Precise Values of the Lifetime of Positronium in Vacuum

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Abstract

In this paper, we calculate the lifetime of ortho-positronium (o-Ps) in vacuum with Euler formula for Basil problem and our previous formulas of the fine-structure constant and the atomic unit of time (t_{au}). Our calculated value of the lifetime of ortho-positronium in vacuum is 142.041358500909 ns in comparison to the most accurate measured value which was 142.043(14) ns and the latest QED calculated value which was 142.04606(20) ns. In addition, with the similar method we also calculate out the lifetime of para-positronium (p-Ps) in vacuum which is 0.1251627502491 ns compared to the QED calculated values 0.125162617(3) ns and 0.125162432(3) ns.

1. Introduction

Positronium (Ps) is the bound state of an electron and a positron (antielectron) and is somewhat like a hydrogen atom (**Fig. 1**). Since positronium contains no protons or neutrons, nuclear interactions can be neglected, and it can be accurately described solely by quantum electrodynamics (QED). Hence, positronium is an ideal system to test QED and to look for deviations that could indicate new physics beyond the standard model.

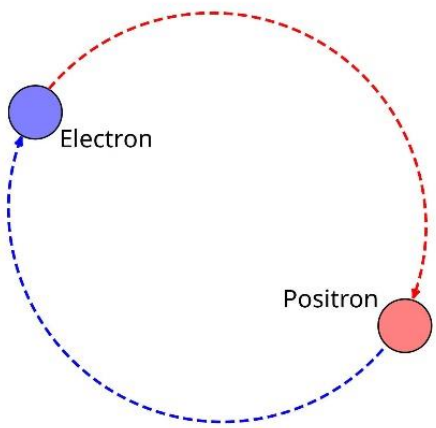

Fig. 1 Positronium

Positronium system is unstable, the electron and positron annihilate each other to predominantly produce two or three gamma rays, depending on the relative spin states of the two particles. The lowest energy orbital state of positronium is 1S, and like with hydrogen, it has a hyperfine structure arising from the relative orientations of the spins of the electron and the positron. The singlet state ($^1S_0$) of Ps with antiparallel spins of the electron and the positron, which is known as para-positronium (o-Ps), decays preferentially into two photons with lifetime about 0.125 ns. The triplet state ($^3S_1$) of Ps with parallel spins of the electron and the positron, which is known as ortho-positronium (o-Ps), decays into three photons with lifetime about 142 ns. The decay rate (or lifetime) can be calculated with QED and measured experimentally.

For ortho-positronium, the QED theoretically calculated values of its lifetime have been improved to yield the decay rate 7.039970(10) μs^{-1} or the lifetime 142.04606(20) ns [1]. The experimentally measured decay rate or the lifetime once had a long history of inconsistency with the theoretical predictions, the so-called o-Ps lifetime puzzle. The most accurate experiments of the decay rate which resulted in 7.0401(7) μs^{-1} or 142.043(14) ns was in agreement with the theoretical predictions [2]. However, more precise experimental measurements would regenerate new inconsistency with the theoretical predictions.

For para-positronium, the theoretically calculated values of its lifetime yielded the decay rate 7989.6060(2) μs^{-1} or the lifetime 0.125162617(3) ns [3] and the decay rate 7989.6178(2) μs^{-1} or the lifetime 0.125162432(3) ns [4].

In this paper, we try to calculate the lifetime of ortho-positronium (o-Ps) to a precision with 15 digits and the lifetime of para-positronium (p-Ps) to a precision with 13 digits by a novel method.

2. Formulas of the Fine-structure Constant and the Speed of Light in Atomic Units Based on 2π-e Formula and the 112th element Cn*

In our previous papers, we gave the formulas of the fine-structure constant and the speed of light in atomic units based on 2π-e formula and the natural end of the elements, i.e., the 112th element Cn* [5-17]. They are listed as follows.

