The Periodic variation of Baryon masses as a function of their Magnetic Moments: Quantum interference in the femtometer scale of Hadrons .

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Abstract: The objective of this paper is to give full consideration to a (never divulged) result which comes straight from *tabulated* data for all baryons of the octet and decuplet when these data are duly analyzed theoretically, namely: *The masses of spin ½ baryons, in proton mass-units, are a simple periodic function of their magnetic moments, in nuclear magneton-units.* As discussed here, this can be attributed to quantum interference of closed currents inside the Particles, as observed in conventional superconducting rings confining magnetic flux.

keywords: Quantum interference, Regularization techniques, Casimir effect.

1) **Introduction, and discussion of the theory in previous work**.

In recent years the author has carried out investigations on the possible application of electrodynamics in the theoretical analysis of Particles properties, like in the determination of their masses, and their relation to magnetic flux confinement inside the Particle. This led to the association of the origin of Particles to a topological transition(to a current loop-state) starting from plane waves in an environment at 10^{13} K (possibly Big Bang conditions) [1,2]. In view of the amount of data available, the analysis has been concentrated on the baryons of the octet and decuplet.

Quantitative agreement between models and data has been achieved adopting a relativistic circular ring of currents model for the baryons[1], in which is included the concept (introduced by Asim Barut in the 1970s [3]) that a proton-state(with mass m_p)surrounded by a cloud of mesons, neutrinos, and electrons is taken as a fundamental element present in all baryons. It is inevitable to notice the potential similarity of this problem to the one of determining the dynamic properties of currents flowing around a conventional superconductor ring, which is confirmed by this investigation.

The ring model is explicitly developed to represent spin ½ particles, since elements of such quasi-proton can circulate in two opposite directions in the ring plane. Making reference to details available in previous publications[1], a Dirac equation is written for the ring-shaped distribution of charge that models a Particle. The motion of momentum around the loop (of perimeter L) is the result of the propagation of local elemental displacements, like in a vibrating string. Bohr-Sommerfeld quantum conditions impose a continuity of phase around the ring that introduces an infinite number of vibrating modes (indexed as k), given by the momenta $p_k = 2\pi\hbar k/L$. The fundamental state is obtained by summing up over these modes. This sum diverges, but the converging part of the solution can be isolated by applying a Regularization ("Reg" below) procedure that extracts the diverging parts, which are associated to the surrounding infinite environment[1]. The Dirac equation and its solution include a magnetic gauge field A , which introduces an amount of magnetic flux ϕ arrested inside the ring. The flux $\phi = A\,L$ is defined in numbers n of magnetic flux quanta $\phi_0 = hc/e$. The solutions of the Dirac hamiltonian provide energies which are associated with the rest energies Mc^2 of the baryons, through the expression(s $\rightarrow$ -1)[1]:

$$M\,c^2 = U_0 + \mathrm{Reg} \sum_k c\{(p_k + e\phi/Lc)^2 + m_p^2c^2\}^{-S/2} \qquad (1)$$

for each baryon of mass M. Here U_0 is the parent state energy of the environment the loops originate from, and is obtained by comparing theory with the mass data for the baryons in the Tables. Equation (1) can be rewritten in dimensionless form. Here the dimensionless parameters adopted are $m' = m_p/m_0$, where $m_0 = 2\pi\hbar/cL$, and $u_0 = U_0/m_p c^2$:

$$M(n)/m_p = u_0 + (1/m')\text{Reg} \sum_k \{(k+n)^2 + m'^2\}^{-S/2} \qquad (2)$$

The second term on the right of (2) corresponds to an internal correlation energy that turns the ring states energetically favorable as compared to the parent state, so that a condensation into ring form takes place. This would be the picture of ring-like Particles formation from a Parent state. The Regularization of the second term in (2) results in(s→ -1, and k + are the positive integers):

$$\text{Reg} \sum_k \{(k+n)^2 + m'^2\}^{-S/2} =$$

$$= \frac{2\sqrt{\pi}}{\Gamma\left(\frac{1-s}{2}\right)} \left(\frac{\Gamma\left(-\frac{s}{2}\right)}{2m'^{\,-s}} + 2\pi^{-\frac{s}{2}} \sum_{k+} \left(\frac{k}{m'}\right)^{\frac{-s}{2}} K_{\frac{s}{2}}(2\pi m' k) \cos(2\pi k n + \delta) \right) \qquad (3)$$

which must be multiplied by $(\pi^{\frac{2s-1}{2}} / \Gamma\left(\frac{s}{2}\right))\, \Gamma\left(\frac{1-s}{2}\right)$ and inserted in (2) (see ref. 1 for details of the Regularization method) to give the masses. One immediately realizes that the $M(n)$ are periodic functions of n, where we have added a phase δ to allow slight changes in topology of the rings as compared to perfect circles. It is predicted from (3) that the mass of spin ½ baryons should be a cosine function of n, provided no other effects modify this parameter. It must be pointed out that the Modified Bessel function K in (3) decays very fast with the argument and thus the only term of the sum that actually contributes is k= 1.

