf{g} = -(Gm/r^2)\hat{\imath}_r$, from Eq. (5), we have:

$$\begin{aligned} E_V &= \frac{1}{2}\left(\frac{1}{4\pi G}\right)\int_\Gamma\left(\frac{Gm}{r^2}\right)^2 dV \\ &= \frac{1}{2}\frac{Gm^2}{4\pi}\int_\Gamma \frac{1}{r^4}dV \\ &= \frac{1}{2}\frac{Gm^2}{4\pi}\int_0^{2\pi}d\theta\int_0^\pi \sin(\phi)d\phi\int_R^\infty \frac{1}{r^4}r^2 d\rho \\ &= \frac{1}{2}\frac{Gm^2}{R} \end{aligned} \tag{14}$$

Using again $m = \Lambda R$, from the second principle, we have:

$$E_V = \frac{1}{2}\frac{Gm(\Lambda R)}{R} = \frac{mG\Lambda}{2} \tag{15}$$

From Eq. (12) we have eventually:

$$E_V = \frac{1}{2}G\Lambda m^2 = \frac{3}{8}mc^2 \tag{16}$$

Which is the contribution of the self energy to the relativistic energy of the particle.

5 The Planck Mass

The radius R is proportional to the mass with a factor $1/\Lambda$:

$$\frac{1}{\Lambda} = \frac{G}{c^2} = 7.42\times 10^{-28}\left[\frac{m}{kg}\right] = 1.12\times 10^{-57}\left[\frac{m}{MeV}\right] \tag{17}$$

which gives a radius R much smaller of the Planck length for the know particles. For a Planck mass m_P the value of R is equal to:

$$R = \overbrace{\frac{4}{3}\frac{G}{c^2}}^{1/\Lambda}\times\overbrace{\sqrt{\frac{\hbar c}{G}}}^{m_P} = \frac{4}{3}\overbrace{\sqrt{\frac{\hbar G}{c^3}}}^{l_p} \tag{18}$$

where l_p is the Planck length.

References

[1] V. Nardozza. *Gravitational and Inertial Mass.* - https://vixra.org/abs/2406.0083 (2024).

[2] V. Nardozza. *Energy Stored in the Gravitational Field.* https://vixra.org/abs/1905.0515 (2019).