$2\pi - e$ formula:

$$2\pi = \left(\frac{e}{e^{\gamma_c}}\right)^2 = e^2 \frac{e^2}{(\frac{2}{1})^3} \frac{e^2}{(\frac{3}{2})^5} \frac{e^2}{(\frac{4}{3})^7} \cdots$$

$$(2\pi)_{Chen-k} = \left(\frac{e}{e^{\gamma_{c-k}}}\right)^2 = e^2 \frac{e^2}{(\frac{2}{1})^3} \frac{e^2}{(\frac{3}{2})^5} \cdots \frac{e^2}{(\frac{k+1}{k})^{2k+1}}$$

Formulas of the fine-structure constant:

$$\alpha_1 = \frac{\lambda_e}{2\pi a_0} = \frac{36}{7(2\pi)_{Chen-112}} \frac{1}{112 + \frac{1}{75^2}} = 1/137.0359990374153791885$$

Relationships with nuclides:

$^{82,83,84}_{36}Kr_{46,47,48}$ $^{100}_{44}Ru_{56}$ $^{112}_{48}Cd_{64}$ $^{136,137,138}_{56}Ba_{80,81,82}$ $^{185,187}_{75}\mathrm{Re}_{110,112}$ $^{188}_{76}Os_{112}$ $^{209}_{83}Bi^*_{126}$

$^{209}_{84}Po^*_{125}$ $^{252}_{99}Es^*_{153}$ $^{285}_{112}Cn^*_{173}$ $^{312}_{125}Ch^{ie}_{187}$ $^{2\cdot157}_{126}Ch^{ie}_{188}$ $^{2\cdot173}_{137}Fy^{ie}_{209}$

$$\alpha_2 = \frac{2\pi r_e}{\lambda_e} = \frac{13(2\pi)_{Chen-278}}{100} \frac{1}{112 - \frac{1}{64\cdot3\cdot29}} = 1/137.0359991118729627581$$

Relationships with nuclides:

$^{56}_{26}Fe_{30}$ $^{63,65}_{29}Cu_{34,36}$ $^{100}_{44}Ru_{56}$ $^{112}_{48}Cd_{64}$ $^{140,142}_{58}Ce_{82,84}$ $^{136,137,138}_{56}Ba_{80,81,82}$ $^{157}_{64}Gd_{93}$ $^{169}_{69}Tm_{100}$

$^{188}_{76}Os_{112}$ $^{223,224}_{87}Fr^*_{136,137}$ $^{257}_{100}Fm^*_{157}$ $^{278}_{109}Mt^*_{169}$ $^{285}_{112}Cn^*_{173}$ $^{2\cdot157}_{126}Ch^{ie}_{188}$ $^{2\cdot173}_{137}Fy^{ie}_{209}$ $^{426}_{169}Ch^{ie}_{257}$

Formulas of the speed of light in atomic units:

$$c_{au} = \frac{4\pi\varepsilon_0\hbar c}{e^2} = \frac{c}{v_e} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_1\alpha_2}}$$

$$= \sqrt{112\left(168 - \frac{1}{3} + \frac{1}{4\cdot141} - \frac{1}{14\cdot112(2\cdot173+1)+13+\frac{7}{72+1/50}}\right)}$$

$$= 2\sqrt{56\left(83 + \frac{157}{188} - \left(\frac{1}{8\cdot141} + \frac{1}{56^2(2\cdot173+1)+26+\frac{7}{36+1/100}}\right)\right)}$$

$$= 137.0359990746441709683$$

$c_{au} = \frac{1}{\sqrt{\alpha_1\alpha_2}}$ is consitent with Maxwell formula of the speed of light $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

Relationships with nuclides:

$^{56}_{26}Fe_{30}$ $^{83,84}_{36}Kr_{47,48}$ $^{100}_{44}Ru_{56}$ $^{107,109}_{47}Ag_{60,62}$ $^{112}_{48}Cd_{64}$ $^{118,119,120}_{50}Sn_{68,69,70}$ $^{136,137,138}_{56}Ba_{80,81,82}$

$^{140,142}_{58}Ce_{82,84}$ $^{157}_{64}Gd_{93}$ $^{166,167,168}_{68}Er_{98,99,100}$ $^{169}_{69}Tm_{100}$ $^{173}_{70}Yb_{103}$ $^{188}_{76}Os_{112}$ $^{200}_{80}Hg_{120}$ $^{209}_{83}Bi^*_{126}$