2) The inclusion of J=3/2 particles data in the analysis.

The data adopted in this analysis comes from ref[4] and is presented in Tables 1 and 2 (in the end of the paper).

Only the octet particles are spin J= ½ particles. One needs to obtain values of mass for the decuplet (J=3/2) particles in a spin ½ state, so that they can be compared to the octet data.

Firstly, in our previous publications the parameter n is defined for spin ½ particles through the equation[1]

$$n = (2c^2\alpha/e^3)\, \mu\, m. \qquad (4)$$

which completely defines n from tabulated data. In particular, one notices that m on the right side is proportional to the ratio n/μ (μ is the magnetic moment, and α is the fine-structure constant), which contains the magnetic part of the mass in the form of a ratio. In principle, n and μ should be proportional to each other, since in the simplest cases flux inside a loop of radius R is given by $2\pi\mu/R$. The radius should not change with spin, and thus magnetic effects on mass given by the n/μ ratio should not change for excited spin states like J= 3/2(that is, depending on spin, n and μ might have different values but keeping their proportionality ratio). Expression (4) is consitent with the expressions of mass obtained by Corben[5] (J proportional to M^2), which depend on spin but not on magnetic properties of hadrons. Furthermore, spin is a global [6,7] property of Mass. It is consistent with Corben's work that the increase in mass of the decuplet particles as compared with the octet particles can be attributed to the greater spin and not to the magnetic contribution in (4). To obtain suitable values for n in the decuplet from eq.(4) and spin ½ we then proceeded as follows. Firstly we recognize from inspection of the tabulated magnetic moments that each of the Δ(including the nucleons), Σ, and Ξ groups of particles has in the decuplet 3/2 spin state a particle with a magnetic moment which is very close to the one of a corresponding particle in the octet ½ spin state. This indicates that it is sensible to assume that an hypothetical ½ spin state decuplet baryon will have about the same mass as the corresponding octet baryon particle. Therefore, from this reasoning one concludes that *n* is obtained for this state by inserting in (4) the octet particle mass and the respective tabulated spin 3/2 magnetic moment since the ratio n/μ should not change with spin. Mass for the hypothetical spin ½ states of the decuplet baryon Ω are obtained by subtracting from the spin 3/2 mass the overall averaged excess of all decuplet particles over the octet masses, which is 244 MeV/c^2 . These masses (in Table 2) are called here *transformed masses* m_t . Then, values of *n* are obtained from eq (4) adopting the transformed masses to simulate spin ½ for decuplet particles, alongside the available values of μ for the decuplet,.

The Tables show that the obtained ratios n/μ are close to unit in dimensionless units for most Particles, which is the expected result for a circulating ring of current. However, the Tables also show there are "jumps" in the values of *n* towards integer values, something typical of the presence of weak-links in superconducting closed currents paths(see, for instance, [8]). Such weak-links are not considered in the perfect-ring circuit model calculations adopted, which resulted in the simple cosinusoidal expressions (1)-(3). For this reason we adopt the dimensionless μ in the analysis below, which might be considered the unperturbed version of the parameter n (cf. eq. 6 of ref.[6], taking $s = 1$), and adequate for comparison with the simple predictions of eq. (1)-(3). By proceeding this way the following plot reveals a relevant new result.