$^{209}_{84}Po^*_{125}$ $^{223,224}_{87}Fr^*_{136,137}$ $^{257}_{100}Fm^*_{157}$ $^{264,265}_{103}Lr^*_{161,162}$ $^{2\cdot137}_{107}Bh^*_{167}$ $^{276}_{108}Hs^*_{168}$ $^{285}_{112}Cn^*_{173}$ $^{300}_{120}Ch^{ie}_{180}$

$^{2\cdot157}_{126}Ch^{ie}_{188}$ $^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ $^{355,357}_{141}Ch^{ie}_{214,216}$ $^{400}_{157}Ch^{ie}_{243}$ $^{420}_{168}Ch^{ie}_{252}$ $^{426}_{169}Ch^{ie}_{257}$ $^{437,438}_{173}Ch^{ie}_{264,265}$

3. Calculations of the Lifetime of Ortho-positronium (o-Ps)

Our method to calculate the lifetime of ortho-positronium is novel and differs from the perturbative calculation with QED. Employing Euler formula for Basil problem and our previous formulas of the fine-structure constant and the atomic unit of time (t_{au}), our calculations are as follows. It is worth noting that the atomic unit of time t_{au}=1.42888432658653278×10^{-17} s was determined in our previous paper [18-22].

Calculation of the lifetime of ortho-positronium (o-Ps)

Calculation with QED [23]: $\tau_{o-Ps} = \frac{1}{2}\frac{9\pi}{\pi^2-9}\frac{\hbar}{m_e c^2 \alpha^6}\frac{1}{1+\Delta}$

Calculted value by QED: 142.04606(20) ns

Measured value: 142.043(14) ns

Euler formula for Basel problem:

$$\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots=\frac{\pi^2}{6} \quad \text{or} \quad \sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6} \quad \text{or} \quad \sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{3}{2}\left(\frac{\pi}{3}\right)^2$$

Definition: $\sum_{n=1}^{k}\frac{1}{n^2}=\frac{3}{2}\left(\frac{\pi_{Euler-k}}{3}\right)^2$

Improved calculation by this work:

$$\tau_{o-Ps}=\frac{3}{2}\frac{\frac{\pi_{Euler-278}}{3}}{\left(\frac{\pi_{Euler-278}}{3}\right)^2-1}\frac{\hbar}{m_e c^2(\alpha_1\alpha_2)\alpha_2^4}$$

In atomic units (au): $\hbar_{au}=1,\ m_{e/au}=1,\ c_{au}^2=\frac{1}{\alpha_1\alpha_2},\ \tau_{o-Ps/au}=\frac{\tau_{o-Ps}}{t_{au}}$

t_{au} is the atomic unit of time, $t_{au}=2.41888432658653278\times10^{-17}s$

So: $\tau_{o-Ps/au}=\frac{3}{2}\frac{\frac{\pi_{Euler-278}}{3}}{\left(\frac{\pi_{Euler-278}}{3}\right)^2-1}\frac{1}{\alpha_2^4}$

In which:

$$\pi_{Euler-278}=3\sqrt{\frac{2}{3}\times1.64134341579444}=3.13816196120701$$

$$\alpha_2=\frac{13(2\pi)_{Chen-278}}{100}\frac{1}{112-\frac{1}{64\cdot3\cdot29}}=\frac{1}{137.035999111873}$$

So: $\tau_{o-Ps/au}=5872184830.82051$

$\tau_{o-Ps}=\tau_{o-Ps/au}t_{au}$

$=5872184830.82051\times2.41888432658653278\times10^{-17}$

$=142.041358500909$ ns

Decay rate: $\Gamma_{o-Ps} = \frac{1}{\tau_{o-Ps}} = 7.04020301237544\ \mu s^{-1}$

Note:

$$\tau_{o-Ps/au} = 137(2\cdot3\cdot137+1)(16\cdot3\cdot5\cdot7\cdot31+1) - \frac{1}{3} + \frac{2}{13}$$

$$= 137(8\cdot103-1)(16\cdot3\cdot5\cdot7\cdot31+1)-1) - \frac{7}{3\cdot13}$$

$$= 137(8\cdot103-1)(2(2\cdot29(2\cdot9\cdot25-1)-1)-1) - \frac{7}{3\cdot13}$$

$$= 137(8\cdot103-1)(2(2\cdot29(64\cdot7+1)-1)-1) - \frac{7}{3\cdot13}$$

$$= 5872184830.82051$$

Relationships with nuclides:

$^{31}_{15}P_{16}$ $^{28,29}_{14}Si_{14,15}$ $^{56,57}_{26}Fe_{30,31}$ $^{63,65}_{29}Cu_{34,36}$ $^{69,71}_{31}Ga_{38,40}$ $^{79,81}_{35}Br_{44,46}$ $^{89}_{39}Y_{50}$

$^{83,84}_{36}Kr_{47,48}$ $^{100}_{44}Ru_{56}$ $^{103}_{45}Rh_{58}$ $^{107,109}_{47}Ag_{60,62}$ $^{112}_{48}Cd_{64}$ $^{118,119,120}_{50}Sn_{68,69,70}$

$^{136,137}_{56}Ba_{80,81}$ $^{140,142}_{58}Ce_{82,84}$ $^{157}_{64}Gd_{93}$ $^{167,168}_{68}Er_{99,100}$ $^{169}_{69}Tm_{100}$ $^{173}_{70}Yb_{103}$

$^{185,187}_{75}\mathrm{Re}_{110,112}$ $^{188}_{76}Os_{112}$ $^{200}_{80}Hg_{120}$ $^{209}_{83}Bi^{*}_{126}$ $^{209}_{84}Po^{*}_{125}$ $^{223,224}_{87}Fr^{*}_{136,137}$

$^{231,232}_{91}Pa^{*}_{140,141}$ $^{264,265}_{103}Lr^{*}_{161,162}$ $^{257}_{100}Fm^{*}_{157}$ $^{2\cdot137}_{107}Bh^{*}_{167}$ $^{276}_{108}Hs^{*}_{168}$ $^{278}_{109}Mt^{*}_{169}$

$^{285}_{112}Cn^{*}_{173}$ $^{200}_{120}Ch^{ie}_{180}$ $^{314}_{126}Ch^{ie}_{188}$ $^{2\cdot173}_{137}Fy^{ie}_{209}$ $^{426}_{169}Ch^{ie}_{257}$ $^{437,438}_{173}Ch^{ie}_{264,265}$

In the above calculations, we suppose that the composition factors of the lifetime of ortho-positronium in atomic units ($\tau_{o\text{-}Ps/au}$) should relate to nuclides. And there are the characteristic factors 137 and 103 appearing, which indicate our calculation should be correct and precise. The reason for this relationship is that ortho-positronium should be regarded as a special nuclide among all nuclides, like a mount among mountains, and a mount is like mountains because of their fractal forms.

4. Calculations of the Lifetime of Para-positronium

With the similar method, we calculate the lifetime of para-positronium as follows.

Calculation of the lifetime of para-positronium (p-Ps)

Calculation with QED [23]: $\tau_{o-Ps} = \frac{2\hbar}{m_e c^2 \alpha^5} \frac{1}{1+\Delta}$

Calculted values by QED: 0.125162617(3) ns or 0.125162432(3) ns

$$\tau_{p-Ps/au} = \frac{2}{\alpha_2^{\ 3}} \times (1 + \frac{1}{11\cdot17 - \frac{11}{14}}) = 2\times137.035999111873^3 \times (1 + \frac{1}{11\cdot17 - \frac{11}{14}})$$

$$= 5174399.985705$$

$$\tau_{p-Ps} = \tau_{p-Ps/au} t_{au} = 5174399.985705 \times 2.41888432658652378 \times 10^{-17}$$
$$= 0.1251627502491 \text{ ns}$$