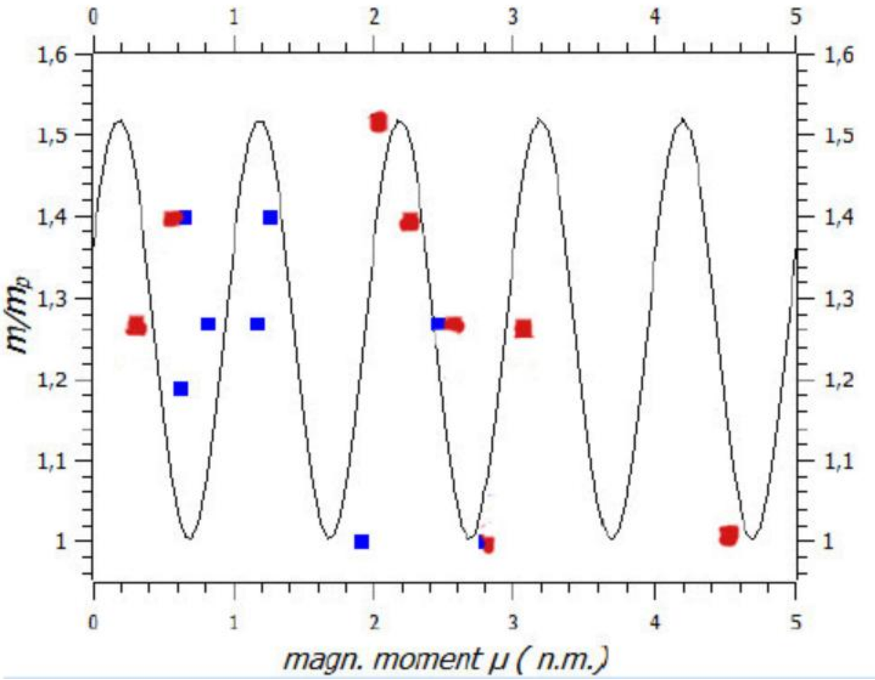

Figure 1: Plot of the ratio m/m_p from Tables 1(blue points) and 2 (red points, m_t adopted for Table 2 data) using the magnetic moments in the Tables in place of n in eq. (2-3). The phase-displced cosine curve is the plot of eqs.(2)- (3) with μ in place of n (cf. ref. [6])**. One notices the good fit obtained for ten points above μ= 1.**

3) **Analysis and Conclusion**.

The curve in Figure 1 clearly displays the periodic behavior of eqs. (2)-(3) in the fit of the data, better for magnetic moments μ between 1 and 5 magnetons(taken as representing numbers of flux quanta unaffected by tunneling through weak-links in the current path). The dimensionless parameters are $m' = m_p/m_0$, where $m_0 = 2\pi\hbar/cL$, and $u_0 = U_0/m_pc^{2.}$. The fit gives $m' = 0.36$ and $u_0 = 2.5$, which slightly correct numbers obtained in previous publications(a ½ factor is included here in the k=0 additive tern in (3) as compared to the expression in ref. [1]). This u_0 parameter corresponds to U_0= 2340 MeV, which is consistent with the peak position in the plot of cosmic rays protons flux analyzed in previous work[2]. Below μ= 1 there is a scattering of data and the analysis loses precision if a single set of adjusting parameters is adopted. Above μ= 1 magneton a simple phase-displaced cosine reasonably fits the data, as discussed in the previous sections. It might be expected however that the phase δ in eq. (3) should not be exactly the same for all baryons, and the fit should not be as good as the one obtained. The good result reinforces Barut's proposal that the differences between these particles are not related to differences between inner constituents, quite independently adding up their individual effects as point sources. Following Barut, an individual proton state and rest mass dominates, with mass modulated by interference of circulating clouds of charge, which manifest in terms of the magnetic flux trapped in the structure.

The periodic dependence of energy of currents (here determining the rest masses of baryons) as a function of trapped flux in a ring or tube-shaped superconductor was initially analyzed theoretically by Byers and Yang[9](cf. their Theorem 1), as a consequence of quantum interference of currents circulating around the trapped flux. The present work shows that existing tabulated data can be organized to display a similar periodic effect happening inside baryons. Both treatments have in common the imposition of gauge invariance and continuity of wavefunctions around the magnetic flux confined inside the structure.

In conclusion, this paper divulges the following (revealing) results, immediately obtainable from tabulated data: *the masses of spin ½ baryons, in proton mass-units, are a simple periodic function of their magnetic moments, in nuclear magneton-units*. This is in accordance with predictions by Byers and Yang[9] for the periodic behavior of the energy of currents in multiply-connected superconductors trapping magnetic flux.

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Table 1: Baryon octet data(magnetic moments μ from ref. [4]). According to eq.(4) (gaussian units): n= 1.16x10^{47} μm.

	abs μ (n.m.)	μ(erg/G) x 10^{23}	m(Mev/c^2)	m(g) x 10^{24}	n eq.(4)
p	2.79	1.41	939	1.67	2.73
n	1.91	0.965	939	1.67	1.9
Σ^+	2.46	1.24	1189	2.12	3
Σ^0	0.82(theor.)	0.414	1192	2.12	1
Σ^-	1.16	0.586	1197	2.12	1.5
Ξ^0	1.25	0.631	1314	2.34	1.7
Ξ^-	0.65	0.328	1321	2.34	0.9
Λ	0.61	0.308	1116	1.98	0.7

Table 2: Baryon decuplet data **in a spin ½ state** **(magn. moments μ from ref. [4]). According to eq.(4): $n = 1.16\text{x}10^{47}\ \mu\, m_t$ (gaussian units). See text for the definition of m_t.**

	abs μ (n.m.)	μ(erg/G) x 10^{23}	m_t(Mev/c^2) (**see text**)	m_t(g) x 10^{24}	*n*
Δ^{++}	4.52	2.28	939	1.67	4.4
Δ^{+}, Δ^{-}	2.81,2.81	1.42	939	1.67	2.8 , 2.8
Σ^{+}	3.09	1.56	1189	2.12	3.8
Σ^{0}	0.27	0.136	1192	2.12	0.33
Σ^{-}	2.54	1.28	1197	2.12	3,1
Ξ^{0}	0.55	0.28	1314	2.34	0.76
Ξ^{-}	2.25	1.14	1321	2.34	3,1
Ω^{-}	2.02	1.02	1428	2.54	3