Decay rate: $\Gamma_{p-Ps} = \dfrac{1}{\tau_{p-Ps}} = 7989.597528097\ \mu s^{-1}$

Note: $\tau_{p-Ps} = 128 \cdot 3 \cdot 25 \cdot 49 \cdot 11 - \dfrac{1}{70 - \dfrac{1}{22}} = 5174399.985705$

Relationships with nuclides:

$^{23}_{11}Na_{12}$ $^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ $^{56}_{26}Fe_{30}$ $^{64,66}_{30}Zn_{34,36}$ $^{79,81}_{35}Br_{44,46}$ $^{88}_{38}Sr_{50}$

$^{100}_{44}Ru_{56}$ $^{112}_{48}Cd_{64}$ $^{113,116}_{49}In_{48,46}$ $^{118,119,120}_{50}Sn_{68,69,70}$ $^{136,137}_{56}Ba_{80,81}$ $^{157}_{64}Gd_{93}$

$^{168}_{68}Er_{100}$ $^{169}_{69}Tm_{100}$ $^{173}_{70}Yb_{103}$ $^{185,187}_{75}\mathrm{Re}_{110,112}$ $^{200}_{80}Hg_{120}$ $^{209}_{83}Bi^{*}_{126}$ $^{209}_{84}Po^{*}_{125}$

$^{223,224}_{87}Fr^{*}_{136,137}$ $^{257}_{100}Fm^{*}_{157}$ $^{285}_{112}Cn^{*}_{173}$ $^{200}_{120}Ch^{ie}_{180}$ $^{2\cdot173}_{137}Fy^{ie}_{209}$ $^{400}_{157}Ch^{ie}_{253}$

5. Discussion and Conclusion

In the above formulas of the fine-structure constant and the lifetime of ortho-positronium in atomic units ($\tau_{o\text{-}Ps/au}$) there are some characteristic and repeatedly factors such as 112, 137 and 278 which are shown as follows.

$$\alpha_1 = \frac{36}{7(2\pi)_{Chen-112}} \frac{1}{112 + \dfrac{1}{75^2}} = 1/137.035999037415379188 5$$

$$\alpha_2 = \frac{13(2\pi)_{Chen-278}}{100} \frac{1}{112 - \dfrac{1}{64 \cdot 3 \cdot 29}} = 1/137.0359991118729627581$$

$$c_{au} = \sqrt{112(168 - \frac{1}{3} + \frac{1}{4 \cdot 141} - \frac{1}{14 \cdot 112(2 \cdot 173 + 1) + 13 + \dfrac{7}{72 + 1/50}})}$$
$$= 137.035999074644170968 3$$

$$\tau_{o-Ps/au} = \frac{3}{2} \frac{\dfrac{\pi_{Euler-278}}{3}}{(\dfrac{\pi_{Euler-278}}{3})^2 - 1} \frac{1}{\alpha_2^4} = 5872184830.82051$$

$$\tau_{o-Ps/au} = 137(2 \cdot 3 \cdot 137 + 1)(112 \cdot 3 \cdot 5 \cdot 31 + 1) - \frac{1}{3} + \frac{2}{13}$$
$$= 137(8 \cdot 103 - 1)(2(2 \cdot 29(4 \cdot 112 + 1) - 1) - 1) - \frac{7}{3 \cdot 13}$$
$$= 5872184830.82051$$

And in our previous formulas of the anomalous magnetic moments of electron, muon and tauon [24, 25], there are some characteristic factors such as 109 and 278 which are shown as follows.

$$a_e = \frac{\alpha_2\gamma_1}{(2\pi)_{Chen-109}} = \frac{13(2\pi)_{Chen-278}}{100(2\pi)_{Chen-109}} \frac{1+\frac{1}{3\cdot 47\cdot 73\cdot 137}}{112-\frac{1}{64\cdot 3\cdot 29}}$$

$$= 0.00115965218058117$$

$$a_\mu = \frac{\alpha_2\gamma_1\gamma_2}{(2\pi)_{Chen-109}} = \frac{13(2\pi)_{Chen-278}}{100(2\pi)_{Chen-109}} \frac{(1+\frac{1}{3\cdot 47\cdot 73\cdot 137})(1+\frac{1}{5\cdot 37})}{112-\frac{1}{64\cdot 3\cdot 29}}$$

$$= 0.00116592057075$$

$$a_\tau = \frac{\alpha_2\gamma_1\gamma_2\gamma_3}{(2\pi)_{Chen-109}} = \frac{13(2\pi)_{Chen-278}}{100(2\pi)_{Chen-109}} \frac{(1+\frac{1}{3\cdot 47\cdot 73\cdot 137})(1+\frac{1}{5\cdot 37})(1+\frac{1}{103})}{112-\frac{1}{64\cdot 3\cdot 29}}$$

$$= 0.00117724018794$$

These miraculous coincidences strongly indicate the following relationships.

$$(2\pi)_{Chen-112} \Leftrightarrow {}^{285}_{112}Cn^*_{173}$$

$$(2\pi)_{Chen-109} + \begin{cases}(2\pi)_{Chen-278} \\ \pi_{Euler-278}\end{cases} \Leftrightarrow {}^{278}_{109}Mt^*_{169}$$

These relationships can be illustrated with the following analogical graphics (**Fig. 2**). From Chengdu to Shanghai, there are two ways, by train and by airplane, the former corresponds to the all elements and nuclides, the latter corresponds to our 2π-e formula and Euler formula for Basel problem. Chengdu and Shanghai have two airports respectively, which correspond to the elements/nuclides H, He and Mt^*, Cn^* respectively. This also explains the formulas could only correspond to the natural ends of the elements, i.e., Mt^*, Cn^* [26, 27].

$2\pi - e$ formula:

$$2\pi = (\frac{e}{e^{\gamma_c}})^2 = e^2 \frac{e^2}{(\frac{2}{1})^3} \frac{e^2}{(\frac{3}{2})^5} \frac{e^2}{(\frac{4}{3})^7} \cdots$$

$$(2\pi)_{Chen-k} = (\frac{e}{e^{\gamma_{c-k}}})^2 = e^2 \frac{e^2}{(\frac{2}{1})^3} \frac{e^2}{(\frac{3}{2})^5} \cdots \frac{e^2}{(\frac{k+1}{k})^{2k+1}}$$

Euler formula for Basel problem:

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6} \text{ or } \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} \text{ or } \sum_{n=1}^{k} \frac{1}{n^2} = \frac{3}{2}(\frac{\pi_{Euler-k}}{3})^2$$

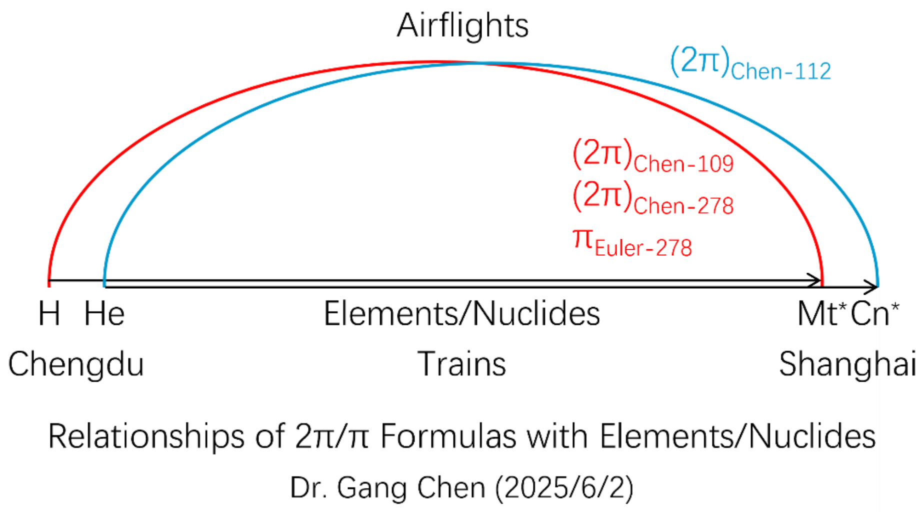

Fig. 2

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